Gravity in Flatland
Black holes in lower dimensions

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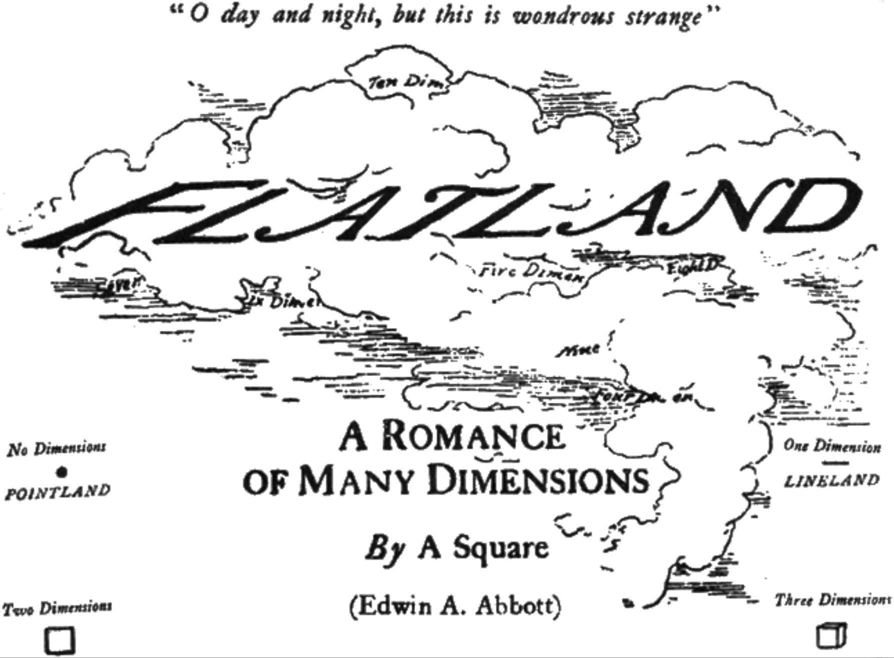

"O day and night, but this is wondrous strange"
FLATLAND
A ROMANCE
OF MANY DIMENSIONS
By A Square
(Edwin A. Abbott)
No Dimensions
POINTLAND
One Dimension
LINELAND
Two Dimensions
Three Dimensions

Outline

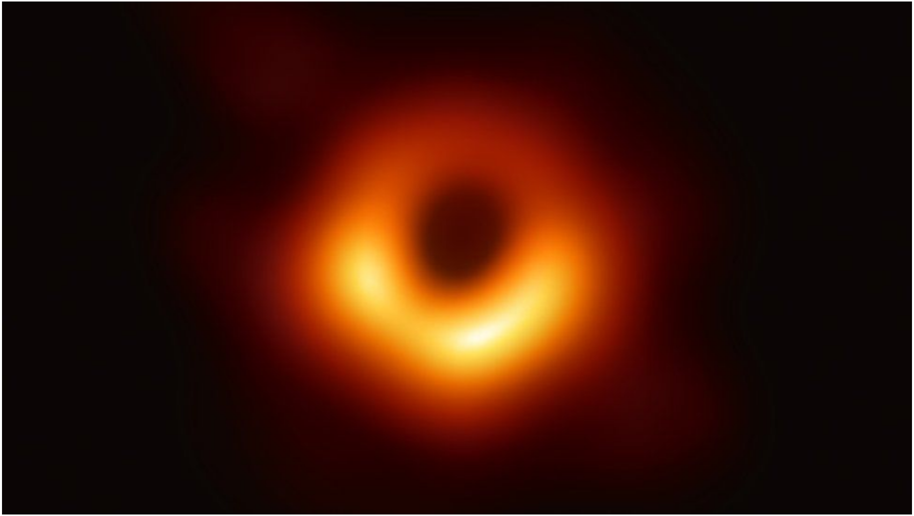
Motivation

Gravity in three dimensions

Gravity in two dimensions

Black holes hide key secrets to Nature

Seeing is believing...



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Some open issues in gravity

- IR (classical gravity)

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  - asymptotic symmetries
  - soft physics
  - near horizon symmetries

Take-away slogan

Equivalence principle needs modification

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  - numerous conceptual issues
  - **black hole** evaporation and unitarity
  - **black hole** microstates

Take-away homework

Find 'hydrogen-atom' of quantum gravity

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  - asymptotic symmetries
  - soft physics
  - near horizon symmetries
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  - numerous conceptual issues
  - **black hole** evaporation and unitarity
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- UV/IR (holography)

See book by Erdmenger or lecture notes 1807.09872

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  - **black hole** evaporation and unitarity
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- UV/IR (holography)
  - AdS/CFT and applications (see Erdmenger, Meyer and collaborators)
  - precision holography
  - generality of holography

Take-away question(s)

(When) is quantum gravity in $D+1$ dimensions equivalent to (which) quantum field theory in D dimensions?

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> - all issues above can be addressed in lower dimensions
> - lower dimensions technically simpler
> - hope to resolve conceptual problems

Gravity in various dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
- 5D: 50 (35 Weyl and 15 Ricci)
- 4D: 20 (10 Weyl and 10 Ricci)

Caveat: just counting tensor components can be misleading as measure of complexity

Example: large D limit actually simple for some problems (Emparan et al.)

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- 2D: 1 (Ricci scalar)
- 1D: 0 (space or time but not both $\Rightarrow$ no lightcones)

> Apply as mantra the slogan "as simple as possible, but not simpler"

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> - Simplest gravitational theories with **BHs** in 2D
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- 3D: lowest dimension exhibiting **BHs** and gravitons
- Simplest gravitational theories with **BHs** and gravitons in 3D
- Lowest dimension for Einstein gravity (**BHs** but no gravitons)

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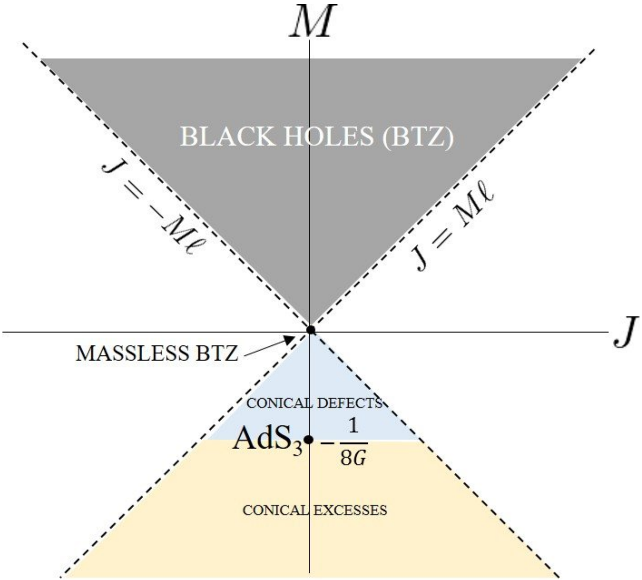
Gravity in three dimensions

Gravity in two dimensions

Spectrum of BTZ **black holes** and related physical states

Could this **black hole** be the 'hydrogen atom' for quantum gravity?



Choice of theory

- **Choice of bulk action**
  Pick Einstein–Hilbert action with negative cc ($\Lambda = -1/\ell^2$)

$$I_{\text{EH}}[g] = -\frac{1}{16\pi G}\int_{\mathcal{M}} \mathrm{d}^3x\sqrt{-g}\left(R + \frac{2}{\ell^2}\right)$$

  Usually choose also topology of $\mathcal{M}$, e.g. cylinder

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  - rotating (BTZ) **black hole** solutions analogous to Kerr

$$\mathrm{d}s^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{\ell^2 r^2}\ \mathrm{d}t^2 + \frac{\ell^2 r^2\,\mathrm{d}r^2}{(r^2 - r_+^2)(r^2 - r_-^2)} + r^2\left(\mathrm{d}\varphi - \frac{r_+ r_-}{\ell r^2}\ \mathrm{d}t\right)^2$$

t: time, $\varphi \sim \varphi + 2\pi$: angular coordinate, r: radial coordinate
$r \to \infty$: asymptotic region
$r \to r_+ \geq r_-$: **black hole** horizon
$r \to r_- \geq 0$: inner horizon
$r_+ \to r_- > 0$: extremal BTZ
$r_- \to 0$: non-rotating BTZ

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  - Bekenstein–Hawking entropy

$$S_{\text{BH}} = \frac{A}{4G} = \frac{\pi r_+}{2G}$$

    Hawking–Unruh temperature: $T = (r_+^2 - r_-^2)/(2\pi r_+ \ell^2)$

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  Crucial to define theory — yields spectrum of 'edge states'
  Pick whatever suits best to describe relevant physics

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> - Goal: understand holography beyond AdS/CFT
> - Explain first in general how edge states emerge

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Definition of asymptotic symmetries

All boundary condition preserving gauge transformations (bcpgt’s) modulo trivial gauge transformations

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Asymptotic symmetries in gravity

- Impose some bc's at (asymptotic or actual) boundary:

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- bcpgt's generated by asymptotic Killing vectors ξ:

$$\mathcal{L}_\xi g_{\mu\nu} \stackrel{!}{=} \mathcal{O}(\delta g_{\mu\nu})$$

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$$\xi^\mu(r_b,\, x^i) = \xi^\mu_{(0)}(r_b,\, x^i) + \text{subleading terms}$$

  $\xi^\mu_{(0)}(r_b,\, x^i)$: generates asymptotic symmetries/changes physical state
  subleading terms: generate trivial diffeos

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Definition of asymptotic symmetry algebra

Lie bracket quotient algebra of asymptotic Killing vectors modulo trivial diffeos

Canonical boundary charges

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  time-independent Schrödinger equation:

$$-\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi(x) = E\psi(x)$$

  look for (normalizable) bound state solutions, $E < 0$
  - Dirichlet bc's: no bound states
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  - Neumann bc's: no bound states
  - Robin bc's

    $$\big(\psi + \alpha\psi'\big)\big|_{x=0^+} = 0 \qquad \alpha \in \mathbb{R}^+$$

    lead to one bound state

    $$\psi(x)\big|_{x\geq 0} = \sqrt{\frac{2}{\alpha}}\, e^{-x/\alpha}$$

    with energy $E = -1/\alpha^2$, localized exponentially near $x = 0$

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- changing boundary conditions can change physical spectrum
- to distinguish asymptotic symmetries from trivial gauge trafos: either use Noether's second theorem and covariant phase space analysis or perform Hamiltonian analysis in presence of boundaries

  Some references:
  - covariant phase space: Lee, Wald '90, Iyer, Wald '94 and Barnich, Brandt '02
  - review: see Compère, Fiorucci '18 and refs. therein
  - canonical analysis: Arnowitt, Deser, Misner '59, Regge, Teitelboim '74 and Brown, Henneaux '86
  - review: see Bañados, Reyes '16 and refs. therein

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- to distinguish asymptotic symmetries from trivial gauge trafos: perform Hamiltonian analysis in presence of boundaries
- in Hamiltonian language: gauge generator $G[\epsilon]$ varies as

$$\delta G[\epsilon] = \int_\Sigma (\text{bulk term})\, \epsilon\, \delta\Phi - \int_{\partial\Sigma} (\text{boundary term})\, \epsilon\, \delta\Phi$$

  not functionally differentiable in general (Σ: constant time slice)

  Φ: shorthand for phase space variables
  ϵ: smearing function/parameter of gauge trafos
  δ: arbitrary field variation

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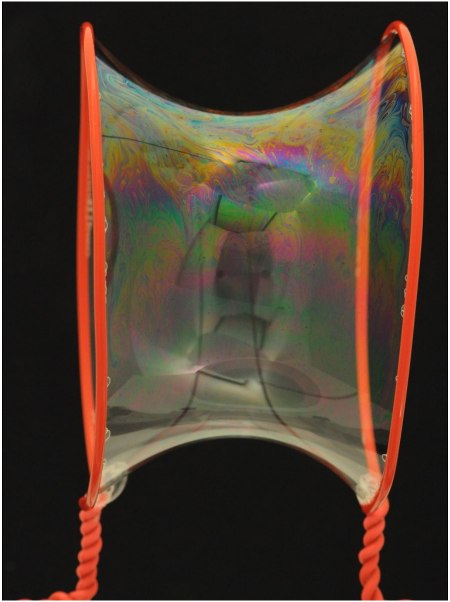
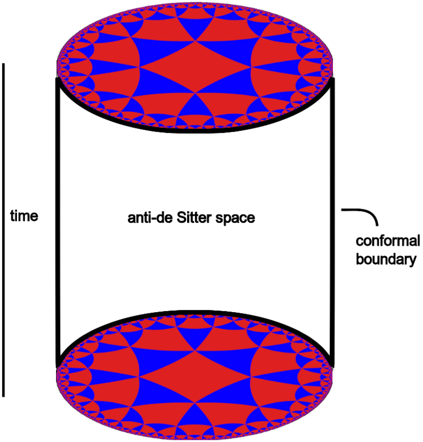
Trivial gauge transformations generated by some ϵ with $Q[\epsilon] = 0$

Soap bubble metaphor for AdS_3





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- Introduce also Fourier modes for charges $L_n^\pm = Q[l_n^\pm]$
- Canonical realization of asymptotic symmetries

$$i\{L_n^\pm,\, L_m^\pm\} = (n-m)\, L_{n+m}^\pm + \frac{c_{\text{BH}}}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

with central charge

$$c_{\text{BH}} = \frac{3\ell}{2G}$$

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- Given some bc's it is easy to determine asymptotic Killing vectors
- Brown–Henneaux imposed following bc's

$$\mathrm{d}s^2 = \mathrm{d}r^2 + e^{2r/\ell}\,\mathrm{d}x^+\,\mathrm{d}x^- + \mathcal{O}(1)\,\mathrm{d}x^{+\,2} + \mathcal{O}(1)\,\mathrm{d}x^{-\,2} + \dots$$

- Metrics above preserved by asymptotic Killing vectors

$$\xi = \varepsilon^+(x^+)\partial_+ + \varepsilon^-(x^-)\partial_- + \dots$$

- Introducing (Fourier) modes $l_n^\pm \sim \xi(\varepsilon^\pm = e^{inx^\pm})$ yields ASA

$$[l_n^\pm,\, l_m^\pm]_{\text{Lie}} = (n-m)\, l_{n+m}^\pm$$

- Introduce also Fourier modes for charges $L_n^\pm = Q[l_n^\pm]$
- Canonical realization of asymptotic symmetries

$$i\{L_n^\pm,\, L_m^\pm\} = (n-m)\, L_{n+m}^\pm + \frac{c_{\text{BH}}}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

with central charge

$$c_{\text{BH}} = \frac{3\ell}{2G}$$

- Dual field theory, if it exists, must be CFT_2!

Some checks of AdS_3/CFT_2

Every AdS_3 gravity observable must correspond to some CFT_2 observable

ok, fine, so what about...

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- e.g. 5-point stress-tensor correlator in CFT_2 Bagchi, DG, Merbis '15

$$\text{CFT}_2: \qquad \langle T_{++}(z_1)T_{++}(z_2)T_{++}(z_3)T_{++}(z_4)T_{++}(z_5)\rangle = \frac{4c\, g_5(\gamma,\, \zeta)}{\prod_{1\leq i\leq 5} z_{ij}}$$

$\gamma = z_{12}z_{34}/(z_{13}z_{24})$, $\zeta = z_{25}z_{34}/(z_{35}z_{24})$, $z_{ij} = z_i - z_j$ and

$$g_5(\gamma,\, \zeta) = \frac{\gamma+\zeta}{2(\gamma-\zeta)} - \frac{\gamma^2-\gamma\zeta+\zeta^2}{\gamma(\gamma-1)\zeta(\zeta-1)(\gamma-\zeta)}\big([\gamma(\gamma-1)+1][\zeta(\zeta-1)+1]-\gamma\zeta\big)$$

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- on gravity side given by 5th functional variation of action w.r.t. metric
- result on gravity side

$$\frac{\delta^5 I_{\mathrm{EH}}[g_{\mu\nu}]}{\delta g^{++}(z_1)\delta g^{++}(z_2)\delta g^{++}(z_3)\delta g^{++}(z_4)\delta g^{++}(z_5)} = \frac{4c\, g_5(\gamma,\, \zeta)}{\prod_{1\leq i\leq 5} z_{ij}}$$

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$$S_{\rm CFT_2} = S_{\rm Cardy} = 2\pi\,\sqrt{\frac{c}{6}\,(M+J)} + 2\pi\,\sqrt{\frac{c}{6}\,(M-J)}$$

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$$S_{\mathrm{BH}} = \frac{A}{4G} = \frac{\pi r_+}{2G} = 2\pi\sqrt{\frac{\ell}{4G}\,(M+J)} + 2\pi\sqrt{\frac{\ell}{4G}\,(M-J)}$$

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- entropy formulas coincide for

$$c = \frac{3\ell}{2G}$$

  matches precisely Brown–Henneaux result $c = c_{\mathrm{BH}}$

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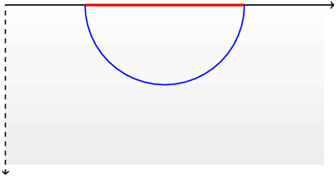
- Ryu–Takayanagi prescription: EE = length of geodesic anchored at boundary entangling region



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- ...boundary conditions different from Brown–Henneaux?

Different boundary conditions may lead to other symmetries, hence no AdS_3/CFT_2!

Brief history of boundary conditions in AdS_3 (and their ASAs)

- Brown–Henneaux '86

$$[L^\pm_n,\, L^\pm_m] = (n-m)\, L^\pm_{n+m} + \frac{c_{\text{BH}}}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

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$$[J_n,\, J_m] = \frac{k}{2}\, n\, \delta_{n+m,\,0}$$

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$$[J^a_n,\, J^b_m] = (a-b)\, J^{a+b}_{n+m} - k\, n\, \kappa_{ab}\, \delta_{n+m,\,0}$$

$a, b = -1, 0, 1$

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- DG–Riegler '16: yes and yes

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(How) does this work in higher dimensions? Don't know (yet)!

What about non-AdS holography?

Key question

(When) is quantum gravity in $D+1$ dimensions equivalent to (which) quantum field theory in D dimensions?

What about flat space holography?

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Let us be modest and refine this question:

More modest question

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See work by Bagchi et al.

What about flat space holography?

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(How) does holography work in flat space?

See work by Bagchi et al.

Would like concrete model for flat space holography

Outline

Motivation

Gravity in three dimensions

Gravity in two dimensions

Selected list of models

Black holes in (A)dS$_2$, asymptotically flat or arbitrary spaces (Wheeler property)

Model	$U(X)$	$V(X)$
1. Schwarzschild (1916)	$-\frac{1}{2X}$	$-\lambda^2$
2. Jackiw-Teitelboim (1984)	0	ΛX
3. Witten **Black Hole** (1991)	$-\frac{1}{X}$	$-2b^2 X$
4. CGHS (1992)	0	-2Λ
5. $(A)dS_2$ ground state (1994)	$-\frac{a}{X}$	BX
6. Rindler ground state (1996)	$-\frac{a}{X}$	BX^a
7. **Black Hole** attractor (2003)	0	BX^{-1}
8. Spherically reduced gravity ($N > 3$)	$-\frac{N-3}{(N-2)X}$	$-\lambda^2 X^{(N-4)/(N-2)}$
9. All above: ab-family (1997)	$-\frac{a}{X}$	BX^{a+b}
10. Liouville gravity	a	$be^{\alpha X}$
11. Reissner-Nordström (1916)	$-\frac{1}{2X}$	$-\lambda^2 + \frac{Q^2}{X}$
12. Schwarzschild-$(A)dS$	$-\frac{1}{2X}$	$-\lambda^2 - \ell X$
13. Katanaev-Volovich (1986)	α	$\beta X^2 - \Lambda$
14. BTZ/Achucarro-Ortiz (1993)	0	$\frac{Q^2}{X} - \frac{J}{4X^3} - \Lambda X$
15. KK reduced CS (2003)	0	$\frac{1}{2}X(c - X^2)$
16. KK red. conf. flat (2006)	$-\frac{1}{2}\tanh(X/2)$	$A \sinh X$
17. 2D type 0A string **Black Hole**	$-\frac{1}{X}$	$-2b^2 X + \frac{b^2 q^2}{8\pi}$
18. exact string **Black Hole** (2005)	lengthy	lengthy

Choice of theory (review: see hep-th/0204253)

- Choice of bulk action
  Einstein–Hilbert action not useful

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  Dilaton gravity in two dimensions ($X =$ dilaton):

  $$I[X,\, g_{\mu\nu}] = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \big[XR - U(X)(\nabla X)^2 - 2V(X)\big]$$

  - kinetic potential $U(X)$ and dilaton potential $V(X)$
  - constant dilaton and linear dilaton solutions
  - all solutions known in closed form globally for all choices of potentials
  - simple choice (Jackiw–Teitelboim):

    $$U(X) = 0 \qquad V(X) = \Lambda X$$

  - for negative $\Lambda = -1/\ell^2$ leads to AdS_2 solutions

Choice of theory (review: see hep-th/0204253)

- Choice of bulk action

  JT model:

$$I_{\text{JT}}[X,\, g_{\mu\nu}] = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \left[XR - 2\Lambda X\right]$$

  Leads to (A)dS_2 solutions

$$R = 2\Lambda$$

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- Flat space choice of bulk action
  CGHS model

$$I_{\text{CGHS}}[X,\, g_{\mu\nu}] = \frac{1}{16\pi\, G_2}\int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \left[XR - 2\Lambda\right]$$

  Leads to flat solutions

$$R = 0$$

  Flat space holography proposal: Afshar, González, DG, Vassilevich '19

Interlude: SYK in one slide (Kitaev '15; Maldacena, Stanford '16)

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$$G(\tau) \sim \text{sign}(\tau)/\sin^{2\Delta}(\pi\tau/\beta) \qquad \text{conformal weight } \Delta = 1/4$$

- $SL(2, \mathbb{R})$ covariant $x \to (ax + b)/(cx + d)$ with $x = \tan(\pi\tau/\beta)$

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- $SL(2,\mathbb{R})$ covariant $x \to (ax+b)/(cx+d)$ with $x = \tan(\pi\tau/\beta)$
- effective action at large N and large J: Schwarzian action

$$\Gamma[h] \sim -\frac{N}{J}\int_0^\beta \mathrm{d}\tau\,\left[\dot{h}^2 + \tfrac{1}{2}\{h;\,\tau\}\right] \qquad \{h;\,\tau\} = \frac{\dddot{h}}{\dot{h}} - \frac{3}{2}\,\frac{\ddot{h}^2}{\dot{h}^2}$$

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$$G(\tau) \sim \text{sign}(\tau)/\sin^{1/2}(\pi\tau/\beta)$$

- $SL(2, \mathbb{R})$ covariant $x \to (ax + b)/(cx + d)$ with $x = \tan(\pi\tau/\beta)$
- effective action at large N and large J: Schwarzian action

$$\Gamma[h] \sim -\frac{N}{J}\int\limits_0^\beta \mathrm{d}\tau\,\big[\dot{h}^2 + \tfrac{1}{2}\,\{h;\,\tau\}\big] \qquad \{h;\,\tau\} = \frac{\dddot{h}}{\dot{h}} - \frac{3}{2}\,\frac{\ddot{h}^2}{\dot{h}^2}$$

- Schwarzian action also follows from JT gravity

Flat space holography and complex SYK 1911.05739

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$$[L_n,\,L_m] = (n-m)\,L_{n+m}$$
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$$[J_n,\,J_m] = 0$$

and the two-dimensional Maxwell symmetries (L_1, L_0, J_{-1}, J_0)

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Concrete model for flat space holography

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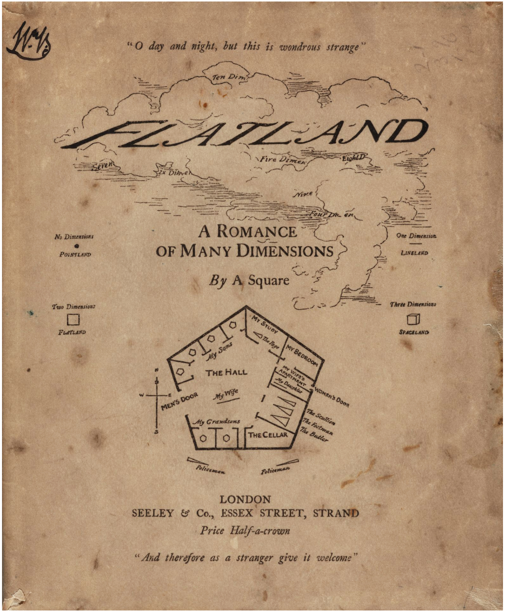
> - Numerous open questions in gravity and holography
> - Many can be addressed in lower dimensions
> - If you are stuck in higher D try $D = 3$ or $D = 2$

Thank you for your attention!



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- abelian Chern–Simons action (on cylinder)

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Note: topological QFT with no local physical degrees of freedom

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- Fourier modes $J_n \sim \oint \mathcal{J} e^{in\varphi}$ yield $u(1)_k$ current algebra, $i\{J_n,\, J_m\} = \frac{k}{2}\, n\, \delta_{n+m,\, 0}$

Edge states

see e.g. Halperin '82, Witten '89, or Balachandran, Chandar, Momen '94

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- theories with no local physical degrees of freedom can have edge states! $\Rightarrow$ perhaps cleanest example of holography