Seminar Talk (TU Wien)

June 2005

An action for the exact string black hole

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supported by an Erwin-Schrödinger fellowship, project J-2330-N08 of the Austrian Science Foundation (FWF)

Based upon: *DG,* `hep-th/0501208`

Outline

1. Actions
2. The exact string BH
3. Gravity as gauge theory: $2^{\text{nd}} \to 1^{\text{st}}$ order
4. First order action, relation to PSM
5. All classical solutions (locally)
6. Global structure (Penrose diagrams)
7. Mass, temperature, entropy
8. Action for the ESBH
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10. Open questions

1. Actions

EH:

$$S = \int \mathrm{d}^D x \sqrt{-g} R + \text{surface}$$

R: Ricci scalar
g: determinant of metric $g_{\mu\nu}$

JBD/scalar-tensor theories/low energy strings:

$$S = \int \mathrm{d}^D x \sqrt{-g} \left(XR - U(X)(\nabla X)^2 + 2V(X) \right)$$

X: "dilaton field"
U, V: (arbitrary) potentials
"dilaton gravity in D dimensions"

Note: higher powers in curvature, e.g.

$$S = \frac{1}{2} \int \mathrm{d}^D x \sqrt{-g} R^2$$

$\equiv$ to dilaton gravity ($U = 0$, $V = -X^2/4$)

Often exponential representation:

$$X = e^{-2\phi}$$

Low energy effective actions

NLSM (target space metric: $g_{\mu\nu}$, coordinates: x^μ)

$$S^\sigma \propto \int \mathrm{d}^2\xi\sqrt{-h}\left(g_{\mu\nu}h^{ij}\partial_i x^\mu \partial_j x^\nu + \alpha'\phi\mathcal{R} + \text{tachyon}\right)$$

set B-field zero. neglect tachyon for the time being. conformal invariance:

$$2\pi T_i^i = \beta^\phi \mathcal{R} + \beta^g_{\mu\nu} h^{ij}\partial_i x^\mu \partial_j x^\nu \stackrel{!}{=} 0$$

thus, β-functions must vanish. LO:

$$\frac{16\pi^2}{\alpha'}\beta^\phi = -4b^2 - 4(\nabla\phi)^2 + 4\Box\phi + R$$
$$\beta^g_{\mu\nu} = R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi$$

conditions $\beta^\phi = 0 = \beta^g_{\mu\nu}$ follow from

$$S = \int \mathrm{d}^D x\sqrt{-g}e^{-2\phi}\left(R + 4(\nabla\phi)^2 - 4b^2\right)$$

for $D = 2$: "Witten BH" and CGHS model:
$2D$ dilaton gravity ($U = -1/X$, $V = -2b^2 X$)
Note: $b^2 = (26 - D)/(6\alpha')$

CFT description as $SL(2,\mathbb{R})/U(1)$ gauged WZW
review: K. Becker, `hep-th/9404157`

Relation to $SL(2,\mathbb{R})/U(1)$ gauged WZW

NLSM: complicated on generic backgrounds; easier: background is group manifold on which string propagates

For $g \in SL(2,\mathbb{R})$:

$$S_{WZW} = \frac{k}{8\pi}\int_\Sigma \mathrm{d}^2\xi\sqrt{h}h^{ij}\mathrm{tr}\left(g^{-1}\partial_i g g^{-1}\partial_j g\right)+\Gamma(g)$$

with the Wess-Zumino term

$$\Gamma(g) = \frac{ik}{12\pi}\int_{\mathrm{d}^{-1}\Sigma} \mathrm{d}^3\zeta\epsilon^{abc}\mathrm{tr}\left(g^{-1}\partial_a g g^{-1}\partial_b g g^{-1}\partial_c g\right)$$

Global $SL(2,\mathbb{R})\times SL(2,\mathbb{R})$ symmetry $g \to agb^{-1}$

Euclidean BH: gauge compact abel. subg.
Minkowskian BH: gauge non-compact a. s.
need to introduce gauge field!

conserved currents from $SL(2,\mathbb{R})$ symmetry yield current algebra (Kac-Moody algebra) with level k

minisuperspace approach: keep only zero-mode algebra!

$$[J_3, J_\pm] = \pm J_\pm\,, \quad [J_+, J_-] = -2J_3$$

get L_0^L, L_0^R from the group theory of $SL(2,\mathbb{R})$

2. The exct string BH

R. Dijkgraaf, H. Verlinde, and E. Verlinde, Nucl. Phys. **B371** *(1992) 269–314.*

in $2D$: only physical propagating degree of freedom: tachyon!

tachyon action ($V(T) = -2T^2 + \mathcal{O}(T^3)$):

$$S^T = \int \mathrm{d}^D x \sqrt{-g} e^{-2\phi} \left((\nabla T)^2 - V(T) \right)$$

CFT: tachyon defined through zero modes L_0^L, L_0^R of stress tensors for left and right movers:

$$S^T = \int \mathrm{d}^D x \sqrt{-g} e^{-2\phi} \left(T(L_0^L + L_0^R) T - V(T) \right)$$

"standard result": $L_0^L + L_0^R =$ Laplacian:

$$(L_0^L + L_0^R) T = -\frac{e^{2\phi}}{\sqrt{-g}} \partial_\mu \left(g^{\mu\nu} e^{-2\phi} \sqrt{-g} \partial_\nu T \right)$$

strategy of DVV: determine L_0^L, L_0^R with CFT methods (for any level k), then use identification with Laplacian above to obtain metric and dilaton field

Witten BH arises as limit $k \to \infty$, while for $k \to 2$ AdS_2 emerges ("JT model")

Geometry of the ESBH

2D line element (Minkowskian):

$$\mathrm{d}s^2 = f^2(x)\,\mathrm{d}\tau^2 - \mathrm{d}x^2\,,$$

with

$$f(x) = \frac{\tanh(bx)}{\sqrt{1 - p\tanh^2(bx)}}\,.$$

The corresponding expression for the dilaton,

$$\phi = \phi_0 - \ln\cosh(bx) - \frac{1}{4}\ln\left(1 - p\tanh^2(bx)\right),$$

contains an integration constant ϕ_0.

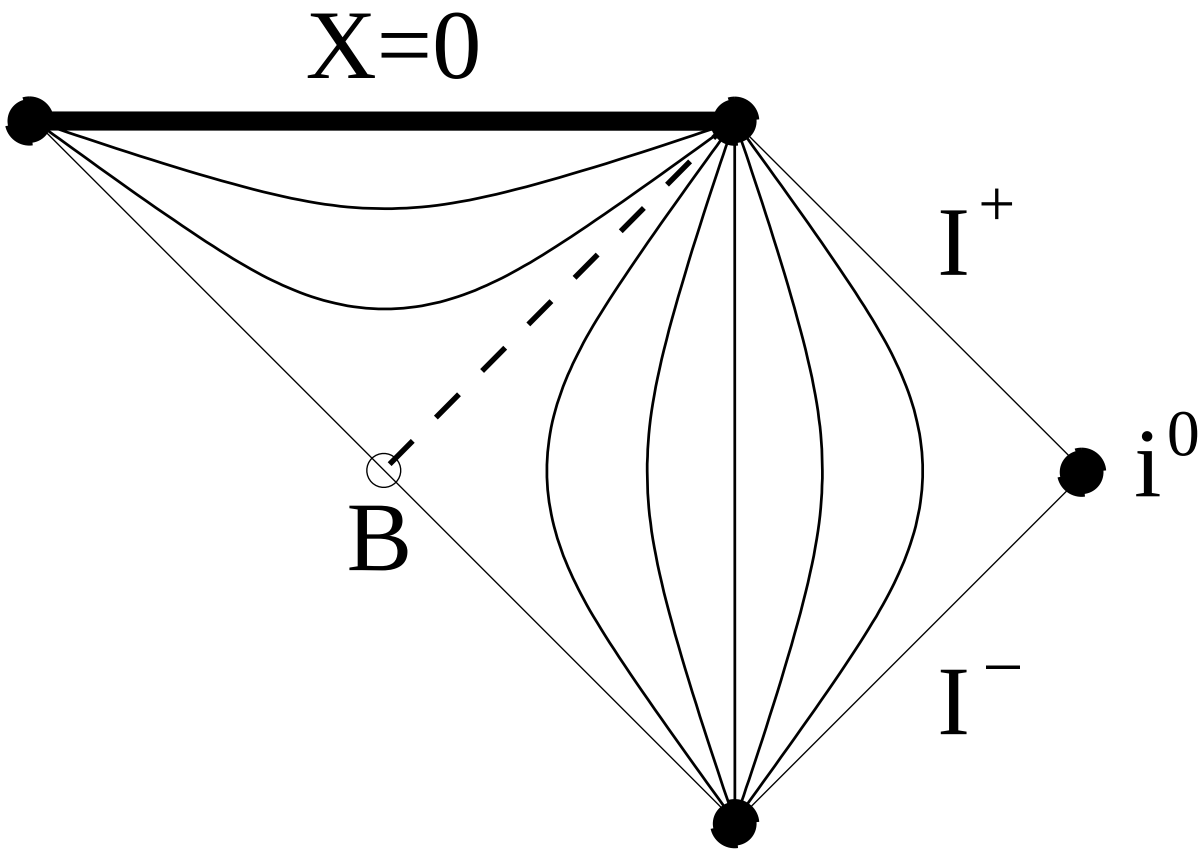
Additionally, there are the following relations between constants, string-coupling α', level k and dimension D of string target space:

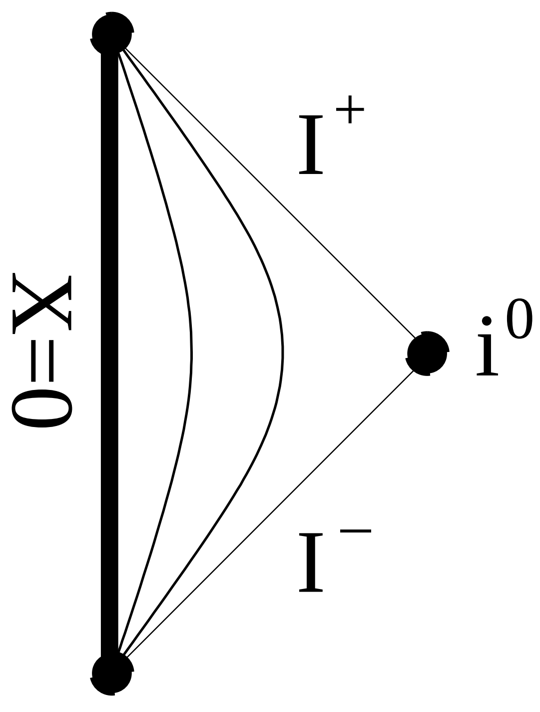
$$\alpha' b^2 = \frac{1}{k-2}\,, \quad p := \frac{2}{k} = \frac{2\alpha' b^2}{1 + 2\alpha' b^2}\,, \quad D - 26 + 6\alpha' b^2 = 0\,.$$

For $D = 2$ one obtains $p = 8/9$; for $p = 0$ one recovers the Witten BH; for $p = 1$ the JT model is obtained.

Winding/momentum mode duality (ESNS):

$$bx \to bx + i\pi/2$$





i^0, I^+, I^- and B denote spatial infinity, future light-like infinity, past light-like infinity and the bifurcation point, respectively; the Killing horizon is denoted by the dashed line

Why an action?

- Needed for mass definition ("Gibbons-Hawking term") – clarify which, if any, of previous mass definitions is correct

- Needed for entropy – get insight into thermodynamics of a non-perturbative BH solution of string theory

- Supersymmetrization

- Quantization

- Get a new theory in this way which is interesting on its own and which generalizes the CGHS model – may couple geometric action to some matter action, study critical collapse, etc.

Thermodynamics of ESBH: as early as *Gibbons/Perry,* `hep-th/9204090`, but also recently *Davis/McNees* `hep-th/0411121`

Unsolved for 13 years: either very difficult or not very intersting... third option: solution simple once appropriate tools are employed!

3. Gravity as gauge theory: $2^{\text{nd}} \to 1^{\text{st}}$

JT model: (A)dS$_2$ ($SO(1,2)$): *C. Teitelboim, PL* **B126** *(1983) 41*, *R. Jackiw, NP* **B252** *(1985) 343*

$$[P_a, P_b] = \Lambda \epsilon_{ab} J \,, \quad [P_a, J] = \epsilon_{ab} P^b$$

1^{st} order form: *K. Isler, C. Trugenberger, PRL* **63** *(1989) 834*, *A. Chamseddine, D. Wyler, PL* **B228** *(1989) 75*

$$L = X_A F^A = X_a (De)^a + X(\text{d}\omega + \frac{1}{2}\Lambda \epsilon_{ab} e^a e^b)$$

$SO(1,2)$ connection: $A = e^a P_a + \omega J$,
$F = \text{d}A + \frac{1}{2}[A, A]$, e^a, ω: "Cartan variables",
X_A: Lagr. mult. (trafo under coadjoint rep.)

CGHS: central extended Poincaré ($ISO(1,1)$): *D. Cangemi, R. Jackiw,* `hep-th/9203056`

$$[P_a, P_b] = \epsilon_{ab} I \,, \quad [P_a, J] = \epsilon_{ab} P^b \,, \quad [I, J] = 0 = [I, P_a]$$

again first order with $L = X_A F^A$ possible without central extension: *Verlinde, MG VI*
cf. also *A. Achúcarro,* `hep-th/9207108`

other important pre-cursors:

W. Kummer, D.J. Schwarz, PR **D45** *(1992) 3628*

N. Ikeda, `hep-th/9312059`: (non-linear) gauge formulation for $U = 0$ but generic $V(X)$

4. First order action, relation to PSM

First order action:

P. Schaller, T. Strobl, hep-th/9405110

$$S^{(FOG)} = \int_{\mathcal{M}_2} \left[X_a T^a + XR + \epsilon \mathcal{V}(X^a X_a, X)\right] \tag{1}$$

$T^a = (De)^a$: torsion 2-form
$R^a{}_b = \epsilon^a{}_b R = \epsilon^a{}_b \, \mathrm{d}\omega$: curvature 2-form
$\epsilon = -\frac{1}{2}\epsilon_{ab} e^a \wedge e^b$: volume 2-form
X: "dilaton" (Lagrange mult. f. curvature)
X^a: auxiliary fields (— " — torsion)
$\mathcal{V}$: potential defining the model (as before)

Relation to second order:
dilaton: $X = X$
kinetic term: $(\nabla X)^2 = -X^a X_a$
metric: $g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu$
connection: Levi-Civitá $= \omega -$torsion part

Technical Note: Use light-cone components ($\eta_{+-} = 1 = \eta_{-+}$, $\eta_{++} = 0 = \eta_{--}$); define $\epsilon^{\pm}{}_{\pm} = \pm 1$

$$T^{\pm} = (\mathrm{d} \pm \omega) \wedge e^{\pm}, \quad \epsilon = e^+ \wedge e^-, \quad X^a X_a = 2X^+ X^-$$

Typically: $\mathcal{V}(X^+X^-, X) = X^+X^- U(X) + V(X)$

Equivalence to specific type of Poisson-σ model
[*P. Schaller, T. Strobl*, hep-th/9405110]

$$\mathcal{S}_{gPSM} = \int_{\mathcal{M}_2} \mathrm{d}X^I \wedge A_I + \frac{1}{2} P^{IJ} A_J \wedge A_I \, .$$

- gauge field 1-forms: $A_I = (\omega, e_a)$, connection, Zweibeine
- target space coordinates: $X^I = (X, X^a)$, dilaton, auxiliary fields
- target space: Poisson manifold
- Poisson tensor: odd dimension $\rightarrow$ kernel!
- Jacobi: $P^{IL}\partial_L P^{JK} + \mathit{perm}\,(IJK) = 0$

$$P^{IJ} = \begin{pmatrix} 0 & X^+ & -X^- \\ -X^+ & 0 & \mathcal{V} \\ X^- & -\mathcal{V} & 0 \end{pmatrix}$$

Equations of motion (first order):

$$\mathrm{d}X^I + P^{IJ} A_J = 0$$
$$\mathrm{d}A_I + \frac{1}{2}(\partial_I P^{JK}) A_K \wedge A_J = 0$$

Gauge symmetries:

$$\delta X^I = P^{IJ} \varepsilon_J$$
$$\delta A_I = -\mathrm{d}\varepsilon_I - \left(\partial_I P^{JK}\right) \varepsilon_K A_J$$

Note 1: if P^{IJ} linear: Lie-Algebra! Otherwise: non-linear gauge symmetries

Note 2: on-shell equivalent to diffeomorphisms+local Lorentz trafos for specific Poisson tensor on previous page

Remark: Schouten-Nijenhuis bracket:

$$[X^I, X^J]_{SN} = P^{IJ}$$

Note 1: like non-commutative geometry

Note 2: Jacobi identity for bracket equivalent to non-linear identity for Poisson tensor on previous page

$\star$: *M. Kontsevich,* q-alg/9709040, path integral approach: *A. Cattaneo, G. Felder,* math.qa/9902090

5. All classical solutions (locally)

Ansatz: $X^+ \neq 0$ in a patch $\to$ $e^+ = X^+ Z$

Summary of EOM for dilaton gravity:

$$\begin{aligned}
\delta\omega :&\quad \mathrm{d}X + X^- e^+ - X^+ e^- = 0\,,\\
\delta e^{\mp} :&\quad (\mathrm{d}\pm\omega)X^{\pm} \mp \mathcal{V} e^{\pm} = 0\,,\\
\delta X :&\quad \mathrm{d}\omega + \epsilon \frac{\partial \mathcal{V}}{\partial X} = 0\,,\\
\delta X^{\mp} :&\quad (\mathrm{d}\pm\omega)e^{\pm} + \epsilon\, \frac{\partial \mathcal{V}}{\partial X^{\mp}} = 0\,.
\end{aligned}$$

1. use $\delta\omega$ to get $e^- = \mathrm{d}X/X^+ + X^- Z$
2. read off $\epsilon = e^+ \wedge e^- = Z \wedge \mathrm{d}X$
3. use δe^- to get $\omega = -\mathrm{d}X^+/X^+ + Z\mathcal{V}$
4. use δX^- to get $\mathrm{d}Z = \mathrm{d}X \wedge Z U(x)$
5. define "integrating factor":

$$I(X) := \exp \int^X U(X')\, \mathrm{d}X'$$

6. obtain $Z =: \hat{Z} I(X)$ with $\mathrm{d}\hat{Z} = 0 \to \hat{Z} = \mathrm{d}u$
7. use $g_{\mu\nu} = e^+_\mu e^-_\nu + e^-_\mu e^+_\nu$

general solution for the line element:

$$\mathrm{d}s^2 = I(X)\left(2\,\mathrm{d}u\,\mathrm{d}X + 2X^+X^- I(X)\,\mathrm{d}u^2\right)$$

$X^+X^- = 0$: apparent horizon!

Conservation law:

T. Banks, M. O'Loughlin, NP **B362** *(1991) 649;*
V. Frolov, PR **D46** *(1992) 5383; R. Mann,* `hep-th/9206044`

later generalized by "Vienna group"

W. Kummer+students; for a review cf. e.g.

DG, W. Kummer, D. Vassilevich, `hep-th/0204253`

Derivation in absence of matter: EOMs $X^+\delta e^+ + X^-\delta e^-$ using also the EOM $\delta\omega$ establishes

$$\mathrm{d}(X^+X^-) + \mathcal{V}\,\mathrm{d}X = 0$$

for "standard" $\mathcal{V} = X^+X^-U(X) + V(X)$:

$$\mathcal{C} = I(X)X^+X^- + w(X)\,, \quad \mathrm{d}\mathcal{C} = 0$$

with

$$w(X) := \int^X I(X')V(X')\,\mathrm{d}X'$$

"Generalized Birkhoff theorem" (always 1 Killing)

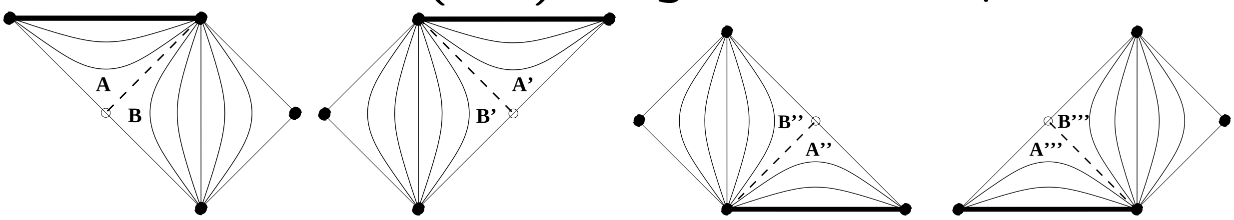
Inserting into line element, $\mathrm{d}r = I(X)\,\mathrm{d}X$:

$$\mathrm{d}s^2 = 2\,\mathrm{d}u\,\mathrm{d}r + 2I(X(r))\,(\mathcal{C} - w(X(r)))\,\mathrm{d}u^2 \tag{2}$$

Eddington-Finkelstein patch!

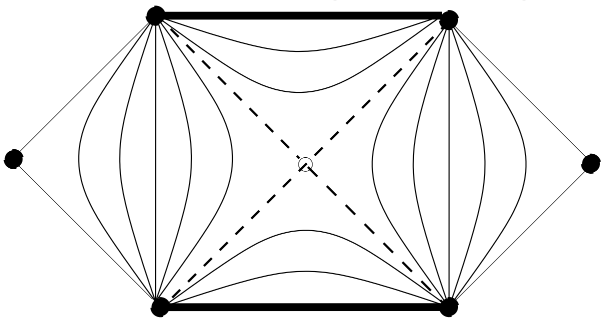
6. Global structure (Penrose diagrams)

Carter-Penrose (CP) diagram of EF patches:



for Schwarzschild-like solutions

Global CP: glue together EF patches:



Only point not covered by EF patches:
bifurcation point: $X^+ = 0 = X^-$

M. Walker, J. Math. Phys. **11** *(1970) 2280*, *T. Klösch, T. Strobl*, `gr-qc/9508020`, `gr-qc/9511081`

open regions with $X^+ = 0 = X^-$: $X = \text{const.}$
"Constant Dilaton Vacua" (very simple)

$$V(X_{CDV}) = 0\,, \qquad R \propto V'(X_{CDV}) = \text{const.}$$

only Minkowski, Rindler, (A)dS

7. Mass, temperature, entropy

Naively from surface gravity:

$$T_H = \frac{1}{2\pi}\Big|w'(X)\Big|_{X=X_h} . \qquad (3)$$

Note: independent from $I(X)$; don't need action

With minimally coupled matter: same result; e.g. from trace anomaly

S. Christensen, S. Fulling, PR **D15** *(1977) 2088*

$$< T^\mu{}_\mu > \propto R\,, \qquad \nabla_\mu T^{\mu\nu} = 0$$

Matter coupled ("non-minimally") to dilaton: Non-conservation equation!

W. Kummer, D. Vassilevich, gr-qc/9907041

$$\nabla^\mu T_{\mu\nu} = -(\partial_\nu \Phi)\frac{1}{\sqrt{-g}}\frac{\delta W}{\delta \Phi}\,, \qquad X = e^{-2\Phi}$$

Mass-to-temperature law: need action!
Sometimes mutually contradicting results for "ADM mass" (e.g. 2D string theory)

clarified in appendix of *DG, D. Mayerhofer,* gr-qc/0404013

BH Entropy

BH: "Bekenstein-Hawking" or "Black Hole"

Simple thermodynamic considerations:

$$dS = \frac{dM}{T}$$

J. Gegenberg, G. Kunstatter, D. Louis-Martinez, gr-qc/9408015

$$S = 2\pi\, X|_{\mathcal{C}=w(X)} \equiv \frac{A}{4} \qquad (4)$$

confirmed by more elaborate derivations (CFT methods, near horizon conformal symmetry, Cardy-formula)

S. Carlip gr-qc/9906126, gr-qc/0203001, hep-th/0408123, *S. Solodukhin*, hep-th/9812056, *M. Cadoni, S. Mignemi,* hep-th/9810251

Note: need action!

open question: counting of microstates is fine, but what are actually the microstates of 2D dilaton gravity?

“Vienna School”

Brauer

Fuchs

Hausner

Lehmden

8. Action for the ESBH

extensive review on $2D$ dilaton gravity:
DG, W. Kummer, D. Vassilevich, `hep-th/0204253`

nogo result *DG, D. Vassilevich*, `hep-th/0210060`

$\mathbb{R}$

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circumvent it by allowing matter – but: don't want propagating physical degrees of freedom!

suggestive: consider (abelian) gauge field
result: it works! *DG,* hep-th/0501208

$$S_{ESBH} = \int_{\mathcal{M}_2} \Big[X_a T^a + \Phi R + BF + \epsilon \Big(X^+ X^- U(\Phi) + V(\Phi) \Big) \Big] , \tag{14}$$

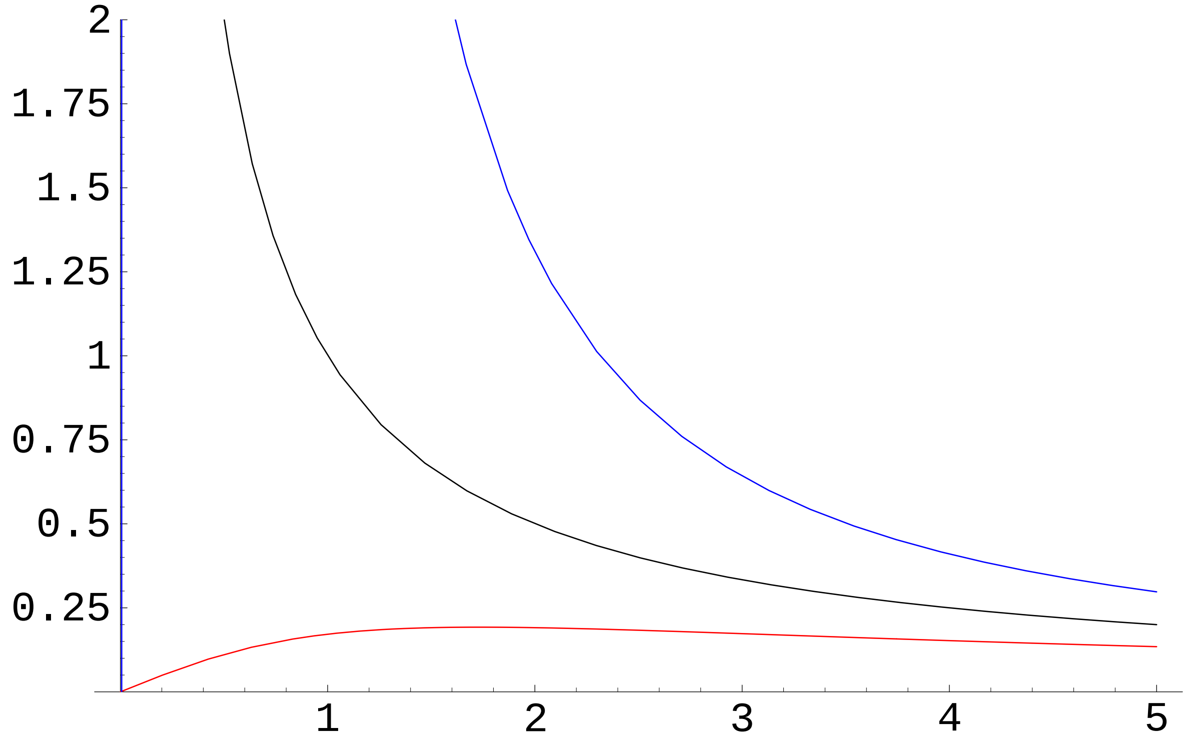
with $\Phi_\pm = \gamma \pm \operatorname{arcsinh} \gamma$ and $\gamma = X/B$
$+$: ESBH, $-$: ESNS
The potentials read

$$V = -2b^2 \gamma , \quad U_\pm = -\frac{1}{\gamma N_\pm(\gamma)} , \tag{15}$$

with an irrelevant scale parameter $b \in \mathbb{R}^+$ and

$$N_\pm(\gamma) = 1 + \frac{2}{\gamma} \left(\frac{1}{\gamma} \pm \sqrt{1 + \frac{1}{\gamma^2}} \right) . \tag{16}$$

Note that $N_+ N_- = 1$.



Plot of U as a function of γ

Red: ESBH, Blue: ESNS, Black: Witten BH

Asymptotic ("weak coupling") limit ($\gamma \to \infty$):
Witten BH: $U = -1/\Phi$, $V = -2b^2\Phi$
valid for both branches (ESBH, ESNS)

Strong coupling limit ($\gamma \to 0$):
ESBH branch: JT model ($U = 0$, $V = -b^2\Phi$)
ESNS branch: 5D Schwarzschild!
($U = -2/(3\Phi)$, $V = -2b^2(6\Phi)^{1/3}$)

9. Mass and entropy of the ESBH

Constants of motion:
$U(1)$-charge: value of DVV dilaton at origin
mass: determined by level k! ($M_{ADM} = bk$)

Hawking temperature $T_H = \frac{b}{2\pi}\sqrt{1 - \frac{2b}{M_{ADM}}}$

Note: for mass knowledge of action pivotal!
the same holds for entropy!

$$S = 2\pi\Phi|_{\text{horizon}} = 2\pi\,(x + \operatorname{arcsinh} x)$$

with $x := 2\sqrt{M(M-1)}$ and $M = k/2$
Limit of large mass ($k \to \infty$)

$$S|_{M\gg 1} = S_{LO} + 2\pi \ln S_{LO} + \mathcal{O}(1)$$

with $S_{LO} = 4\pi M$

Limit of small mass ($k \to 2$)

$$S|_{M=1+\varepsilon} = 8\pi\sqrt{\varepsilon}\,(1 + \mathcal{O}(\varepsilon))$$

Specific heat

Result:

$$C := \frac{\mathrm{d}M_{ADM}}{\mathrm{d}T_H} = \frac{16\pi^2}{b} M^2 T_H$$

Low temperatures ($T_H \to 0$, $M \to 1$): like electron gas with Sommerfeld constant $\gamma = 16\pi^2/b$

High masses: Witten BH limit! Note: for Witten BH: $1/C = 0$; thus, corrections are highly non-trivial!

$$C = \frac{2\pi}{b^2} M_{ADM}^2 + \mathcal{O}(M_{ADM})$$

Specific heat is positive and proportional to M_{ADM}^2. Up to numerical coefficient the *same* result has been obtained for the quantum corrected Witten BH in *DG, W. Kummer and D. Vassilevich*, `hep-th/0305036`!

Why is the mass given by the level?

Technically: see hep-th/0501208
result: $\mathcal{C}^{(g)} = -bk = -M_{ADM}$

Physically: conservation law in presence of matter:

$$\underbrace{\mathrm{d}\mathcal{C}^{(g)}}_{\text{geometry}} + \underbrace{W^{(m)}}_{\text{matter}} = 0$$

addition of matter "deforms" $\mathcal{C}^{(g)}$ in generic dilaton gravity

Thus, matter has to "deform" the level k

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But this is precisely what happens: the addition of matter generically changes the central charge and hence the level k

Similar interpretation in *V.A. Kazakov and A.A. Tseytlin,* hep-th/0104138 to explain Hawking temperature of the ESBH

$$T_H = \frac{b}{2\pi}\sqrt{1 - \frac{2}{k}}$$

10. Open questions

- Supplementary thermodynamical considerations (free energy, entropy, alternative mass definition)

- Microstates?

- Logarithmic corrections from thermal fluctuations (negative sign!)

- Understand duality $AdS_2 \leftrightarrow$ Schwarzschild$_5$

- Supersymmetrization

- Coupling to matter! (critical collapse, tachyon, 2D type 0A/0B strings, quantization)