Near Horizon Soft Hairs as Black Hole Microstates

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Two simple punchlines

1. Heisenberg algebra

$$[X_n,\, P_m] = i\,\delta_{n,\,m}$$

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2. Black hole microstates identified as specific "soft hair" descendants

based on work with

- **Hamid Afshar** [IPM Teheran]
- Stephane Detournay [ULB]
- Wout Merbis [TU Wien]
- Blagoje Oblak [ULB]
- Alfredo Perez [CECS Valdivia]
- Stefan Prohazka [TU Wien]
- **Shahin Sheikh-Jabbari** [IPM Teheran]
- David Tempo [CECS Valdivia]
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Outline

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Motivation

Near horizon boundary conditions

Explicit construction of BTZ microstates

Discussion

Black hole microstates

Bekenstein–Hawking

$$S_{\rm BH} = \frac{A}{4G_N}$$

- Motivation: microscopic understanding of generic black hole entropy

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- Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...)

warped CFT: Detournay, Hartman, Hofman '12
Galilean CFT: Bagchi, Detournay, Fareghbal, Simon '13; Barnich '13

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- Main idea: consider near horizon symmetries for non-extremal horizons

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- Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...)
- Main idea: consider near horizon symmetries for non-extremal horizons
- Near horizon line-element with Rindler acceleration a:

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

Meaning of coordinates:

- ρ: radial direction ($\rho = 0$ is horizon)
- $\varphi \sim \varphi + 2\pi$: angular direction
- v: (advanced) time

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  suggestion in `1511.08687`

  We make this choice in this talk!

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- Work in 3d Einstein gravity in Chern–Simons formulation

$$I_{\text{CS}} = \pm \sum_{\pm} \frac{k}{4\pi} \int \langle A^{\pm} \wedge \mathrm{d}A^{\pm} + \tfrac{2}{3} A^{\pm} \wedge A^{\pm} \wedge A^{\pm} \rangle$$

with $sl(2)$ connections $A^{\pm}$ and $k = \ell/(4G_N)$ with AdS radius $\ell = 1$

Outline

Diagonal gauge

Standard trick: partially fix gauge

$$A^{\pm} = b_{\pm}^{-1}(\rho)\,\big(\mathrm{d}+\mathfrak{a}_{\pm}(x^0,\, x^1)\big)b_{\pm}(\rho)$$

with some group element $b \in SL(2)$ depending on radius ρ with $\delta b = 0$

Drop $\pm$ decorations in most of talk

Manifold topologically a cylinder or torus, with radial coordinate ρ and boundary coordinates $(x^0, x^1) \sim (v, \varphi)$

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- Precise boundary conditions (ζ: chemical potential):

$$\mathfrak{a} = (\mathcal{J}\;\mathrm{d}\varphi + \zeta\;\mathrm{d}v)\;L_0 \qquad \delta\mathfrak{a} = \delta\mathcal{J}\;\mathrm{d}\varphi\,L_0$$

  and $b = \exp\left(\frac{1}{\zeta}\,L_+\right)\cdot\exp\left(\frac{\rho}{2}\,L_-\right)$. (assume constant ζ for simplicity)

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state-dependent functions $\mathcal{J}^\pm = \gamma \pm \omega$, chemical potentials $\zeta^\pm = -a \pm \Omega$

For simplicity set $\Omega = 0$ and $a = \text{const.}$ in metric above

EOM imply $\partial_v \mathcal{J}^\pm = \pm\partial_\varphi \zeta^\pm$; in this case $\partial_v \mathcal{J}^\pm = 0$

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- Recover desired near horizon metric

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- $\gamma = \gamma(\varphi)$: "black flower"

Canonical boundary charges

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- Zero mode charges: mass and angular momentum

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Background independent result for Chern–Simons yields

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Meaningful near horizon boundary conditions and non-trivial theory!

Near horizon symmetry algebra

- Near horizon symmetry algebra = all near horizon boundary conditions preserving trafos, modulo trivial gauge trafos

  Most general trafo

  $$\delta_\epsilon \mathfrak{a} = \mathrm{d}\epsilon + [\mathfrak{a},\, \epsilon] = \mathcal{O}(\delta\mathfrak{a})$$

  that preserves our boundary conditions for constant ζ given by

  $$\epsilon = \epsilon^+ L_+ + \eta L_0 + \epsilon^- L_-$$

  with

  $$\partial_v \eta = 0$$

  implying

  $$\delta_\epsilon \mathcal{J} = \partial_\varphi \eta$$

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- Expand charges in Fourier modes

$$J_n^{\pm} = \frac{k}{4\pi} \oint \mathrm{d}\varphi \, e^{in\varphi} \mathcal{J}^{\pm}(\varphi)$$

What should we expect?

  - Virasoro? (spacetime is locally AdS_3)
  - BMS_3? (Rindler boundary similar to scri)
  - warped conformal algebra? (this is what we found for Rindleresque holography and what Donnay, Giribet, Gonzalez, Pino found in their near horizon analysis)

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$$[J_n^\pm, J_m^\pm] = \pm\tfrac{1}{2} kn\delta_{n+m,\,0} \qquad [J_n^+, J_m^-] = 0$$

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- Map

$$P_0 = J_0^+ + J_0^- \qquad P_n = \tfrac{i}{kn}\,(J_{-n}^+ + J_{-n}^-) \text{ if } n \neq 0 \qquad X_n = J_n^+ - J_n^-$$

  yields Heisenberg algebra (with Casimirs X_0, P_0)

$$[X_n,\, X_m] = [P_n,\, P_m] = [X_0,\, P_n] = [P_0,\, X_n] = 0$$

$$[X_n,\, P_m] = i\delta_{n,m} \quad \text{if } n \neq 0$$

Map to asymptotic variables

- Usual asymptotic AdS_3 connection with chemical potential μ:

$$\hat{A} = \hat{b}^{-1}\big(\mathrm{d} + \hat{\mathfrak{a}}\big)\hat{b} \qquad \hat{\mathfrak{a}}_\varphi = L_+ - \tfrac{1}{2}\,\mathcal{L}\,L_-$$

$$\hat{b} = e^{\rho L_0} \quad \hat{\mathfrak{a}}_t = \mu L_+ - \mu' L_0 + \big(\tfrac{1}{2}\,\mu'' - \tfrac{1}{2}\,\mathcal{L}\mu\big)\,L_-$$

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- Get Virasoro with non-zero central charge $\delta\mathcal{L} = 2\mathcal{L}\varepsilon' + \mathcal{L}'\varepsilon - \varepsilon'''$

Remarks on asymptotic and near horizon variables

- Asymptotic spin-2 currents fulfill Virasoro algebra, but charges obey still Heisenberg algebra

$$\delta Q = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\varepsilon\,\delta\mathcal{L} = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\eta\,\delta\mathcal{J}$$

Reason: asymptotic "chemical potentials" μ depend on near horizon charges $\mathcal{J}$ and chemical potentials ζ

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  Solved automatically from map to asymptotic observables; reminder:

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Near horizon boundary conditions natural for near horizon observer

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- Denote "near horizon" generators with calligraphic letters

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- Call this "near horizon symmetry algebra" (note: independent from ℓ)

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- Will exploit this property to provide cut-off on soft hair spectrum!

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Proposed map between near horizon and asymptotic generators

- Suggestive proposal (see Bañados 9811162)

$$cL_0^\pm = \mathcal{L}_0^\pm - \frac{1}{24}$$

Note: in 9811162 whole Virasoro algebras are mapped, so even for $n \neq 0$

$$cL_n^\pm = \mathcal{L}_{nc}^\pm$$

we use much weaker map above between zero modes as ansatz

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- Microstates = all states in near horizon Hilbert space obeying equations above

Horizon fluffs as microstates

We are now ready to identify all BTZ microstates

- Vector space $\mathcal{V}_{\mathcal{B}}$ of BTZ microstates defined by

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- Agrees with Bekenstein–Hawking and Cardy formula

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Generalization to four dimensions

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Compare with near horizon construction of Donnay, Giribet, Gonzalez, Pino '15

- Near horizon algebra similar to but different from BT-BMS$_4$:

$$[\mathcal{Y}_n^\pm,\, \mathcal{Y}_m^\pm] = (n-m)\,\mathcal{Y}_{n+m}^\pm$$
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- Making AKVs in DGGP state-dependent to leading order relates their canonical boundary charges to Heisenberg boundary charges

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- Near horizon algebra similar to but different from BT-BMS$_4$:

$$[\mathcal{Y}^\pm_n,\, \mathcal{Y}^\pm_m] = (n-m)\,\mathcal{Y}^\pm_{n+m}$$
$$[\mathcal{Y}^+_l,\, \mathcal{T}_{(n,m)}] = -n\,\mathcal{T}_{(n+l,\,m)}$$
$$[\mathcal{Y}^-_l,\, \mathcal{T}_{(n,m)}] = -m\,\mathcal{T}_{(n,\,m+l)}$$

- Intriguing algebraic observation: introducing again

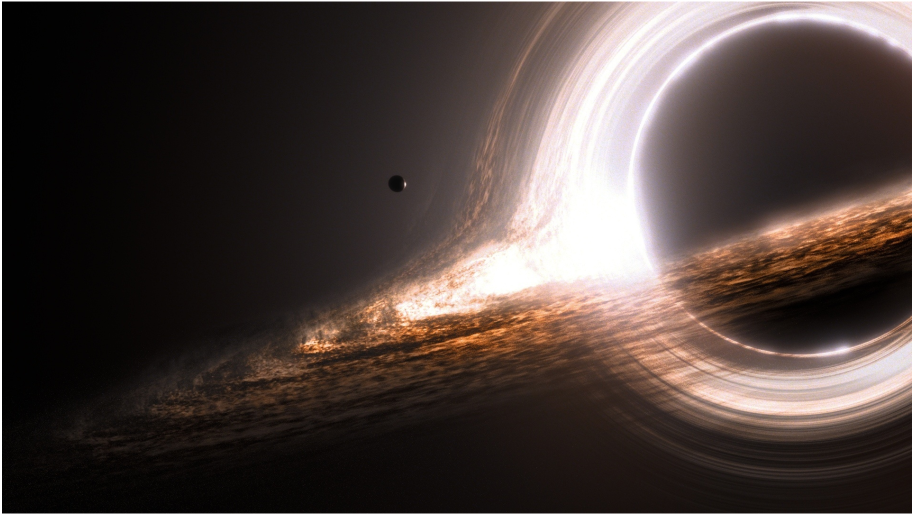
$$[\mathcal{J}^\pm_n,\, \mathcal{J}^\pm_m] = \tfrac{1}{2}\,n\,\delta_{n,-m} = -[\mathcal{K}^\pm_n,\, \mathcal{K}^\pm_m]$$

  recovers 4d algebra above by "Sugawara construction"

$$\mathcal{T}_{(n,\,m)} = \big(\mathcal{J}^+_n + \mathcal{K}^+_n\big)\big(\mathcal{J}^-_m + \mathcal{K}^-_m\big)$$
$$\mathcal{Y}^\pm_n = \sum_{p\in\mathbb{Z}}\big(\mathcal{J}^\pm_{n-p} + \mathcal{K}^\pm_{n-p}\big)\big(\mathcal{J}^\pm_p - \mathcal{K}^\pm_p\big)$$

- Making AKVs in DGGP state-dependent to leading order relates their canonical boundary charges to Heisenberg boundary charges
- Highly non-trivial indication for existence of soft Heisenberg hair in 4d

Microstates of non-extremal Kerr?



Main challenge: how to provide (controlled) cut-off on soft hair spectrum in four dimensions?

Thanks for your attention!

- H. Afshar, D. Grumiller and M.M. Sheikh-Jabbari "Near Horizon Soft Hairs as Microstates of Three Dimensional Black Holes," 1607.00009.
- H. Afshar, S. Detournay, D. Grumiller, W. Merbis, A. Perez, D. Tempo and R. Troncoso "Soft Heisenberg hair on black holes in three dimensions," Phys.Rev. **D93** (2016) 101503(R); 1603.04824.

Thanks to Bob McNees for providing the LaTeX beamerclass!

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- Spectral flow and discrete conic spaces generated by $\mathcal{J}_r^\pm$ ($r = 1, 2, \dots c - 1$), the "horizon fluffs"

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- Mismatch in coefficients; not sure yet if bug or feature