3D quantum gravity?

Daniel Grumiller

Center for Theoretical Physics
Massachusetts Institute of Technology
and
Institute for Theoretical Physics
Vienna University of Technology

Mitchell Institute for Fundamental Physics, Texas A & M,
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MARIE CURIE ACTIONS

with R. Jackiw and N. Johansson: 0805.2610, 0806.4185, 0808.2575, ...

Outline

Why 3D?

Which 3D theory?

How to quantize 3D gravity?

What next?

Outline

Quantum gravity

The Holy Grail of theoretical physics

There is a lot we do know about quantum gravity already

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- Non-perturbative understanding of quantum gravity?

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- Experimental signatures? Data?

Gravity in lower dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
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For a review see `DG & Meyer` and Refs. therein

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> - 3D: lowest dimension exhibiting BHs and gravitons

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> - 3D: lowest dimension exhibiting BHs and gravitons
> - Study gravity in 3D!

Outline

Attempt 1: Einstein–Hilbert

As simple as possible... but not simpler!

Let us start with the simplest attempt. `Einstein-Hilbert` action:

$$I_{\rm EH}=\frac{1}{16\pi\,G}\int \mathrm{d}^3x\sqrt{-g}\,R$$

Equations of motion:

$$R_{\mu\nu}=0$$

Ricci-flat and therefore Riemann-flat – locally trivial!

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Properties of `Einstein-Hilbert`

- No gravitons (recall: in D dimensions $D(D-3)/2$ gravitons)
- No BHs
- `Einstein-Hilbert` in 3D is too simple for us!

Attempt 2: Topologically massive gravity

Deser, Jackiw and Templeton found a way to introduce gravitons!

Let us now add a gravitational Chern–Simons term. TMG action:

$$I_{\text{TMG}} = I_{\text{EH}} + \frac{1}{16\pi\, G} \int \mathrm{d}^3x\sqrt{-g}\, \frac{1}{2\mu}\, \varepsilon^{\lambda\mu\nu}\, \Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu \Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\, \Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)$$

Equations of motion:

$$R_{\mu\nu} + \frac{1}{\mu}\, C_{\mu\nu} = 0$$

with the Cotton tensor defined as

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Properties of TMG

- Gravitons! Reason: third derivatives in Cotton tensor!
- No BHs
- TMG is slightly too simple for us!

Attempt 3: Einstein–Hilbert–AdS

Bañados, Teitelboim and Zanelli (and Henneaux) taught us how to get 3D BHs

Add negative cosmological constant to `Einstein-Hilbert` action:

$$I_{\Lambda\text{EH}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x\sqrt{-g}\,\big(R + \frac{2}{\ell^2}\big)$$

Equations of motion:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}R - \frac{1}{\ell^2}\,g_{\mu\nu} = 0$$

Particular solutions: BTZ BH with line-element

$$\mathrm{d}s^2_{\text{BTZ}} = -\frac{(r^2-r_+^2)(r^2-r_-^2)}{\ell^2 r^2}\,\mathrm{d}t^2 + \frac{\ell^2 r^2}{(r^2-r_+^2)(r^2-r_-^2)}\,\mathrm{d}r^2 + r^2\,\big(\mathrm{d}\phi - \frac{r_+ r_-}{\ell r^2}\,\mathrm{d}t\big)^2$$

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Properties of `Einstein-Hilbert-AdS`

- No gravitons
- Rotating BH solutions that asymptote to AdS_3!
- Adding a negative cosmological constant produces BH solutions!

Cosmological topologically massive gravity

CTMG is a 3D theory with BHs and gravitons!

We want a 3D theory with gravitons and BHs and therefore take CTMG action

$$I_{\rm CTMG} = \frac{1}{16\pi\, G}\, \int \mathrm{d}^3x\sqrt{-g}\, \Big[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\, \varepsilon^{\lambda\mu\nu}\, \Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)\Big]$$

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Properties of CTMG

- Gravitons!
- BHs!
- CTMG is just perfect for us. Study this theory!

Einstein sector of the classical theory

Solutions of Einstein's equations

$$G_{\mu\nu} = 0 \qquad \leftrightarrow \qquad R = -\frac{6}{\ell^2}$$

also have vanishing Cotton tensor

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and therefore are solutions of CTMG.

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Line-element of pure AdS:

$$\mathrm{d}s^2_{\mathrm{AdS}} = \bar{g}_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \ell^2\big(-\cosh^2\rho\,\mathrm{d}\tau^2 + \sinh^2\rho\,\mathrm{d}\phi^2 + \mathrm{d}\rho^2\big)$$

Isometry group: $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$
Useful to introduce light-cone coordinates $u = \tau + \phi$, $v = \tau - \phi$

AdS$_3$-algebra of Killing vectors

A technical reminder

The $SL(2,\mathbb{R})_L$ generators

$$L_0 = i\partial_u$$

$$L_{\pm 1} = ie^{\pm iu}\left[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_u - \frac{1}{\sinh 2\rho}\partial_v \mp \frac{i}{2}\,\partial_\rho\right]$$

obey the algebra

$$[L_0, L_{\pm 1}] = \mp L_{\pm 1}\,, \qquad [L_1, L_{-1}] = 2L_0$$

and have the quadratic Casimir

$$L^2 = \frac{1}{2}(L_1 L_{-1} + L_{-1} L_1) - L_0^2$$

The $SL(2,\mathbb{R})_R$ generators $\bar{L}_0, \bar{L}_{\pm 1}$ obey same algebra, but with

$$u \leftrightarrow v\,, \qquad L \leftrightarrow \bar{L}$$

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Solutions of CTMG with

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Few exact solutions of this type are known.

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Perhaps most interesting solution:

- Warped AdS (stretched/squashed), see `Bengtsson & Sandin`

Line-element of space-like warped AdS:

$$\mathrm{d}s^2_{\text{warped AdS}} = \frac{\ell^2}{\nu^2+3}\left(-\cosh^2\rho\,\mathrm{d}\tau^2 + \frac{4\nu^2}{\nu^2+3}\,(\mathrm{d}u + \sinh\rho\,\mathrm{d}\tau)^2 + \mathrm{d}\rho^2\right)$$

Sidenote: null-warped AdS in holographic duals of cold atoms:

$$\mathrm{d}s^2_{\text{null warped AdS}} = \ell^2\left(\frac{dy^2 + 2\,\mathrm{d}x^+\,\mathrm{d}x^-}{y^2} \pm \frac{(\mathrm{d}x^-)^2}{y^4}\right)$$

CTMG as particle mechanics problem

Stationary and axi-symmetric solutions

Stationarity plus axi-symmetry:

- Two commuting Killing vectors

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Reduced action (`Clement`):

$$I_{\rm C}[\zeta, X^i] \sim \int {\rm d}\rho \left[\frac{\zeta}{2}\,\dot{X}^i\dot{X}^j\eta_{ij} - \frac{2}{\zeta\ell^2} + \frac{\zeta^2}{2\mu}\,\epsilon_{ijk}\,X^i\dot{X}^j\ddot{X}^k\right]$$

Here ζ is a Lagrange-multiplier and $X^i = (T, X, Y)$ a Lorentzian 3-vector

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It could be rewarding to investigate this mechanical problem systematically and numerically!

CTMG at the chiral point
...abbreviated as CCTMG

Definition: CTMG at the chiral point is CTMG with the tuning

$$\mu \, \ell = 1$$

between the cosmological constant and the Chern–Simons coupling.

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Why special?
Calculating the central charges of the dual boundary CFT yields

$$c_L = \frac{3}{2G}\left(1 - \frac{1}{\mu\,\ell}\right), \qquad c_R = \frac{3}{2G}\left(1 + \frac{1}{\mu\,\ell}\right)$$

Thus, at the chiral point we get

$$c_L = 0\,, \qquad c_R = \frac{3}{G}$$

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Notes:

- Abbreviate "CTMG at the chiral point" as CCTMG
- CCTMG is also known as "chiral gravity"

Gravitons around AdS_3 in CTMG

Linearization around AdS background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

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leads to linearized EOM that are third order PDE

$$G^{(1)}_{\mu\nu} + \frac{1}{\mu}\, C^{(1)}_{\mu\nu} = (\mathcal{D}^R \mathcal{D}^L \mathcal{D}^M h)_{\mu\nu} = 0 \tag{1}$$

with three mutually commuting first order operators

$$(\mathcal{D}^{L/R})_\mu{}^\nu = \delta_\mu^\nu \pm \ell\, \varepsilon_\mu{}^{\alpha\nu} \bar{\nabla}_\alpha\,, \qquad (\mathcal{D}^M)_\mu{}^\nu = \delta_\mu^\nu + \frac{1}{\mu} \varepsilon_\mu{}^{\alpha\nu} \bar{\nabla}_\alpha$$

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Three linearly independent solutions to (1):

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At chiral point left (L) and massive (M) branches coincide!

Degeneracy at the chiral point

Will be quite important later!

`Li, Song & Strominger` found all solutions of linearized EOM.

- Primaries: $L_0, \bar{L}_0$ eigenstates $\psi^{L/R/M}$ with

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$$h_{\mu\nu} = \mathrm{Re}\, \psi_{\mu\nu}$$

- At chiral point: L and M branches degenerate. Get new solution (DG & Johansson)

$$\psi_{\mu\nu}^{\text{new}} = \lim_{\mu\ell \to 1} \frac{\psi_{\mu\nu}^{M}(\mu\ell) - \psi_{\mu\nu}^{L}}{\mu\ell - 1}$$

with property

$$\big(\mathcal{D}^L \psi^{\text{new}}\big)_{\mu\nu} = \big(\mathcal{D}^M \psi^{\text{new}}\big)_{\mu\nu} \neq 0\,, \qquad \big((\mathcal{D}^L)^2 \psi^{\text{new}}\big)_{\mu\nu} = 0$$

Sign oder nicht sign?

That is the question. Choosing between Skylla and Charybdis.

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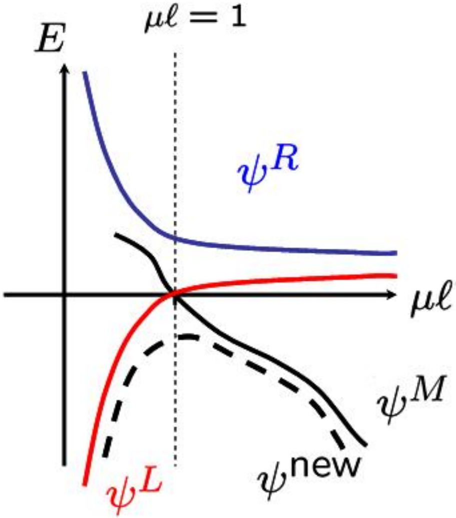
- Either way need a mechanism to eliminate unwanted negative energy objects – either the gravitons or the BHs
- Even at chiral point the problem persists because of the logarithmic mode. See Figure. (Figure: thanks to `N. Johansson`)

Energy for all branches:



Outline

Witten's attempt

Different approach (without gravitons!):

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- Naive remark 1: 3D gravity is trivial

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Different approach (without gravitons!):

- Naive remark 1: 3D gravity is trivial
- Naive remark 2: 3D gravity is non-renormalizable
- Synthesis of naive remarks: 3D quantum gravity may exist as non-trivial theory

Witten's attempt

Different approach (without gravitons!):

- Naive remark 1: 3D gravity is trivial
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Various suggestions to interpret this problem: need cosmic strings, need sum over complex geometries, 3D quantum gravity does not exist by itself

Li, Song & Strominger attempt

Is CCTMG dual to a chiral CFT?

Interesting observations:

1. If left-moving sector is trivial, $Z_L = 1$, then problem of holomorphic factorization
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But:

Disagrees with results by `Carlip, Deser, Waldron & Wise`!

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Such a Jordan form of $L_0, \bar{L}_0$ is defining property of a logarithmic CFT!

Note: called "logarithmic CFT" because some correlators take the form

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- Currently unknown which of these alternatives is realized!

Viability of the logarithmic mode, part 1

Explicit solution for logarithmic mode (DG & Johansson)

Is the logarithmic mode really there?

Collect in the following suggestions how the logarithmic mode could drop out of the physical spectrum and show that none of them is realized.

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Before starting, here is the explicit form of the logarithmic mode:

$$h_{\mu\nu}^{\text{new}} = \frac{\sinh\rho}{\cosh^3\rho}\left(c\,\tau - s\ln\cosh\rho\right)\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}_{\mu\nu} - \tanh^2\rho\left(s\,\tau + c\ln\cosh\rho\right)\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -a^2 \end{pmatrix}_{\mu\nu} \tag{2}$$

with

$$c = \cos(2u)\,, \qquad s = \sin(2u)\,, \qquad a = \frac{1}{\sinh\rho\,\cosh\rho}$$

Viability of the logarithmic mode, part 2

Physical mode with negative energy

Suggestion 1

The logarithmic mode is pure gauge?

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The logarithmic mode is pure gauge? No!

h^{new} does not solve linearized Einstein equations. Thus is not pure gauge.
Note: confirmed by `Sachs` who considered logarithmic quasi-normal modes

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Suggestion 2

Logarithmic mode has infinite energy and thus must be discarded? No!

$$E^{\text{new}} = -\frac{47}{1152 G\,\ell^3}$$

Energy is finite and negative.
Thus logarithmic mode leads to instability but cannot be discarded.

Viability of the logarithmic mode, part 3

Boundary conditions beyond Brown–Henneaux

Suggestion 3

New mode is not a small perturbation?

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New mode is not a small perturbation? It is!

h^{new} diverges asymptotically like ρ, but AdS background diverges asymptotically like $e^{2\rho}$. Thus h^{new} is really a small perturbation.

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Solution is asymptotically AdS

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + \left(\gamma_{ij}^{(0)} e^{2\rho/\ell} + \gamma_{ij}^{(1)} \rho + \gamma_{ij}^{(2)} + \gamma_{ij}^{(4)} e^{-2\rho/\ell} + \dots\right) \mathrm{d}x^i \, \mathrm{d}x^j$$

but violates `Brown-Henneaux` boundary conditions! $(\gamma_{ij}^{(1)}\big|_{\text{BH}} = 0)$

`Henneaux et al.` showed precedents where this may happen in 3D

New boundary conditions replacing `Brown-Henneaux` (`DG & Johansson`)

Viability of the logarithmic mode, part 4

Brown–York boundary stress tensor

Suggestion 5

New mode leads to ill-defined `Brown-York` boundary stress tensor?

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Total action including boundary terms (Kraus & Larsen)

$$I_{\text{total}} = I_{\text{CTMG}} + \frac{1}{8\pi G} \int \mathrm{d}^2x \sqrt{-\gamma} \left(K - \frac{1}{\ell} \right)$$

Its first variation leads to Brown-York boundary stress-tensor:

$$\delta I_{\text{total}} \big|_{\text{EOM}} = \frac{1}{32\pi G} \int \mathrm{d}^2x \sqrt{-\gamma^{(0)}}\, T^{ij}\, \delta\gamma_{ij}^{(0)}$$

DG & Johansson: T_{ij} is finite, traceless and chiral:

$$T_{ij} = -\frac{\ell}{16\pi\, G} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}_{ij}$$

Note: coincides with Brown-York boundary stress-tensor of global AdS_3

Viability of the logarithmic mode, part 5

Artifact of linearization?

Suggestion 6

Maybe some non-linear "magic" kills the new mode?

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DG, Jackiw & Johansson: classical phase space analysis of CCTMG

$$N = \frac{1}{2}\left(2 \times D - 2 \times N_1 - N_2\right) = \frac{1}{2}\left(2 \times 18 - 2 \times 14 - 6\right) = 1$$

- N: number of physical degrees of freedom (per point)
- D: number of canonical pairs in full phase space
- $N_{1(2)}$: number of linearly independent first (second) class constraints

confirmed by Carlip (see also Gibbons, Pope & Sezgin)

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- Conclusion 1: logarithmic mode passed all tests so far
- Conclusion 2: CCTMG is unstable; dual CFT probably logarithmic

Outline

Why 3D?

Which 3D theory?

How to quantize 3D gravity?

What next?

Chiral vs. logarithmic

Pivotal open question: does dual CFT exist? is it chiral or logarithmic?

To Do

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- Need different mechanism of truncation!

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ad logarithmic:

- straightforward but somewhat lengthy calculation
- expand metric around AdS background up to second order:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h^{\text{new}}_{\mu\nu} + h^{(2)}_{\mu\nu}$$

  EOM lead to linear PDE for $h^{(2)}_{\mu\nu}$:

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- Might be rewarding exercise for a student

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Two observations:

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Strominger et al. :

Suggestive to consider warped AdS as possible groundstate of (C)CTMG

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Thank you for your attention!

Some recent literature on cosmological topologically massive gravity

- W. Li, W. Song and A. Strominger, JHEP **0804** (2008) 082, 0801.4566.
- D. Grumiller and N. Johansson, JHEP **0807** (2008) 134, 0805.2610.
- S. Carlip, S. Deser, A. Waldron and D. Wise, Phys.Lett. **B666** (2008) 272, 0807.0486
- G.W. Gibbons, C. N. Pope and E. Sezgin, Class.Quant.Grav. **25** (2008) 205005, 0807.2613.
- G. Giribet, M. Kleban and M. Porrati, JHEP **0810** (2008) 045, 0807.4703.
- S. Carlip, S. Deser, A. Waldron and D. K. Wise, 0803.3998.
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- S. Carlip, 0807.4152
- A. Strominger, 0808.0506.
- D. Grumiller and N. Johansson, 0808.2575.

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