Gravity and holography in lower dimensions

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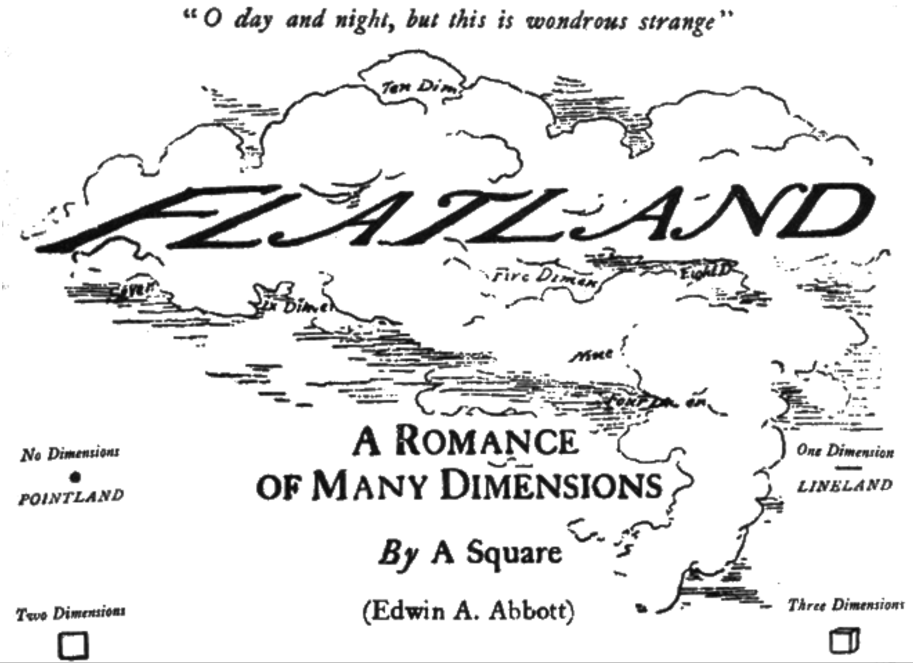
“O day and night, but this is wondrous strange”
FLATLAND
A ROMANCE
OF MANY DIMENSIONS
By A Square
(Edwin A. Abbott)
No Dimensions
POINTLAND
One Dimension
LINELAND
Two Dimensions
Three Dimensions

Outline

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Some open issues in gravity

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  - soft physics
  - near horizon symmetries

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- all issues above can be addressed in lower dimensions
- lower dimensions technically simpler
- hope to find “hydrogen atom” of quantum gravity

Gravity in various dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
- 5D: 50 (35 Weyl and 15 Ricci)
- 4D: 20 (10 Weyl and 10 Ricci)

Caveat: just counting tensor components can be misleading as measure of complexity

Example: large D limit actually simple for some problems (Emparan et al.)

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- 2D: 1 (Ricci scalar)
- 1D: 0 (space or time but not both $\Rightarrow$ no lightcones)

Apply as mantra the slogan "as simple as possible, but not simpler"

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- 3D: lowest dimension exhibiting BHs and gravitons*
- Simplest gravitational theories with BHs and gravitons in 3D

* at least off-shell; in higher derivative theories also on-shell

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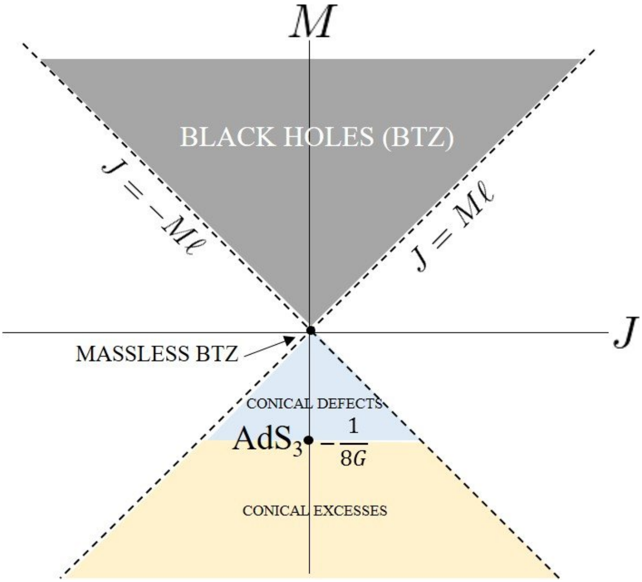
Spectrum of BTZ black holes and related physical states



Choice of theory

- Choice of bulk action
  Pick Einstein–Hilbert action with negative cc ($\Lambda = -1/\ell^2$)

$$I_{\text{EH}}[g] = -\frac{1}{16\pi G}\int_{\mathcal{M}} \mathrm{d}^3x\sqrt{-g}\left(R + \frac{2}{\ell^2}\right)$$

  Usually choose also topology of $\mathcal{M}$, e.g. cylinder

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  Main features:

  - no local physical degrees of freedom
  - all solutions locally and asymptotically AdS_3
  - rotating (BTZ) black hole solutions analogous to Kerr

$$\mathrm{d}s^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{\ell^2 r^2}\,\mathrm{d}t^2 + \frac{\ell^2 r^2\,\mathrm{d}r^2}{(r^2 - r_+^2)(r^2 - r_-^2)} + r^2\left(\mathrm{d}\varphi - \frac{r_+ r_-}{\ell r^2}\,\mathrm{d}t\right)^2$$

  - conserved mass $M = (r_+^2 + r_-^2)/\ell^2$ and angular mom. $J = 2r_+r_-/\ell$
  - Bekenstein–Hawking entropy

$$S_{\text{BH}} = \frac{A}{4G} = \frac{\pi r_+}{2G} = 2\pi\sqrt{\frac{c}{6}\,L_0^+} + 2\pi\sqrt{\frac{c}{6}\,L_0^-}$$

Cardy formula with $c = 3\ell/(2G)$ and $L_0^{\pm} = (\ell M \pm J)/(8G)$

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- Choice of formulation
  Pick Cartan formulation ($R^a = \mathrm{d}\omega^a + \frac{1}{2}\,\epsilon^a{}_{bc}\,\omega^b \wedge \omega^c$)

$$I_{\text{EHP}}[e^a,\,\omega^a] = \frac{1}{8\pi G}\int_{\mathcal{M}} \left(e_a \wedge R^a + \frac{1}{6\ell^2}\,\epsilon_{abc}\,e^a \wedge e^b \wedge e^c\right)$$

  e^a: dreibein, $\omega^a = \frac{1}{2}\,\epsilon^a{}_{bc}\,\omega^{bc}$: dualized spin-connection

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Rewrite as gauge theory of Chern–Simons type ($k = \ell/(4G)$)

$$I_{\text{CS}}[A] = \frac{k}{4\pi}\int_{\mathcal{M}} \langle A \wedge \mathrm{d}A + \tfrac{2}{3}\,A \wedge A \wedge A\rangle$$

A: so$(2,2)$ connection (Achucarro, Townsend '86; Witten '88)

$$A = e^a P_a + \omega^a J_a \qquad [P_a,\,P_b] = \epsilon_{ab}{}^c\,J_c = [J_a,\,J_b] \quad [J_a,\,P_b] = \epsilon_{ab}{}^c\,P_c$$

bilinear form: $\langle J_a,\,P_b\rangle = \eta_{ab}$, $\langle J_a,\,J_b\rangle = \langle P_a,\,P_b\rangle = 0$
EOM: $F = \mathrm{d}A + A \wedge A = 0 \Rightarrow$ gauge flat connections!
3d gravity $=$ topological gauge theory

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  Crucial to define theory — yields spectrum of 'edge states'
  Pick whatever suits best to describe relevant physics

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  'holographic' ansatz that often works in Chern–Simons formulation:

$$A = b^{-1}\big(\,\mathrm{d} + a\big)b \qquad b = b(r) \qquad a = a_t(t,\,\varphi)\;\mathrm{d}t + a_\varphi(t,\,\varphi)\;\mathrm{d}\varphi$$

with variations constrained as

$$\delta b = 0 \qquad\qquad \delta a = \mathcal{O}(1)$$

all info about physical state captured by boundary connection a!

group element b describes radial dependence of connection

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> - Goal: apply this to specific set of boundary conditions inspired by near horizon physics
> - Explain first in general how edge states emerge

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Definition of asymptotic symmetries

All boundary condition preserving gauge transformations (bcpgt's) modulo trivial gauge transformations

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Asymptotic symmetries in gravity

- Impose some bc's at (asymptotic or actual) boundary:

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$$\xi^\mu(r_b,\,x^i) = \xi^\mu_{(0)}(r_b,\,x^i) + \text{subleading terms}$$

  $\xi^\mu_{(0)}(r_b,\,x^i)$: generates asymptotic symmetries
  subleading terms: generate trivial diffeos

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Definition of asymptotic symmetry algebra

Lie bracket quotient algebra of asymptotic Killing vectors modulo trivial diffeos

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  time-independent Schrödinger equation:

$$-\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi(x) = E\psi(x)$$

  look for (normalizable) bound state solutions, $E < 0$
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  - Robin bc's

$$\left(\psi + \alpha\psi'\right)\big|_{x=0^+} = 0 \qquad \alpha \in \mathbb{R}^+$$

    lead to one bound state

$$\psi(x)\big|_{x\geq 0} = \sqrt{\frac{2}{\alpha}}\, e^{-x/\alpha}$$

    with energy $E = -1/\alpha^2$, localized exponentially near $x = 0$

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- changing boundary conditions can change physical spectrum
- to distinguish asymptotic symmetries from trivial gauge trafos: either use Noether's second theorem and covariant phase space analysis or perform Hamiltonian analysis in presence of boundaries

  Some references:
  - covariant phase space: Lee, Wald '90, Iyer, Wald '94 and Barnich, Brandt '02
  - review: see Compère, Fiorucci '18 and refs. therein
  - canonical analysis: Arnowitt, Deser, Misner '59, Regge, Teitelboim '74 and Brown, Henneaux '86
  - review: see Bañados, Reyes '16 and refs. therein

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- in Hamiltonian language: gauge generator $G[\epsilon]$ varies as

$$\delta G[\epsilon] = \int_\Sigma (\text{bulk term})\, \epsilon\, \delta\Phi - \int_{\partial\Sigma} (\text{boundary term})\, \epsilon\, \delta\Phi$$

  not functionally differentiable in general (Σ: constant time slice)

  Φ: shorthand for phase space variables
  ϵ: smearing function/parameter of gauge trafos
  δ: arbitrary field variation

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Trivial gauge transformations generated by some ϵ with $Q[\epsilon] = 0$

Simple example: abelian Chern–Simons

- abelian Chern–Simons action (on cylinder)

$$I[A] = \frac{k}{4\pi} \int_{\mathbb{R}\times\Sigma} A \wedge \mathrm{d}A$$

Note: topological QFT with no local physical degrees of freedom

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- choice of bc's

$$\lim_{r\to\infty} A = \mathcal{J}(\varphi)\ \mathrm{d}\varphi + \mu\ \mathrm{d}t \qquad \delta\mathcal{J} = \mathcal{O}(1) \quad \delta\mu = 0$$

preserved by $\epsilon = \eta(\varphi) +$ subleading

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- Fourier modes $J_n \sim \oint \mathcal{J} e^{in\varphi}$ yield $u(1)_k$ current algebra, $i\{J_n,\, J_m\} = \frac{k}{2}\, n\, \delta_{n+m,\,0}$

Edge states

see e.g. Halperin '82, Witten '89, or Balachandran, Chandar, Momen '94

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- boundary charges (if non-trivial) thus generate edge states

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$$|\text{edge}(\{n_i\})\rangle = \prod_{\{n_i>0\}} J_{-n_i}|0\rangle$$

e.g.

$$|\text{edge}(\{1,1,42\})\rangle = J_{-1}^2 J_{-42}|0\rangle$$

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- theories with no local physical degrees of freedom can have edge states! $\Rightarrow$ perhaps cleanest example of holography

Outline

Motivation for near horizon boundary conditions

Old idea by Strominger '97 and Carlip '98

Main idea

Impose existence of non-extremal horizon as boundary condition on state space

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Explicit form of near horizon boundary conditions

See Donnay, Giribet, Gonzalez, Pino '15 and Afshar et al '16

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Postulates of near horizon boundary conditions:

1. Rindler approximation

$$\mathrm{d}s^2 = -\kappa^2 r^2\,\mathrm{d}t^2 + \mathrm{d}r^2 + \Omega_{ab}(t,\,x^c)\,\mathrm{d}x^a\,\mathrm{d}x^b + \dots$$

$r \to 0$: Rindler horizon

κ: surface gravity

Ω_{ab}: metric transversal to horizon

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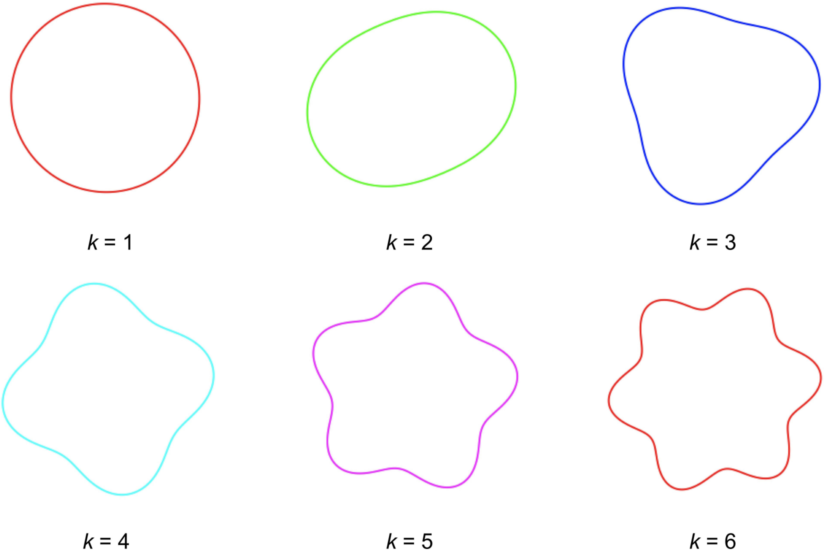
4. Remaining terms fixed by consistency of canonical boundary charges

Black holes can be deformed into black flowers Afshar et al. 16

Horizon can get excited by area preserving shear-deformations



Near horizon symmetries = "asymptotic symmetries" for near horizon bc's

Restrict for the time being to AdS_3 black holes (BTZ)

Simplification in 3d:

$$\mathrm{d}s^2 = \Big[- \kappa^2 r^2 \, \mathrm{d}t^2 + \mathrm{d}r^2 + \gamma^2(\varphi) \, \mathrm{d}\varphi^2 + 2\kappa \, \omega(\varphi) \, r^2 \, \mathrm{d}t \, \mathrm{d}\varphi \Big] \big(1 + \mathcal{O}(r^2)\big)$$

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- Trivial or non-trivial?
  Answer provided by boundary charges!

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- Non-trivial diffeo!
- Canonical analysis yields

$$Q^\pm[\epsilon^\pm] \sim \oint \mathrm{d}\varphi\, \epsilon^\pm(\varphi)\, \big(\gamma(\varphi) \pm \omega(\varphi)\big)$$

where $\epsilon^\pm$ are functions appearing in asymptotic Killing vectors

charge conservation follows from on-shell relations $\partial_t \gamma = 0 = \partial_t \omega$

explains last word in title: γ and ω are hair of black hole

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- Near horizon symmetry algebra Fourier modes $\mathcal{J}_n^\pm = Q^\pm[\epsilon^\pm = e^{in\varphi}]$

$$[\mathcal{J}_n^\pm,\,\mathcal{J}_m^\pm] = \tfrac{1}{2}\,n\,\delta_{n+m,\,0}$$

Two $u(1)$ current algebras! Afshar et al. 16

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$$[\mathcal{J}_n^{\pm},\,\mathcal{J}_m^{\pm}] = \tfrac{1}{2}\,n\,\delta_{n+m,\,0}$$

- Isomorphic to Heisenberg algebras plus center

$$[X_n,\,P_m] = i\,\delta_{n,m} \qquad\qquad [P_0,\,X_n] = 0 = [X_0,\,P_n]$$

$P_0 = \mathcal{J}_0^+ + \mathcal{J}_0^-$, $X_n = \mathcal{J}_n^+ - \mathcal{J}_{-n}^-$, $P_n = 2i/n(\mathcal{J}_{-n}^+ + \mathcal{J}_n^-)$ for $n \neq 0$

Unique features of near horizon boundary conditions

1. All states allowed by bc's have same temperature

   By contrast: asymptotically AdS or flat space bc's allow for black hole states at different masses and hence different temperatures

Unique features of near horizon boundary conditions

1. All states allowed by bc's have same temperature
2. All states allowed by bc's are regular

   (in particular, they have no conical singularities at the horizon in the Euclidean formulation)

   By contrast: for given temperature not all states in theories with asymptotically AdS or flat space bc's are free from conical singularities; usually a unique black hole state is picked

Unique features of near horizon boundary conditions

1. All states allowed by bc's have same temperature
2. All states allowed by bc's are regular

   (in particular, they have no conical singularities at the horizon in the Euclidean formulation)
3. There is a non-trivial reducibility parameter (= Killing vector)

   By contrast: for any other known (non-trivial) bc's there is no vector field that is Killing for all geometries allowed by bc's

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4. Technical feature: in Chern–Simons formulation of 3d gravity simple expressions in diagonal gauge

$$A^{\pm} = b^{\mp 1}\big(\,\mathrm{d}+a^{\pm}\big)\,b^{\pm 1}$$

$$a^{\pm} = L_0\,\Big(\big(\gamma(\varphi) \pm \omega(\varphi)\big)\,\mathrm{d}\varphi + \kappa\,\mathrm{d}t\Big)$$

$$b = \exp\big[\big(L_+ - L_-\big)\,r/2\big]$$

$L_\pm$ are $sl(2,\,\mathbb{R})$ raising/lowering generators
L_0 is $sl(2,\,\mathbb{R})$ Cartan subalgebra generator

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5. Leads to soft Heisenberg hair (see next slides!)

Soft Heisenberg hair for BTZ

- Black flower excitations = hair of black holes
  Algebraically, excitations from descendants

$$|\text{black flower}\rangle \sim \prod_{n_i^\pm > 0} \mathcal{J}^+_{-n_i^+} \mathcal{J}^-_{-n_i^-} |\text{black hole}\rangle$$

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- Near horizon Hamiltonian = boundary charge associated with unit time-translations*

$$H = Q[\partial_t] = \kappa\, P_0$$

commutes with all generators $\mathcal{J}^\pm_n$

* units defined by specifying κ

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Call it "soft Heisenberg hair"

New entropy formula

Express entropy in terms of near horizon charges:

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$$\delta S = \frac{2\pi}{\kappa}\,\delta\big(\kappa P_0\big) \qquad \Rightarrow \qquad T\,\delta S = \delta H$$

with Hawking–Unruh-temperature

$$T = \frac{\kappa}{2\pi}$$

δ refers to any variation of phase space variables allowed by the boundary conditions

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Can we understand entropy law microscopically?

Semi-classical microstates?

Given our soft Heisenberg hair, attack now entropy questions

1. Why only semi-classical input for entropy?
2. What are microstates?
3. Semi-classical construction of microstates?
4. Does counting of microstates reproduce S_{BH}?

Regarding 1. and 3.: may expect decoupling of scales so that description of microstates does not need info about UV completion, but rather only some semi-classical "Bohr-like" input

Evidence for this: universality of BH entropy for large black holes

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- possible resolution: do not consider asymptotic but near horizon observer (i.e., employ near horizon bc's and symmetry algebra)

Fluff proposal (with Afshar, Sheikh-Jabbari '16 and also with Yavartanoo '17)

Semi-classical BTZ black hole microstates as near horizon descendants of vacuum

Highest weight vacuum $|0\rangle$

$$\mathcal{J}_n^{\pm}|0\rangle = 0 \quad \forall n \geq 0$$

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subject to spectral constraint depending on black hole mass M and angular momentum J (measured by asymptotic observer)

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derived from Bohr-type quantization conditions

- quantization of central charge $c = 3/(2G)$ in integers
- quantization of conical deficit angles in integers over c
- black hole/particle correspondence
  (black hole = gas of coherent states of particles on AdS_3)

Outline

Selected list of models

Black holes in (A)dS$_2$, asymptotically flat or arbitrary spaces (Wheeler property)

Model	$U(X)$	$V(X)$
1. Schwarzschild (1916)	$-\frac{1}{2X}$	$-\lambda^2$
2. Jackiw-Teitelboim (1984)	0	ΛX
3. Witten Black Hole (1991)	$-\frac{1}{X}$	$-2b^2 X$
4. CGHS (1992)	0	$-2b^2$
5. $(A)dS_2$ ground state (1994)	$-\frac{a}{X}$	BX
6. Rindler ground state (1996)	$-\frac{a}{X}$	BX^a
7. Black Hole attractor (2003)	0	BX^{-1}
8. Spherically reduced gravity ($N > 3$)	$-\frac{N-3}{(N-2)X}$	$-\lambda^2 X^{(N-4)/(N-2)}$
9. All above: ab-family (1997)	$-\frac{a}{X}$	BX^{a+b}
10. Liouville gravity	a	$be^{\alpha X}$
11. Reissner-Nordström (1916)	$-\frac{1}{2X}$	$-\lambda^2 + \frac{Q^2}{X}$
12. Schwarzschild-$(A)dS$	$-\frac{1}{2X}$	$-\lambda^2 - \ell X$
13. Katanaev-Volovich (1986)	α	$\beta X^2 - \Lambda$
14. BTZ/Achucarro-Ortiz (1993)	0	$\frac{Q^2}{X} - \frac{J}{4X^3} - \Lambda X$
15. KK reduced CS (2003)	0	$\frac{1}{2}X(c - X^2)$
16. KK red. conf. flat (2006)	$-\frac{1}{2}\tanh(X/2)$	$A \sinh X$
17. 2D type 0A string Black Hole	$-\frac{1}{X}$	$-2b^2 X + \frac{b^2 q^2}{8\pi}$
18. exact string Black Hole (2005)	lengthy	lengthy

Choice of theory (review: see hep-th/0204253)

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  Dilaton gravity in two dimensions ($X =$ dilaton):

  $$I[X,\, g_{\mu\nu}] = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \big[XR - U(X)(\nabla X)^2 - 2V(X)\big]$$

  - kinetic potential $U(X)$ and dilaton potential $V(X)$
  - constant dilaton and linear dilaton solutions
  - all solutions known in closed form globally for all choices of potentials
  - simple choice (Jackiw–Teitelboim):

    $$U(X) = 0 \qquad V(X) = \Lambda X$$

  - for negative $\Lambda = -1/\ell^2$ leads to AdS_2 solutions

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- **Choice of bulk action**
  JT model:

$$I_{\rm JT}[X,\,g_{\mu\nu}] = \frac{1}{16\pi\,G_2}\int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\;[XR - 2\Lambda X]$$

- **Choice of formulation**
  Use again Cartan formulation

$$I_{\rm Cartan}[e^a,\,\omega,\,X^a,\,X] = \frac{1}{8\pi G_2}\int_{\mathcal{M}} \big(X^a T_a + XR - \epsilon_{ab}e^a\wedge e^b\,\Lambda X\big)$$

  torsion 2-form $T^a = \mathrm{d}e^a + \epsilon^a{}_b\omega\wedge e^b$ and curvature 2-form $R = \mathrm{d}\omega$

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Rewrite as gauge theory of BF-type ($k = 1/(4G_2)$):

$$I_{\rm BF}[\mathcal{X},\, A] = \frac{k}{2\pi}\int_{\mathcal{M}} \langle \mathcal{X}\, F\rangle$$

$F = \mathrm{d}A + A \wedge A$ with $A \in \mathsf{sl}(2, \mathbb{R})$; co-adjoint scalars $\mathcal{X}$

$A = e^a P_a + \omega J$ with $[P_a,\, J] = \epsilon_a{}^b P_b$ and $[P_a,\, P_b] = \Lambda \epsilon_{ab}\, J$

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  $F = \mathrm{d}A + A\wedge A$ with $A \in \mathsf{sl}(2,\mathbb{R})$; co-adjoint scalars $\mathcal{X}$

- Choice of boundary conditions
  Analogous to AdS_3:

$$A = b^{-1}(\mathrm{d}{+}a)\,b \qquad\qquad \mathcal{X} = b^{-1}\,x\,b$$

  with $b = b(\rho)$, $a = a_\tau(\tau)\,\mathrm{d}\tau$, $x = x(\tau)$, $\delta b = 0$ and $\delta a = \mathcal{O}(1) = \delta x$

Outline

Interlude: SYK in one slide (Kitaev '15; Maldacena, Stanford '16)

Sachdev–Ye–Kitaev model = strongly interacting quantum system solvable at large N (N is number of Majorana fermions ψ^a)

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$$G(\tau) \sim \text{sign}(\tau)/\sin^{2\Delta}(\pi\tau/\beta) \qquad \text{conformal weight } \Delta = 1/4$$

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- effective action at large N and large J: Schwarzian action

$$\Gamma[\tau] \sim -\frac{N}{J}\int\limits_0^\beta \mathrm{d}u\,\left[\dot{\tau}^2 + \tfrac{1}{2}\{\tau;\,u\}\right] \qquad \{\tau;\,u\} = \frac{\dddot{\tau}}{\dot{\tau}} - \frac{3}{2}\frac{\ddot{\tau}^2}{\dot{\tau}^2}$$

Boundary and integrability conditions for JT

See DG, McNees, Salzer, Valcárcel, Vassilevich '17 and González, DG, Salzer '18

- Analogous to Brown–Henneaux bc's in AdS_3:

$$a_\tau = L_1 + \mathcal{L}(\tau)\, L_{-1} \qquad\qquad b = \exp\left(\rho\, L_0\right)$$

L_n: usual sl(2) generators

$$[L_n,\, L_m] = (n-m)\, L_{n+m}$$

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$$a_\tau = f_\tau\, x + g^{-1} \partial_\tau g$$

with $g = \exp\left(-\frac{1}{2}\, y' L_{-1}\right) \exp\left(\ln(y) L_0\right)$ where $f_\tau = 1/y$

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note: boundary action given by

$$I_{\partial\mathcal{M}} \sim \int \mathrm{d}\tau f_\tau\, \mathrm{Tr}\left(x^2\right) \sim \int \mathrm{d}\tau f_\tau\, C$$

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- defining inverse diffeo, $f^{-1}(u) := \tau(u)$ and inserting into Casimir

$$\Gamma|_{F=0}[\tau] = -\frac{k\,\bar{y}}{2\pi}\int_0^\beta \mathrm{d}u\,\left[\dot{\tau}^2\mathcal{L} + \tfrac{1}{2}\,\{\tau;\, u\}\right] \qquad \{\tau;\, u\} = \frac{\dddot{\tau}}{\dot{\tau}} - \frac{3}{2}\,\frac{\ddot{\tau}^2}{\dot{\tau}^2}$$

yields Schwarzian action, with $k \sim N$ and $1/\bar{y} \sim J$

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  - Gauge theory formulation convenient
  - Boundary conditions crucial
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> - Numerous open questions in gravity and holography
> - Many can be addressed in lower dimensions
> - If you are stuck in higher D try $D = 3$ or $D = 2$

Thank you for your attention!