Introduction to black holes in two dimensions

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Outline

1. Gravity in 2D
   - Models in 2D
   - Generic dilaton gravity action
2. Classical and Semi-Classical Black Holes
   - Classical black holes
   - Semi-Classical black holes and thermodynamics
3. Quantum and Virtual Black Holes
   - Path integral quantization
   - S-matrix

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What is a black hole?
Fishy analogy (Bill Unruh)

Above: black hole
(NASA picture)
Left: Waterfall

Analogy:
Infinity $\leftrightarrow$ Lake

Horizon $\leftrightarrow$ Point of no return

Singularity $\leftrightarrow$ Waterfall

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Riemann (Weyl+Ricci): $\frac{N^2(N^2-1)}{12}$ components in N dimensions

- 4D: 20 (10 Weyl and 10 Ricci)
- 5D: 50 (35 Weyl and 15 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
- 11D: 1210 (1144 Weyl and 66 Ricci)
- 3D: 6 (Ricci)
- 2D: 1 (Ricci scalar) $\rightarrow$ Lowest dimension with curvature
- 1D: 0

But: 2D EH $I_{EH} = \kappa \int \mathrm{d}^2x \sqrt{g} R$: no equations of motion!

Number of graviton modes: $\frac{N(N-3)}{2}$

Stuck already in the formulation of the model?

Have to go beyond Einstein-Hilbert in 2D!

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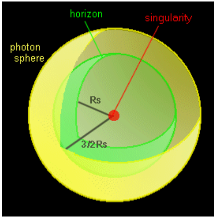
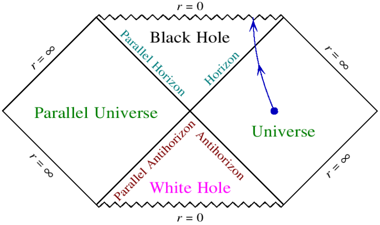
Artistic impression



Spherical symmetry



Carter-Penrose diagram of Schwarzschild

- Spherical symmetry reduces 4D to 2D
- 2D: Time and surface radius
- Exact solution of Einstein equations: Schwarzschild
- Schwarzschild: "Hydrogen atom of General Relativity"
- Quantize in 2D!

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Spherical reduction

Line element adapted to spherical symmetry:

$$ds^2 = \underbrace{g_{\mu\nu}^{(N)}}_{\text{full metric}} dx^\mu\, dx^\nu = \underbrace{g_{\alpha\beta}(x^\gamma)}_{2D \text{ metric}} dx^\alpha\, dx^\beta - \underbrace{\phi^2(x^\alpha)}_{\text{surface area}} d\Omega^2_{S_{N-2}}\,,$$

Insert into N-dimensional EH action $I_{EH} = \kappa \int d^N x \sqrt{-g^{(N)}} R^{(N)}$:

$$I_{EH} = \kappa \underbrace{\frac{2\pi^{(N-1)/2}}{\Gamma(\frac{N-1}{2})}}_{N-2\text{ sphere}} \int d^2x \underbrace{\sqrt{-g}\phi^{N-2}}_{\text{determinant}} \underbrace{\left[R + \frac{(N-2)(N-3)}{\phi^2}\Big((\nabla\phi)^2 - 1\Big)\right]}_{\text{Ricci scalar}}$$

Cosmetic redefinition $X \propto (\lambda\phi)^{N-2}$:

$$I_{EH} \propto \int d^2x \sqrt{-g} \underbrace{\left[XR + \frac{N-3}{(N-2)X}(\nabla X)^2 - \lambda^2 X^{(N-4)/(N-2)}\right]}_{\text{Scalar}-\text{tensor theory a.k.a. dilaton gravity}}$$

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Second order formulation

Similar action arises from string theory, from other kinds of dimensional reduction, from intrinsically 2D considerations, ...
Generic action:

$$I_{2DG} = \kappa \int d^2x \sqrt{-g} \Big[XR + U(X)(\nabla X)^2 - V(X) \Big] \qquad (1)$$

Special case $U = 0, V = X^2$: EOM $R = 2X$

$$I \propto \int d^2x \sqrt{-g} R^2$$

Similarly $f(R)$ Lagrangians related to (1) with $U = 0$
String context: $X = e^{-2\phi}$, with ϕ as string dilaton
Conformal trafo to different model with $\tilde{U}(X) = 0$:
$\tilde{V}(X) = \frac{d}{dX} \underbrace{w(X) := V(X) e^{Q(X)}}_{\text{conformally invariant}}$, with $Q(X) := \int^X dy U(y)$

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Selected list of models

Model	$U(X)$	$V(X)$	$w(X)$
1. Schwarzschild (1916)	$-\frac{1}{2X}$	$-\lambda^2$	$-2\lambda^2\sqrt{X}$
2. Jackiw-Teitelboim (1984)	0	ΛX	$\frac{1}{2}\Lambda X^2$
3. Witten BH (1991)	$-\frac{1}{X}$	$-2b^2X$	$-2b^2X$
4. CGHS (1992)	0	$-2b^2$	$-2b^2X$
5. $(A)dS_2$ ground state (1994)	$-\frac{a}{X}$	BX	$a\neq 2:\ \frac{B}{2-a}X^{2-a}$
6. Rindler ground state (1996)	$-\frac{a}{X}$	BX^a	BX
7. BH attractor (2003)	0	BX^{-1}	$B\ln X$
8. SRG ($N>3$)	$-\frac{N-3}{(N-2)X}$	$-\lambda^2X^{(N-4)/(N-2)}$	$-\lambda^2\frac{N-2}{N-3}X^{(N-3)/(N-2)}$
9. All above: *ab*-family (1997)	$-\frac{a}{X}$	BX^{a+b}	$b\neq -1:\ \frac{B}{b+1}X^{b+1}$
10. Liouville gravity	a	$be^{\alpha X}$	$a\neq-\alpha:\ \frac{b}{a+\alpha}e^{(a+\alpha)X}$
11. Reissner-Nordström (1916)	$-\frac{1}{2X}$	$-\lambda^2+\frac{Q^2}{X}$	$-2\lambda^2\sqrt{X}-2Q^2/\sqrt{X}$
12. Schwarzschild-$(A)dS$	$-\frac{1}{2X}$	$-\lambda^2-\ell X$	$-2\lambda^2\sqrt{X}-\frac{2}{3}\ell X^{3/2}$
13. Katanaev-Volovich (1986)	α	$\beta X^2-\Lambda$	$\int^X e^{\alpha y}(\beta y^2-\Lambda)\,dy$
14. Achucarro-Ortiz (1993)	0	$\frac{Q^2}{X}-\frac{J}{4X^3}-\Lambda X$	$Q^2\ln X+\frac{J}{8X^2}-\frac{1}{2}\Lambda X^2$
15. Scattering trivial (2001)	generic	0	const.
16. KK reduced CS (2003)	0	$\frac{1}{2}X(c-X^2)$	$-\frac{1}{8}(c-X^2)^2$
17. exact string BH (2005)	lengthy	$-\gamma$	$-(1+\sqrt{1+\gamma^2})$
18. Symmetric kink (2005)	generic	$-X\Pi_{i=1}^n(X^2-X_i^2)$	lengthy
19. KK red. conf. flat (2006)	$-\frac{1}{2}\tanh(X/2)$	$A\sinh X$	$4A\cosh(X/2)$
20. 2D type 0A	$-\frac{1}{X}$	$-2b^2X+\frac{b^2q^2}{8\pi}$	$-2b^2X+\frac{b^2q^2}{8\pi}\ln X$

Red: mentioned in abstract Blue: pioneer models

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First order formulation

Gravity as gauge theory (89: Isler, Trugenberger, Chamseddine, Wyler, 91: Verlinde, 92: Cangemi, Jackiw, Achucarro, 93: Ikeda, Izawa, 94: Schaller, Strobl)

Example: Jackiw-Teitelboim model ($U = 0, V = \Lambda X$)

$$[P_a, P_b] = \Lambda \varepsilon_{ab} J \,, \qquad [P_a, J] = \varepsilon_a{}^b P_b \,,$$

Non-abelian BF theory:

$$I_{BF} = \int X_A F^A = \int \Big[X_a \, \mathrm{d}e^a + X_a \varepsilon^a{}_b \omega \wedge e^b + X \, \mathrm{d}\omega + \varepsilon_{ab} e^a \wedge e^b \Lambda X \Big]$$

field strength $F = \mathrm{d}A + [A, A]/2$ contains $SO(1,2)$ connection $A = e^a P_a + \omega J$, coadjoint Lagrange multipliers X_A

Generic first order action:

$$I_{2DG} \propto \int \Big[X_a \underbrace{T^a}_{\text{torsion}} + X \underbrace{R}_{\text{curvature}} + \underbrace{\epsilon}_{\text{volume}} (X^a X_a U(X) + V(X)) \Big] \qquad (2)$$

$T^a = \mathrm{d}e^a + \varepsilon^a{}_b \omega \wedge e^b$, $R = \mathrm{d}\omega$, $\epsilon = \varepsilon_{ab} e^a \wedge e^b$

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Classical solutions

Light-cone components and Eddington-Finkelstein gauge (87: Polyakov, 92: Kummer, Schwarz, 96: Klösch, Strobl)

Constant dilaton vacua:

$$X = \text{const.}\,, \quad V(X) = 0\,, \quad R = V'(X)$$

- Minkowski, Rindler or (A)dS only
- isolated solutions (no constant of motion)

Generic solutions in EF gauge $\omega_0 = e_0^+ = 0$, $e_0^- = 1$:

$$ds^2 = 2e^{Q(X)}\,du\,dX + \underbrace{e^{Q(X)}\big(w(X) + M\big)}_{\text{Killing norm}}\,du^2 \tag{3}$$

- Birkhoff theorem: at least one Killing vector ∂_u
- one constant of motion: mass M
- dilaton is coordinate x^0 (residual gauge trafos!)

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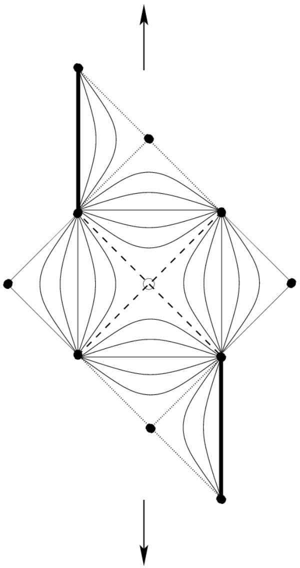
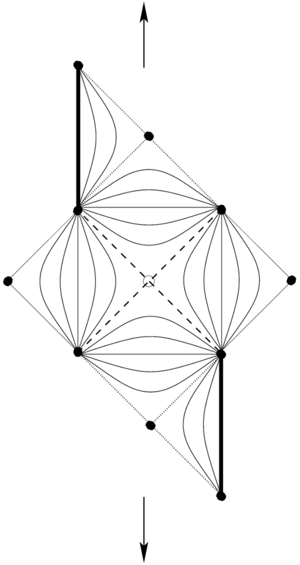


Key ingredient: Killing norm!

- for each zero $w(X) + M = 0$: Killing horizon
- multiple zeros: extremal horizons (BPS)
- glue together basic EF-patches
- caveat: bifurcation points
- check geodesics for (in)completeness
- Simple example: Carter-Penrose diagram on the left: Killing norm $1 - 2M/r + Q^2/r^2$ (RN)

Global structure

Simple algorithm exists to construct all possible global structures (Israel, Walker)



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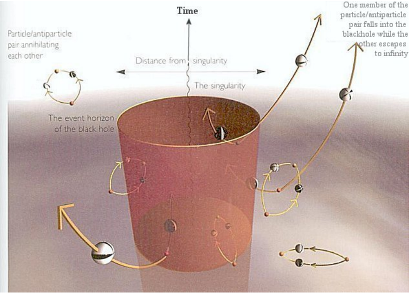

- conformal anomaly
  $< T^\mu{}_\mu > \propto R$
- conservation equation
  $\nabla_\mu < T^{\mu\nu} > = 0$
- boundary conditions (Unruh, Hartle-Hawking, Boulware)
- get flux from trace

2D Stefan-Boltzmann (flux $\propto T_H^2$):

$$T_H = \frac{1}{2\pi} |w'(X)|_{X=X_h} = \text{surface gravity}$$

other thermodynamical quantities of interest:
entropy: X on horizon, specific heat: w'/w'' on horizon

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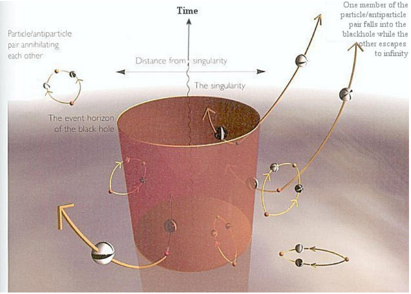
Hawking radiation

Quantization on fixed background; method by Christensen and Fulling



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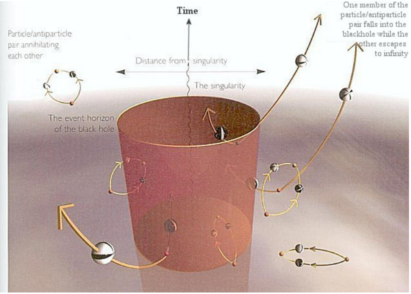

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Thermodynamics from Euclidean path integral

Partition function:

$$Z = \int \mathcal{D}g\mathcal{D}X e^{-\frac{1}{\hbar}\Gamma[g_{\mu\nu},X]}$$

Euclidean action:

$$\Gamma[g_{\mu\nu},X] = I_{\text{bulk}}[g_{\mu\nu},X] + I_{\text{GHY}}[\gamma_{\mu\nu},X] + I_{\text{counter}}[\det(\gamma_{\mu\nu}),X]$$

Analogy in QM:

$$\Gamma[p,q] = \underbrace{\int \mathrm{d}t[-q\dot{p} - H(p,q)]}_{\text{bulk term}} + \underbrace{qp|_{t_i}^{t_f}}_{\text{Gibbons-Hawking-York}} + \underbrace{C(q)|_{t_i}^{t_f}}_{\text{counter term}}$$

- Bulk term: "usual" action
- GHY: boundary conditions
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- Counter term: consistency of path integral

MARIE CURIE ACTIONS

- toy models (2D strings, black hole evaporation, . . .)
- classically: spherically symmetric sector of general relativity (critical collapse)
- semi-classically: near horizon geometry effectively 2D (Carlip, Wilczek, . . .)
- thermodynamics: investigation using the Ruppeiner formalism (Aman, Bengtsson, Pidokrajt, Ruppeiner, . . .)
- solid state analogues: cigar shaped Bose-Einstein condensate as Jackiw-Teitelboim (Fedichev+Fischer); perfect fluid in Laval nozzle (Unruh, Schützhold, Barcelo, Liberati, Visser, Volovik, Cadoni, Mignemi, . . .)
- more speculative ideas: at high energies gravity effectively 2D (Reuter, Ambjorn, Loll)? gravity near the Earth: linear potential, i.e., effectively 2D (Mann, Young)?

instead of these interesting issues focus now on quantum aspects without fixing background

Applications?

The usefulness of Lineland

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Outline

MARIE CURIE ACTIONS

Quantization of specific models

Non-comprehensive history

- 1992: Cangemi, Jackiw (CGHS)
- 1994: Louis-Martinez, Gegenberg, Kunstatter ($U = 0$)
- 1994: Kuchař (Schwarzschild)
- 1995: Cangemi, Jackiw, Zwiebach (CGHS)
- 1997: Kummer, Liebl, Vassilevich (generic geometry)
- 1999: Kummer, Liebl, Vassilevich (minimally coupled scalar, generic geometry)
- 2000-2002: DG, Kummer, Vassilevich (non-minimally coupled scalar, generic geometry)
- 2004: Bergamin, DG, Kummer (minimally coupled matter, generic SUGRA)
- 2006: DG, Meyer (non-minimally coupled fermions, generic geometry)

MARIE CURIE ACTIONS

Non-minimally coupled matter

Prominent example: Einstein-massless Klein-Gordon model (Choptuik)

- no matter: integrability, no scattering, no propagating physical modes
- with matter: no integrability in general, scattering, critical collapse

Massless scalar field S:

$$I_m = \int d^2x \sqrt{-g} F(X) (\nabla S)^2$$

- minimal coupling: $F = \text{const.}$
- non-minimal coupling otherwise
- spherical reduction: $F \propto X$

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MARIE CURIE ACTIONS

Non-perturbative path integral quantization

Integrating out geometry exactly

- constraint analysis $\{G^i(x), G^j(x')\} = G^k C_k{}^{ij} \delta(x - x')$
- BRST charge $\Omega = c^i G_i + c^i c^j C_{ij}{}^k p_k$ (ghosts c^i, p_k)
- gauge fixing fermion to achieve EF gauge

integrating ghost sector yields

$$Z\,[\text{sources}] = \int \mathcal{D}f\,\delta\left(f + i\delta/\delta j_{e_1^+}\right)\tilde{Z}\,[f, \text{sources}]$$

with ($\tilde{S} = S\sqrt{f}$)

$$\tilde{Z}\,[f, \text{sources}] = \int \mathcal{D}\tilde{S}\mathcal{D}(\omega, e^a, X, X^a)\,\det\Delta_{F.P.}\,\exp i(I_{g.f.} + \text{sources})$$

Can integrate over all fields except matter non-perturbatively!

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Can integrate over all fields except matter non-perturbatively!

MARIE CURIE ACTIONS

Non-local effective theory

Convert local gravity theory with matter into non-local matter theory without gravity

Generating functional for Green functions ($F = 1$):

$$\tilde{Z}[f, \text{sources}] = \int \mathcal{D}\tilde{S} \exp i \int (\mathcal{L}^k + \mathcal{L}^v + \mathcal{L}^s) d^2x$$

$$\mathcal{L}^k = \partial_0 S \partial_1 S - E_1^-(\partial_0 S)^2 \,, \; \mathcal{L}^v = -w'(\hat{X}) \,, \; \mathcal{L}^s = \sigma S + j_{e_1^+} \hat{E}_1^+ + \dots \,,$$

$$\tilde{S} = S f^{1/2} \,, \; \hat{E}_1^+ = e^{Q(\hat{X})} \,, \; \hat{X} = \underbrace{a + b x^0}_{X} + \underbrace{\partial_0^{-2}(\partial_0 S)^2}_{\text{non-local}} + \dots \,, \; a = 0 \,, \; b = 1 \,,$$

$$E_1^- = w(X) + M \,, \quad \hat{E}_1^+ = e^{Q(X)} + e^{Q(X)} U(X) \partial_0^{-2} (\partial_0 S)^2 + \dots$$

$$\int \mathcal{D}\tilde{S} \exp i \int \mathcal{L}^k = \exp \underbrace{\left(i/96\pi \int_x \int_y f R_x \Box_{xy}^{-1} R_y \right)}_{\text{Polyakov}}$$

Red: geometry, Magenta: matter, Blue: boundary conditions

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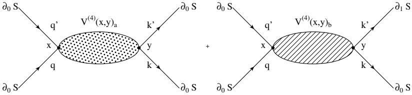
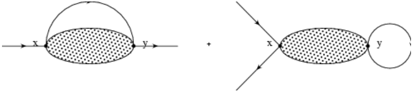
Some Feynman diagrams

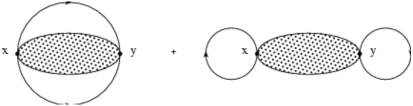
lowest order non-local vertices:



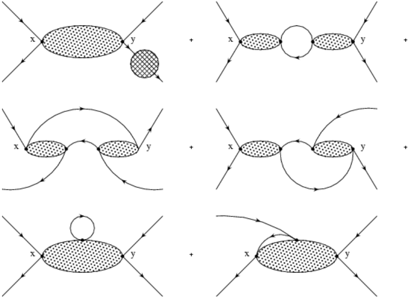
propagator corrections:



vacuum bubbles:



vertex corrections:



- so far: calculated only lowest order vertices and propagator corrections
- partial resummations possible (similar to Bethe-Salpeter)?
- non-local loops vanish to this order

MARIE CURIE ACTIONS

Outline

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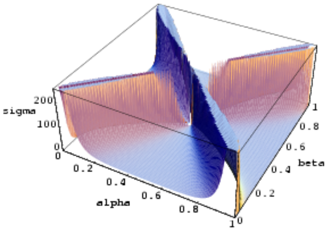
ingoing s-waves $q = \alpha E, q' = (1-\alpha)E$ interact and scatter into outgoing s-waves $k = \beta E, k' = (1-\beta)E$

$$T(q,q';k,k') \propto \tilde{T}\delta(k+k'-q-q')/|kk'qq'|^{3/2} \tag{4a}$$

with $\Pi = (k+k')(k-q)(k'-q)$ and

$$\tilde{T} = \Pi \ln \frac{\Pi^2}{E^6} + \frac{1}{\Pi}\sum_p p^2 \ln \frac{p^2}{E^2} \cdot \left(3kk'qq' - \frac{1}{2}\sum_{r \neq p}\sum_{s \neq r,p} r^2 s^2\right) \tag{4b}$$



- result finite and simple
- monomial scaling with E
- forward scattering poles $\Pi = 0$
- decay of s-waves possible

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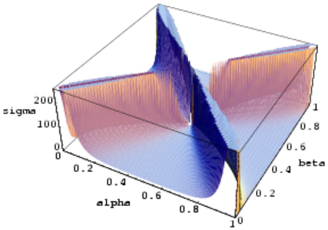

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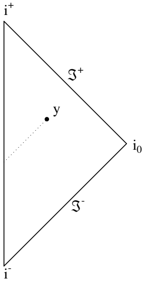
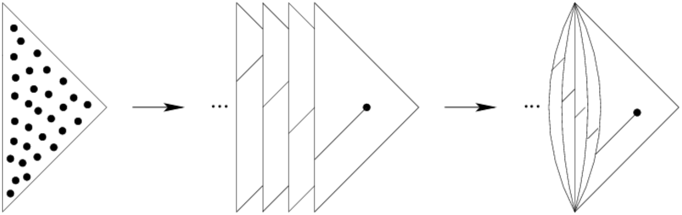
"Intermediate geometry" (caveat: off-shell!):





$$ds^2 = 2\,du\,dr + [1 - \underbrace{\delta(u-u_0)\theta(r_0-r)}_{\text{localized}}(2M/r + ar + d)]\,du^2$$

- Schwarzschild and Rindler terms
- nontrivial part localized
- geometry is non-local (depends on $r, u, \underbrace{r_0, u_0}_{y}$)
- geometry asymptotically fixed (Minkowski)

- J. D. Brown, “LOWER DIMENSIONAL GRAVITY,” World Scientific Singapore (1988).
- A. Strominger, “Les Houches lectures on black holes,” `hep-th/9501071`.
- D. Grumiller, W. Kummer, and D. Vassilevich, “Dilaton gravity in two dimensions,” *Phys. Rept.* **369** (2002) 327–429, `hep-th/0204253`.
- D. Grumiller and R. Meyer, “Ramifications of lineland,” `hep-th/0604049`.