Massive gravity in three dimensions

The $\text{AdS}_3/\text{LCFT}_2$ correspondence

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Outline

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Introduction to 3D gravity

Topologically massive gravity

Logarithmic CFT conjecture

Consequences, Generalizations & Applications

Motivation

- Quantum gravity
  - Address conceptual issues of quantum gravity
  - Black hole evaporation, information loss, black hole microstate counting, virtual black hole production, ...
  - Technically much simpler than 4D or higher D gravity
  - Integrable models: powerful tools in physics (Coulomb problem, Hydrogen atom, harmonic oscillator, ...)
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- Gauge/gravity duality
  - Deeper understanding of black hole holography
  - AdS_3/CFT_2 correspondence best understood
  - Quantum gravity via AdS/CFT? (Witten '07, Li, Song, Strominger '08)
  - Applications to 2D condensed matter systems?
  - Gauge gravity duality beyond standard AdS/CFT: warped AdS, asymptotic Lifshitz, non-relativistic CFTs, logarithmic CFTs, ...

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- Physics
  - Cosmic strings (Deser, Jackiw, 't Hooft '84, '92)
  - Black hole analog systems in condensed matter physics (graphene, BEC, fluids, ...)

Gravity in lower dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
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- 3D: lowest dimension exhibiting BHs and gravitons
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Let us switch off all matter fields and keep only the metric g.

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and the higher derivative Lagrange density

$$\mathcal{L}_{\rm MG}(R_{\mu\nu}) = \sigma R - 2\Lambda + \frac{1}{m^2}\,\big(R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}\,R^2\big) + \mathcal{O}(R^3_{\mu\nu})$$

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Action and equations of motion of topologically massive gravity (TMG)

Consider the action (Deser, Jackiw & Templeton '82)

$$I_{\text{TMG}} = \frac{1}{16\pi\,G}\,\int \mathrm{d}^3x\sqrt{-g}\,\Big[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)\Big]$$

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Equations of motion:

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Some properties of TMG

- Massive gravitons and black holes
- AdS solutions and asymptotic AdS solutions
- warped AdS solutions and warped AdS black holes
- Schrödinger solutions and Schrödinger pp-waves

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Reduced action (Clement '94):

$$I_{\rm C}[e, X^i] \sim \int \mathrm{d}\rho\, e\, \Big[\frac{1}{2}\, e^{-2} \dot{X}^i \dot{X}^j \eta_{ij} - \frac{2}{\ell^2} + \frac{1}{2\mu}\, e^{-3}\, \epsilon_{ijk}\, X^i \dot{X}^j \ddot{X}^k\Big]$$

Here e is the Einbein and $X^i = (T, X, Y)$ a Lorentzian 3-vector

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Classification of solutions:

- Einstein solutions: AdS, BTZ
- warped solutions: warped AdS, warped black holes
- Schrödinger solutions: asymptotic Schrödinger spacetimes, pp-waves
- generic solutions (Ertl, Grumiller & Johansson, '10)

TMG at the chiral point

Definition: TMG at the chiral point is TMG with the tuning

$$\mu \ell = 1$$

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Calculating the central charges of the dual boundary CFT yields

$$c_L = \frac{3\ell}{2G}\,\big(1 - \frac{1}{\mu\,\ell}\big) \qquad c_R = \frac{3\ell}{2G}\,\big(1 + \frac{1}{\mu\,\ell}\big)$$

Thus, at the chiral point we get

$$c_L = 0 \qquad c_R = \frac{3\ell}{G}$$

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- Abbreviate "Cosmological TMG at the chiral point" as CTMG
- CTMG is also known as "chiral gravity"
- Dual CFT: chiral? (conjecture by Li, Song & Strominger '08)
- More adequate name for CTMG: "logarithmic gravity"

Gravitons around AdS_3 in CTMG

Linearization around AdS background.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Line-element $\bar{g}_{\mu\nu}$ of pure AdS:

$$d\bar{s}^2_{\text{AdS}} = \bar{g}_{\mu\nu}\, dx^\mu\, dx^\nu = \ell^2\big(-\cosh^2\rho\, d\tau^2 + \sinh^2\rho\, d\phi^2 + d\rho^2\big)$$

Isometry group: $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$
Useful to introduce light-cone coordinates $u = \tau + \phi$, $v = \tau - \phi$.
The $SL(2,\mathbb{R})_L$ generators

$$L_0 = i\partial_u$$

$$L_{\pm 1} = ie^{\pm iu}\Big[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_u - \frac{1}{\sinh 2\rho}\partial_v \mp \frac{i}{2}\,\partial_\rho\Big]$$

obey the algebra $[L_0, L_{\pm 1}] = \mp L_{\pm 1}$, $[L_1, L_{-1}] = 2L_0$.
The $SL(2,\mathbb{R})_R$ generators $\bar{L}_0, \bar{L}_{\pm 1}$ obey same algebra, but with

$$u \leftrightarrow v\,, \qquad L \leftrightarrow \bar{L}$$

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$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

leads to linearized EOM that are third order PDE

$$G^{(1)}_{\mu\nu} + \frac{1}{\mu}\,C^{(1)}_{\mu\nu} = (\mathcal{D}^R\mathcal{D}^L\mathcal{D}^M h)_{\mu\nu} = 0 \tag{1}$$

with three mutually commuting first order operators

$$(\mathcal{D}^{L/R})_\mu{}^\nu = \delta_\mu^\nu \pm \ell\,\varepsilon_\mu{}^{\alpha\nu}\bar{\nabla}_\alpha \qquad (\mathcal{D}^M)_\mu{}^\nu = \delta_\mu^\nu + \frac{1}{\mu}\varepsilon_\mu{}^{\alpha\nu}\bar{\nabla}_\alpha$$

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Three linearly independent solutions to (1):

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At chiral point left (L) and massive (M) branches coincide!

Degeneracy at the chiral point

Will be quite important later!

Li, Song & Strominger found all normalizable solutions of linearized EOM.

- Primaries: $L_0, \bar{L}_0$ eigenstates $\psi^{L/R/M}$ with

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- At chiral point: L and M branches degenerate. Get log solution (Grumiller & Johansson '08)

$$\psi_{\mu\nu}^{\log} = \lim_{\mu\ell \to 1} \frac{\psi_{\mu\nu}^{M}(\mu\ell) - \psi_{\mu\nu}^{L}}{\mu\ell - 1}$$

with property

$$\big(\mathcal{D}^L \psi^{\log}\big)_{\mu\nu} = \big(\mathcal{D}^M \psi^{\log}\big)_{\mu\nu} \neq 0\,, \qquad \big((\mathcal{D}^L)^2 \psi^{\log}\big)_{\mu\nu} = 0$$

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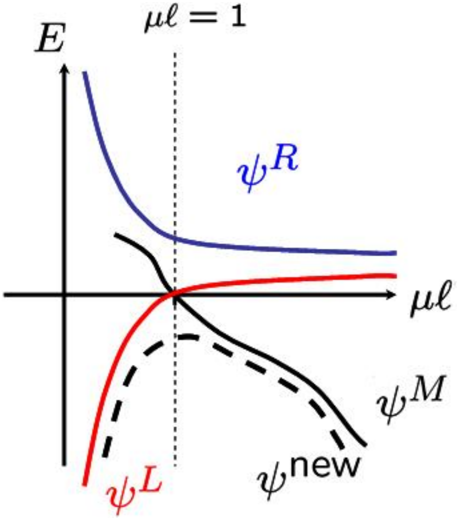
- Even at chiral point the problem persists because of the logarithmic mode. See Figure. (thanks to Niklas Johansson)

Energy for all branches:



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Motivating the conjecture

Log mode exhibits interesting property:

$$H \begin{pmatrix} \psi^{\log} \\ \psi^L \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \psi^{\log} \\ \psi^L \end{pmatrix}$$

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Logarithmic CFT conjecture

CTMG dual to a logarithmic CFT (Grumiller, Johansson '08)

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- Energy of logarithmic mode is finite

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- Brown–York stress tensor is finite and traceless, but not chiral

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- Perturbative solution of linearized EOM, but not pure gauge
- Energy of logarithmic mode is finite

$$E^{\log} = -\frac{47}{1152 G\,\ell^3}$$

  and negative $\to$ instability! (Grumiller & Johansson '08)
- Logarithmic mode is asymptotically AdS

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + \left(\gamma_{ij}^{(0)} e^{2\rho/\ell} + \gamma_{ij}^{(1)} \rho + \gamma_{ij}^{(0)} + \gamma_{ij}^{(2)} e^{-2\rho/\ell} + \dots\right) \mathrm{d}x^i \, \mathrm{d}x^j$$

  but violates Brown–Henneaux boundary conditions! ($\gamma_{ij}^{(1)}\big|_{\mathrm{BH}} = 0$)
- Consistent log boundary conditions replacing Brown–Henneaux (Grumiller & Johansson '08, Martinez, Henneaux & Troncoso '09)
- Brown–York stress tensor is finite and traceless, but not chiral
- Log mode persists non-perturbatively, as shown by Hamilton analysis (Grumiller, Jackiw & Johansson '08, Carlip '08)

Correlators in logarithmic CFTs

- Any CFT has a conserved traceless energy momentum tensor.

$$T_{z\bar{z}} = 0 \qquad T_{zz} = \mathcal{O}^L(z) \qquad T_{\bar{z}\bar{z}} = \mathcal{O}^R(\bar{z})$$

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- The 2- and 3-point correlators are fixed by conformal Ward identities.

$$\langle \mathcal{O}^R(\bar{z})\, \mathcal{O}^R(0) \rangle = \frac{c_R}{2\bar{z}^4}$$

$$\langle \mathcal{O}^L(z)\, \mathcal{O}^L(0) \rangle = \frac{c_L}{2z^4}$$

$$\langle \mathcal{O}^L(z)\, \mathcal{O}^R(0) \rangle = 0$$

$$\langle \mathcal{O}^R(\bar{z})\, \mathcal{O}^R(\bar{z}')\, \mathcal{O}^R(0) \rangle = \frac{c_R}{\bar{z}^2 \bar{z}'^2 (\bar{z} - \bar{z}')^2}$$

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Central charges $c_{L/R}$ determine key properties of CFT.

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- Suppose there is an additional operator $\mathcal{O}^M$ with conformal weights $h = 2 + \varepsilon$, $\bar{h} = \varepsilon$

$$\langle \mathcal{O}^M(z,\bar{z})\, \mathcal{O}^M(0,0) \rangle = \frac{\hat{B}}{z^{4+2\varepsilon}\, \bar{z}^{2\varepsilon}}$$

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- Then energy momentum tensor acquires logarithmic partner $\mathcal{O}^{\log}$

$$\mathcal{O}^{\log} = b_L\, \frac{\mathcal{O}^L}{c_L} + \frac{b_L}{2}\, \mathcal{O}^M$$

where

$$b_L := \lim_{c_L \to 0} -\frac{c_L}{\varepsilon} \neq 0$$

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- Some 2-point correlators:

$$\langle \mathcal{O}^L(z) \mathcal{O}^L(0,0) \rangle = 0$$

$$\langle \mathcal{O}^L(z) \mathcal{O}^{\log}(0,0) \rangle = \frac{b_L}{2z^4}$$

$$\langle \mathcal{O}^{\log}(z,\bar{z}) \mathcal{O}^{\log}(0,0) \rangle = -\frac{b_L \ln(m_L^2|z|^2)}{z^4}$$

"New anomaly" b_L determines key properties of logarithmic CFT.

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- Works at level of 2-point correlators (Skenderis, Taylor & van Rees '09, Grumiller & Sachs '09)
- Works at level of 3-point correlators (Grumiller & Sachs '09)
- Value of new anomaly: $b_L = -c_R = -3\ell/G$

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Recover the result (Grumiller & Hohm '09, Grumiller, Johansson & Zojer, '10)

$$b_L = -\frac{3\ell}{G}$$

1-loop partition function

...yet another non-trivial check (Gaberdiel, Grumiller & Vassilevich '10)

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1	0	1	0	1	0	1	0	1	0	1	0	1	0	...
0	1	0	1	0	1	0	1	0	1	0	1	0	1	...
2	0	2	1	2	1	3	1	3	2	3	2	4	2	...
0	2	1	2	2	3	2	4	3	4	4	5	4	6	...
3	1	4	3	6	4	8	6	10	8	12	10	15	12	...
1	3	3	6	5	9	9	12	12	17	16	21	21	26	...
4	3	8	7	14	13	20	20	29	28	39	38	50	50	...
2	6	7	13	15	22	26	35	39	51	56	70	77	93	...
7	5	15	17	29	32	50	53	76	83	109	119	153	163	...
3	11	15	26	35	52	64	89	106	138	163	203	234	287	...
10	11	27	35	60	73	111	132	183	216	283	328	417	476	...
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...	...	...	...	...	...	...	...	...	...	...	...	...	...	...

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Conclusion: all consistency tests show validity of LCFT conjecture!

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If conjecture true: first example of $AdS_3/LCFT_2$ correspondence!

Consequences for chiral gravity

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Not clear yet if chiral gravity exists!
If it exists: excellent toy model for quantum gravity!

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New massive gravity (Bergshoeff, Hohm & Townsend '09):

$$I_{\text{NMG}} = \frac{1}{16\pi G} \int \mathrm{d}^3x\sqrt{-g}\,\Big[\sigma R + \frac{1}{m^2}\,\Big(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}\,R^2\Big) - 2\lambda m^2\Big]$$

Similar story (Grumiller & Hohm '09, Alishahiha & Naseh '10):

- Linearized EOM around AdS$_3$ ($g = \bar{g} + h$)

$$\big(\mathcal{D}^R\mathcal{D}^L\mathcal{D}^M\mathcal{D}^{\bar{M}}h\big)_{\mu\nu} = 0$$

- Logarithmic point for $\lambda = 3$: $c_L = c_R = 0$
- Massive modes degenerate with left and right boundary gravitons
- 2-point correlators on gravity side match precisely those of a LCFT
- New anomalies: $b_L = b_R = -\sigma 12\ell/G$

Extended generalized massive gravity (Paulos '10)

Reconsider higher curvature theories introduced in the beginning

All actions of type

$$\mathcal{L} = \mathcal{L}_{\text{MG}}(R_{\mu\nu}) + \mathcal{L}_{\text{CS}}$$

with gravitational Chern–Simons term

$$\mathcal{L}_{\text{CS}} = \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^{\rho}{}_{\lambda\sigma}\big(\partial_\mu\Gamma^{\sigma}{}_{\nu\rho} + \frac{2}{3}\,\Gamma^{\sigma}{}_{\mu\tau}\Gamma^{\tau}{}_{\nu\rho}\big)\Big]$$

and the specific higher derivative Lagrange density

$$\mathcal{L}_{\text{MG}}(R_{\mu\nu}) = \sigma R - 2\Lambda + \frac{1}{m^2}\left(R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}\,R^2\right) + \mathcal{O}(R^3_{\mu\nu})$$

have an AdS solution (if $\Lambda_{\text{eff}} < 0$) and linearized equations of motion

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Various degenerations of modes possible $\rightarrow$ log excitations

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Thus, we have infinitely many gravity duals for LCFTs!

Potential applications in condensed matter physics

LCFTs arise in systems with quenched disorder.

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- Quenched disorder: systems with random variable that does not evolve in time

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- Apply $\mathrm{AdS}_3/\mathrm{LCFT}_2$ to describe strongly coupled LCFTs!

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Thanks for your attention!

Some literature

- M. R. Gaberdiel, "An algebraic approach to logarithmic conformal field theory," Int. J. Mod. Phys. **A18** (2003) 4593 hep-th/0111260.
- D. Grumiller and N. Johansson, "Gravity duals for logarithmic conformal field theories," 1001.0002. See also Refs. therein.
- W. Li, W. Song and A. Strominger, "Chiral Gravity in Three Dimensions," JHEP **0804** (2008) 082, 0801.4566.
- D. Grumiller and N. Johansson, "Instability in cosmological topologically massive gravity at the chiral point," JHEP **0807** (2008) 134, 0805.2610.

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