Rindler Holography

Daniel Grumiller

Institute for Theoretical Physics
TU Wien

Solvay workshop 'Holography for black holes and cosmology'
Bruxelles, May 2016

FWF

based on work w. H. Afshar, S. Detournay, W. Merbis,
(B. Oblak), A. Perez, D. Tempo, R. Troncoso

Simple punchline

Heisenberg algebra

$$[X_n,\, P_m] = i\,\delta_{n,\,m}$$

fundamental not only in quantum mechanics
but also in near horizon physics

Outline

Outline

Black hole microstates

Bekenstein–Hawking

$$S_{\rm BH} = \frac{A}{4G_N}$$

- Motivation: microscopic understanding of generic black hole entropy

Black hole microstates

Bekenstein–Hawking

$$S_{\text{BH}} = \frac{A}{4G_N}$$

- Motivation: microscopic understanding of generic black hole entropy
- Microstate counting from CFT_2 symmetries (Strominger, Carlip, ...) using Cardy formula

Black hole microstates

Bekenstein–Hawking

$$S_{\mathrm{BH}} = \frac{A}{4G_N}$$

- Motivation: microscopic understanding of generic black hole entropy
- Microstate counting from CFT_2 symmetries (Strominger, Carlip, ...) using Cardy formula
- Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...)

warped CFT: Detournay, Hartman, Hofman '12
Galilean CFT: Bagchi, Detournay, Fareghbal, Simon '13; Barnich '13

Black hole microstates

Bekenstein–Hawking

$$S_{\text{BH}} = \frac{A}{4G_N}$$

- Motivation: microscopic understanding of generic black hole entropy
- Microstate counting from CFT_2 symmetries (Strominger, Carlip, ...) using Cardy formula
- Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...)
- Main idea: consider near horizon symmetries for non-extremal horizons

Black hole microstates

Bekenstein–Hawking

$$S_{\text{BH}} = \frac{A}{4G_N}$$

- Motivation: microscopic understanding of generic black hole entropy
- Microstate counting from CFT_2 symmetries (Strominger, Carlip, ...) using Cardy formula
- Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...)
- Main idea: consider near horizon symmetries for non-extremal horizons
- Near horizon line-element with Rindler acceleration a:

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

Meaning of coordinates:

- ρ: radial direction ($\rho = 0$ is horizon)
- $\varphi \sim \varphi + 2\pi$: angular direction
- v: (advanced) time

Choices

- Rindler acceleration: state-dependent or chemical potential?

Choices

- Rindler acceleration: state-dependent or chemical potential?
- If state-dependent: need mechanism to fix scale

Recall scale invariance

$$a \to \lambda a \qquad \rho \to \lambda \rho \qquad v \to v/\lambda$$

of Rindler metric

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2$$

Choices

- Rindler acceleration: state-dependent or chemical potential?
- If state-dependent: need mechanism to fix scale — suggestion in `1512.08233`:

$$v \sim v + 2\pi L$$

Works technically but physical interpretation difficult

Recall scale invariance

$$a \to \lambda a \qquad \rho \to \lambda \rho \qquad v \to v/\lambda$$

of Rindler metric

$$\mathrm{d}s^2 = -2a\rho\, \mathrm{d}v^2 + 2\, \mathrm{d}v\, \mathrm{d}\rho + \gamma^2\, \mathrm{d}\varphi^2$$

Choices

- Rindler acceleration: state-dependent or chemical potential?
- If state-dependent: need mechanism to fix scale — suggestion in `1512.08233`:

  $$v \sim v + 2\pi L$$

  Works technically but physical interpretation difficult
- If chemical potential: all states in theory have same (Unruh-)temperature

  $$T_U = \frac{a}{2\pi}$$

Choices

- Rindler acceleration: state-dependent or chemical potential?
- If state-dependent: need mechanism to fix scale — suggestion in `1512.08233`:

  $$v \sim v + 2\pi L$$

  Works technically but physical interpretation difficult
- If chemical potential: all states in theory have same (Unruh-)temperature

  $$T_U = \frac{a}{2\pi}$$

  suggestion in `1511.08687`

  We make this choice in this talk!

Choices

- Rindler acceleration: state-dependent or chemical potential?
- If state-dependent: need mechanism to fix scale — suggestion in `1512.08233`:

$$v \sim v + 2\pi L$$

  Works technically but physical interpretation difficult
- If chemical potential: all states in theory have same (Unruh-)temperature

$$T_U = \frac{a}{2\pi}$$

- Work in 3d Einstein gravity in Chern–Simons formulation

$$I_{\mathsf{CS}} = \pm \sum_{\pm} \frac{k}{4\pi} \int \langle A^{\pm} \wedge \mathrm{d}A^{\pm} + \tfrac{2}{3} A^{\pm} \wedge A^{\pm} \wedge A^{\pm} \rangle$$

  with $sl(2)$ connections $A^{\pm}$ and $k = \ell/(4G_N)$ with AdS radius $\ell = 1$

Outline

Diagonal gauge

Standard trick: partially fix gauge

$$A^{\pm} = b_{\pm}^{-1}(\rho)\left(\mathrm{d}+\mathfrak{a}_{\pm}(x^0,\, x^1)\right)b_{\pm}(\rho)$$

with some group element $b \in SL(2)$ depending on radius ρ with $\delta b = 0$

Drop $\pm$ decorations in most of talk

Manifold topologically a cylinder or torus, with radial coordinate ρ and boundary coordinates $(x^0, x^1) \sim (v, \varphi)$

Diagonal gauge

Standard trick: partially fix gauge

$$A = b^{-1}(\rho)\big(\mathrm{d}+\mathfrak{a}(x^0,\, x^1)\big)b(\rho)$$

with some group element $b \in SL(2)$ depending on radius ρ with $\delta b = 0$

- Standard AdS$_3$ approach: highest weight gauge

$$\mathfrak{a} \sim L_+ + \mathcal{L}(x^0,\, x^1)L_- \qquad b(\rho) = \exp\left(\rho L_0\right)$$

$sl(2)$: $[L_n,\, L_m] = (n-m)L_{n+m}, \quad n, m = -1, 0, 1$

Diagonal gauge

Standard trick: partially fix gauge

$$A = b^{-1}(\rho)\left(\mathrm{d}+\mathfrak{a}(x^0,\, x^1)\right)b(\rho)$$

with some group element $b \in SL(2)$ depending on radius ρ with $\delta b = 0$

- Standard AdS$_3$ approach: highest weight gauge

  $$\mathfrak{a} \sim L_+ + \mathcal{L}(x^0,\, x^1)L_- \qquad b(\rho) = \exp\left(\rho L_0\right)$$

  $sl(2)$: $[L_n,\, L_m] = (n-m)L_{n+m}, \quad n, m = -1, 0, 1$
- For near horizon purposes diagonal gauge useful:

  $$\mathfrak{a} \sim \mathcal{J}(x^0,\, x^1)\, L_0$$

Diagonal gauge

Standard trick: partially fix gauge

$$A = b^{-1}(\rho)\,\big(\mathrm{d}+\mathfrak{a}(x^0,\,x^1)\big)b(\rho)$$

with some group element $b \in SL(2)$ depending on radius ρ with $\delta b = 0$

- Standard AdS$_3$ approach: highest weight gauge

$$\mathfrak{a} \sim L_+ + \mathcal{L}(x^0,\,x^1)L_- \qquad b(\rho) = \exp\left(\rho L_0\right)$$

  $sl(2)$: $[L_n,\,L_m] = (n-m)L_{n+m}, \quad n,m = -1,0,1$

- For near horizon purposes diagonal gauge useful:

$$\mathfrak{a} \sim \mathcal{J}(x^0,\,x^1)\,L_0$$

- Precise boundary conditions (ζ: chemical potential):

$$\mathfrak{a} = \left(\mathcal{J}\ \mathrm{d}\varphi + \zeta\ \mathrm{d}v\right)\,L_0 \qquad \delta\mathfrak{a} = \delta\mathcal{J}\ \mathrm{d}\varphi\,L_0$$

  and $b = \exp\left(\frac{1}{\zeta}\,L_+\right)\cdot\exp\left(\frac{\rho}{2}\,L_-\right)$. (assume constant ζ for simplicity)

Near horizon metric

Using

$$g_{\mu\nu} = \frac{1}{2} \left\langle \left(A_\mu^+ - A_\mu^-\right) \left(A_\nu^+ - A_\nu^-\right) \right\rangle$$

Near horizon metric

Using

$$g_{\mu\nu} = \frac{1}{2} \left\langle \left(A_\mu^+ - A_\mu^-\right) \left(A_\nu^+ - A_\nu^-\right) \right\rangle$$

yields ($f := 1 + \rho/(2a)$)

$$\mathrm{d}s^2 = -2a\rho f \,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho - 2\omega a^{-1}\,\mathrm{d}\varphi\,\mathrm{d}\rho + 4\omega\rho f\,\mathrm{d}v\,\mathrm{d}\varphi + \left[\gamma^2 + \frac{2\rho}{a} f(\gamma^2 - \omega^2)\right]\mathrm{d}\varphi^2$$

state-dependent functions $\mathcal{J}^\pm = \gamma \pm \omega$, chemical potentials $\zeta^\pm = -a \pm \Omega$

For simplicity set $\Omega = 0$ and $a = \text{const.}$ in metric above

EOM imply $\partial_v \mathcal{J}^\pm = \pm\partial_\varphi \zeta^\pm$; in this case $\partial_v \mathcal{J}^\pm = 0$

Near horizon metric

Using

$$g_{\mu\nu} = \frac{1}{2}\left\langle \left(A_\mu^+ - A_\mu^-\right)\left(A_\nu^+ - A_\nu^-\right)\right\rangle$$

yields ($f := 1 + \rho/(2a)$)

$$\mathrm{d}s^2 = -2a\rho f\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho - 2\omega a^{-1}\,\mathrm{d}\varphi\,\mathrm{d}\rho + 4\omega\rho f\,\mathrm{d}v\,\mathrm{d}\varphi + \left[\gamma^2 + \frac{2\rho}{a} f(\gamma^2 - \omega^2)\right]\mathrm{d}\varphi^2$$

state-dependent functions $\mathcal{J}^\pm = \gamma \pm \omega$, chemical potentials $\zeta^\pm = -a \pm \Omega$
Neglecting rotation terms ($\omega = 0$) yields Rindler plus higher order terms:

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

Comments:

- Recover desired near horizon metric

Near horizon metric

Using

$$g_{\mu\nu} = \frac{1}{2} \langle \left(A_\mu^+ - A_\mu^-\right) \left(A_\nu^+ - A_\nu^-\right) \rangle$$

yields ($f := 1 + \rho/(2a)$)

$$\mathrm{d}s^2 = -2a\rho f \,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho - 2\omega a^{-1}\,\mathrm{d}\varphi\,\mathrm{d}\rho$$
$$+ 4\omega\rho f\,\mathrm{d}v\,\mathrm{d}\varphi + \left[\gamma^2 + \frac{2\rho}{a} f(\gamma^2 - \omega^2)\right]\mathrm{d}\varphi^2$$

state-dependent functions $\mathcal{J}^\pm = \gamma \pm \omega$, chemical potentials $\zeta^\pm = -a \pm \Omega$
Neglecting rotation terms ($\omega = 0$) yields Rindler plus higher order terms:

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

Comments:

- Recover desired near horizon metric
- Rindler acceleration a indeed state-independent

Near horizon metric

Using

$$g_{\mu\nu} = \tfrac{1}{2} \left\langle \left(A_\mu^+ - A_\mu^-\right) \left(A_\nu^+ - A_\nu^-\right) \right\rangle$$

yields $(f := 1 + \rho/(2a))$

$$\mathrm{d}s^2 = -2a\rho f \,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho - 2\omega a^{-1}\,\mathrm{d}\varphi\,\mathrm{d}\rho \\ + 4\omega\rho f\,\mathrm{d}v\,\mathrm{d}\varphi + \left[\gamma^2 + \tfrac{2\rho}{a} f(\gamma^2 - \omega^2)\right]\mathrm{d}\varphi^2$$

state-dependent functions $\mathcal{J}^\pm = \gamma \pm \omega$, chemical potentials $\zeta^\pm = -a \pm \Omega$
Neglecting rotation terms ($\omega = 0$) yields Rindler plus higher order terms:

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

Comments:

- Recover desired near horizon metric
- Rindler acceleration a indeed state-independent
- Two state-dependent functions (γ, ω) as usual in 3d gravity

Near horizon metric

Using

$$g_{\mu\nu} = \tfrac{1}{2}\left\langle\left(A_\mu^+ - A_\mu^-\right)\left(A_\nu^+ - A_\nu^-\right)\right\rangle$$

yields ($f := 1 + \rho/(2a)$)

$$\mathrm{d}s^2 = -2a\rho f\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho - 2\omega a^{-1}\,\mathrm{d}\varphi\,\mathrm{d}\rho \\ + 4\omega\rho f\,\mathrm{d}v\,\mathrm{d}\varphi + \left[\gamma^2 + \tfrac{2\rho}{a} f(\gamma^2 - \omega^2)\right]\mathrm{d}\varphi^2$$

state-dependent functions $\mathcal{J}^\pm = \gamma \pm \omega$, chemical potentials $\zeta^\pm = -a \pm \Omega$
Neglecting rotation terms ($\omega = 0$) yields Rindler plus higher order terms:

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

Comments:

- Recover desired near horizon metric
- Rindler acceleration a indeed state-independent
- Two state-dependent functions (γ, ω) as usual in 3d gravity
- $\gamma = \gamma(\varphi)$: "black flower"

Canonical boundary charges

- Canonical boundary charges non-zero for large trafos that preserve boundary conditions
- Zero mode charges: mass and angular momentum

Canonical boundary charges

- Canonical boundary charges non-zero for large trafos that preserve boundary conditions
- Zero mode charges: mass and angular momentum

Background independent result for Chern–Simons yields

$$Q[\eta] = \frac{k}{4\pi} \oint \mathrm{d}\varphi \, \eta(\varphi) \, \mathcal{J}(\varphi)$$

- Finite
- Integrable
- Conserved
- Non-trivial

Canonical boundary charges

- Canonical boundary charges non-zero for large trafos that preserve boundary conditions
- Zero mode charges: mass and angular momentum

Background independent result for Chern–Simons yields

$$Q[\eta] = \frac{k}{4\pi} \oint \mathrm{d}\varphi \, \eta(\varphi) \, \mathcal{J}(\varphi)$$

- Finite
- Integrable
- Conserved
- Non-trivial

Meaningful near horizon boundary conditions and non-trivial theory!

Outline

Near horizon symmetry algebra

- Near horizon symmetry algebra = all near horizon boundary conditions preserving trafos, modulo trivial gauge trafos

  Most general trafo

  $$\delta_\epsilon \mathfrak{a} = \mathrm{d}\epsilon + [\mathfrak{a},\, \epsilon] = \mathcal{O}(\delta\mathfrak{a})$$

  that preserves our boundary conditions for constant ζ given by

  $$\epsilon = \epsilon^+ L_+ + \eta L_0 + \epsilon^- L_-$$

  with

  $$\partial_v \eta = 0$$

  implying

  $$\delta_\epsilon \mathcal{J} = \partial_\varphi \eta$$

Near horizon symmetry algebra

- Near horizon symmetry algebra = all near horizon boundary conditions preserving trafos, modulo trivial gauge trafos
- Expand charges in Fourier modes

$$J_n^\pm = \frac{k}{4\pi} \oint \mathrm{d}\varphi \, e^{in\varphi} \mathcal{J}^\pm(\varphi)$$

What should we expect?

  - Virasoro? (spacetime is locally AdS_3)
  - BMS_3? (Rindler boundary similar to scri)
  - warped conformal algebra? (this is what we found for Rindleresque holography and what Donnay, Giribet, Gonzalez, Pino found in their near horizon analysis)

Near horizon symmetry algebra

- Near horizon symmetry algebra = all near horizon boundary conditions preserving trafos, modulo trivial gauge trafos
- Expand charges in Fourier modes

$$J_n^\pm = \frac{k}{4\pi} \oint \mathrm{d}\varphi \, e^{in\varphi} \mathcal{J}^\pm(\varphi)$$

- Near horizon symmetry algebra

$$\left[J_n^\pm, \, J_m^\pm\right] = \pm\tfrac{1}{2} k n \delta_{n+m,\,0} \qquad \left[J_n^+, \, J_m^-\right] = 0$$

Two $\hat{u}(1)$ current algebras with non-zero levels

Near horizon symmetry algebra

- Near horizon symmetry algebra = all near horizon boundary conditions preserving trafos, modulo trivial gauge trafos
- Expand charges in Fourier modes

$$J_n^{\pm} = \frac{k}{4\pi} \oint \mathrm{d}\varphi \, e^{in\varphi} \mathcal{J}^{\pm}(\varphi)$$

- Near horizon symmetry algebra

$$\left[J_n^{\pm},\, J_m^{\pm}\right] = \pm\tfrac{1}{2} k n \delta_{n+m,\,0} \qquad \left[J_n^{+},\, J_m^{-}\right] = 0$$

  Two $\hat{u}(1)$ current algebras with non-zero levels
- Much simpler than CFT_2, warped CFT_2, Galilean CFT_2, etc.

Near horizon symmetry algebra

- Near horizon symmetry algebra = all near horizon boundary conditions preserving trafos, modulo trivial gauge trafos
- Expand charges in Fourier modes

$$J_n^\pm = \frac{k}{4\pi} \oint \mathrm{d}\varphi \, e^{in\varphi} \mathcal{J}^\pm(\varphi)$$

- Near horizon symmetry algebra

$$[J_n^\pm,\, J_m^\pm] = \pm\tfrac{1}{2} kn\delta_{n+m,\,0} \qquad [J_n^+,\, J_m^-] = 0$$

  Two $\hat{u}(1)$ current algebras with non-zero levels
- Much simpler than CFT$_2$, warped CFT$_2$, Galilean CFT$_2$, etc.
- Map

$$P_0 = J_0^+ + J_0^- \qquad P_n = \tfrac{i}{kn}\,(J_{-n}^+ + J_{-n}^-) \text{ if } n \neq 0 \qquad X_n = J_n^+ - J_n^-$$

  yields Heisenberg algebra (with Casimirs X_0, P_0)

$$[X_n,\, X_m] = [P_n,\, P_m] = [X_0,\, P_n] = [P_0,\, X_n] = 0$$
$$[X_n,\, P_m] = i\delta_{n,m} \quad \text{if } n \neq 0$$

Soft hair

- Vacuum descendants $|\psi(q)\rangle$

$$|\psi(q)\rangle \sim \prod (J^+_{-n_i^+})^{m_i^+} \prod (J^-_{-n_i^-})^{m_i^-} |0\rangle$$

Soft hair

- Vacuum descendants $|\psi(q)\rangle$

$$|\psi(q)\rangle \sim \prod (J^+_{-n_i^+})^{m_i^+} \prod (J^-_{-n_i^-})^{m_i^-} |0\rangle$$

- Hamiltonian

$$H := Q[\epsilon^\pm|_{\partial_v}] = aP_0$$

commutes with all generators of algebra

Soft hair

- Vacuum descendants $|\psi(q)\rangle$

$$|\psi(q)\rangle \sim \prod (J^+_{-n_i^+})^{m_i^+} \prod (J^-_{-n_i^-})^{m_i^-} |0\rangle$$

- Hamiltonian

$$H := Q[\epsilon^{\pm}|_{\partial_v}] = aP_0$$

  commutes with all generators of algebra

- Energy of vacuum descendants

$$E_\psi = \langle\psi(q)|H|\psi(q)\rangle = E_{\text{vac}}\langle\psi(q)|\psi(q)\rangle = E_{\text{vac}}$$

  same as energy of vacuum

Soft hair

- Vacuum descendants $|\psi(q)\rangle$

$$|\psi(q)\rangle \sim \prod (J^+_{-n_i^+})^{m_i^+} \prod (J^-_{-n_i^-})^{m_i^-} |0\rangle$$

- Hamiltonian

$$H := Q[\epsilon^\pm|_{\partial_v}] = aP_0$$

commutes with all generators of algebra

- Energy of vacuum descendants

$$E_\psi = \langle\psi(q)|H|\psi(q)\rangle = E_{\text{vac}}\langle\psi(q)|\psi(q)\rangle = E_{\text{vac}}$$

same as energy of vacuum

- Same conclusion true for descendants of any state!

Soft hair

- Vacuum descendants $|\psi(q)\rangle$

$$|\psi(q)\rangle \sim \prod (J^{+}_{-n_i^+})^{m_i^+} \prod (J^{-}_{-n_i^-})^{m_i^-} |0\rangle$$

- Hamiltonian

$$H := Q[\epsilon^{\pm}|_{\partial_v}] = aP_0$$

commutes with all generators of algebra

- Energy of vacuum descendants

$$E_\psi = \langle\psi(q)|H|\psi(q)\rangle = E_{\text{vac}}\langle\psi(q)|\psi(q)\rangle = E_{\text{vac}}$$

same as energy of vacuum

- Same conclusion true for descendants of any state!

Soft hair = zero energy excitations on horizon

Outline

Macroscopic entropy

- Zero-mode solutions with constant chemical potentials: BTZ

$$J_0^\pm = \frac{k}{2}(r_+ \pm r_-)$$

Macroscopic entropy

- Zero-mode solutions with constant chemical potentials: BTZ

$$J_0^\pm = \frac{k}{2}(r_+ \pm r_-)$$

- Generic soft hairy black holes (or "black flowers") from softly boosting BTZ

Macroscopic entropy

- Zero-mode solutions with constant chemical potentials: BTZ

$$J_0^\pm = \frac{k}{2}(r_+ \pm r_-)$$

- Generic soft hairy black holes (or "black flowers") from softly boosting BTZ
- Soft hairy black holes remain regular and have same energy as BTZ (for other boundary conditions generically not true)

Macroscopic entropy

- Zero-mode solutions with constant chemical potentials: BTZ

$$J_0^\pm = \tfrac{k}{2}(r_+ \pm r_-)$$

- Generic soft hairy black holes (or "black flowers") from softly boosting BTZ
- Soft hairy black holes remain regular and have same energy as BTZ (for other boundary conditions generically not true)
- Macroscopic entropy

$$S = 2\pi(J_0^+ + J_0^-) = \frac{A}{4G_N}$$

calculated directly in Chern–Simons formulation

Macroscopic entropy

- Zero-mode solutions with constant chemical potentials: BTZ

$$J_0^{\pm} = \frac{k}{2}(r_+ \pm r_-)$$

- Generic soft hairy black holes (or "black flowers") from softly boosting BTZ
- Soft hairy black holes remain regular and have same energy as BTZ (for other boundary conditions generically not true)
- Macroscopic entropy

$$S = 2\pi(J_0^+ + J_0^-) = \frac{A}{4G_N}$$

- No contribution from soft hair charges

Macroscopic entropy

- Zero-mode solutions with constant chemical potentials: BTZ

$$J_0^\pm = \frac{k}{2}(r_+ \pm r_-)$$

- Generic soft hairy black holes (or "black flowers") from softly boosting BTZ
- Soft hairy black holes remain regular and have same energy as BTZ (for other boundary conditions generically not true)
- Macroscopic entropy

$$S = 2\pi(J_0^+ + J_0^-) = \frac{A}{4G_N}$$

- No contribution from soft hair charges
- Suggestive that microstate counting should work

Macroscopic entropy

- Zero-mode solutions with constant chemical potentials: BTZ

$$J_0^\pm = \tfrac{k}{2}(r_+ \pm r_-)$$

- Generic soft hairy black holes (or "black flowers") from softly boosting BTZ
- Soft hairy black holes remain regular and have same energy as BTZ (for other boundary conditions generically not true)
- Macroscopic entropy

$$S = 2\pi(J_0^+ + J_0^-) = \frac{A}{4G_N}$$

- No contribution from soft hair charges
- Suggestive that microstate counting should work

Before addressing microstates consider map to aymptotic variables

Map to asymptotic variables

- Usual asymptotic AdS_3 connection with chemical potential μ:

$$\hat{A} = \hat{b}^{-1}\big(\mathrm{d}+\hat{\mathfrak{a}}\big)\hat{b} \qquad \hat{\mathfrak{a}}_\varphi = L_+ - \tfrac{1}{2}\,\mathcal{L}\,L_-$$

$$\hat{b} = e^{\rho L_0} \quad \hat{\mathfrak{a}}_t = \mu L_+ - \mu' L_0 + \big(\tfrac{1}{2}\,\mu'' - \tfrac{1}{2}\,\mathcal{L}\mu\big)\,L_-$$

Map to asymptotic variables

- Usual asymptotic AdS$_3$ connection with chemical potential μ:

$$\hat{A} = \hat{b}^{-1}\big(\mathrm{d}+\hat{\mathfrak{a}}\big)\hat{b} \qquad \hat{\mathfrak{a}}_\varphi = L_+ - \tfrac{1}{2}\,\mathcal{L}\,L_-$$
$$\hat{b} = e^{\rho L_0} \quad \hat{\mathfrak{a}}_t = \mu L_+ - \mu' L_0 + \big(\tfrac{1}{2}\,\mu'' - \tfrac{1}{2}\,\mathcal{L}\mu\big)\,L_-$$

- Gauge trafo $\hat{\mathfrak{a}} = g^{-1}\,(\mathrm{d}+\mathfrak{a})\,g$ with

$$g = \exp\left(xL_+\right)\cdot\exp\left(-\tfrac{1}{2}\mathcal{J}\,L_-\right)$$

where $\partial_v x - \zeta x = \mu$ and $x' - \mathcal{J}\,x = 1$

Map to asymptotic variables

- Usual asymptotic AdS_3 connection with chemical potential μ:

$$\hat{A} = \hat{b}^{-1}\big(\mathrm{d} + \hat{\mathfrak{a}}\big)\hat{b} \qquad \hat{\mathfrak{a}}_\varphi = L_+ - \tfrac{1}{2}\,\mathcal{L}\,L_-$$
$$\hat{b} = e^{\rho L_0} \quad \hat{\mathfrak{a}}_t = \mu L_+ - \mu' L_0 + \big(\tfrac{1}{2}\,\mu'' - \tfrac{1}{2}\,\mathcal{L}\mu\big)\,L_-$$

- Gauge trafo $\hat{\mathfrak{a}} = g^{-1}\,(\mathrm{d} + \mathfrak{a})\,g$ with

$$g = \exp\,(x L_+) \cdot \exp\,(-\tfrac{1}{2}\mathcal{J}\,L_-)$$

where $\partial_v x - \zeta x = \mu$ and $x' - \mathcal{J}\,x = 1$
- Near horizon chemical potential transforms into combination of asymptotic charge and chemical potential!

$$\mu' - \mathcal{J}\,\mu = -\zeta$$

Map to asymptotic variables

- Usual asymptotic AdS_3 connection with chemical potential μ:

$$\hat{A} = \hat{b}^{-1}\big(\mathrm{d}+\hat{\mathfrak{a}}\big)\hat{b} \qquad \hat{\mathfrak{a}}_\varphi = L_+ - \tfrac{1}{2}\,\mathcal{L}\,L_-$$
$$\hat{b} = e^{\rho L_0} \quad \hat{\mathfrak{a}}_t = \mu L_+ - \mu' L_0 + \big(\tfrac{1}{2}\,\mu'' - \tfrac{1}{2}\,\mathcal{L}\mu\big)\,L_-$$

- Gauge trafo $\hat{\mathfrak{a}} = g^{-1}\,(\mathrm{d}+\mathfrak{a})\,g$ with

$$g = \exp\left(xL_+\right)\cdot\exp\left(-\tfrac{1}{2}\mathcal{J}\,L_-\right)$$

  where $\partial_v x - \zeta x = \mu$ and $x' - \mathcal{J}\,x = 1$

- Near horizon chemical potential transforms into combination of asymptotic charge and chemical potential!

$$\mu' - \mathcal{J}\,\mu = -\zeta$$

- Asymptotic charges: twisted Sugawara construction with near horizon charges

$$\mathcal{L} = \tfrac{1}{2}\mathcal{J}^2 + \mathcal{J}'$$

Map to asymptotic variables

- Usual asymptotic AdS$_3$ connection with chemical potential μ:

$$\hat{A} = \hat{b}^{-1}\big(\mathrm{d}+\hat{\mathfrak{a}}\big)\hat{b} \qquad \hat{\mathfrak{a}}_\varphi = L_+ - \tfrac{1}{2}\,\mathcal{L}\,L_-$$

$$\hat{b} = e^{\rho L_0} \quad \hat{\mathfrak{a}}_t = \mu L_+ - \mu' L_0 + \big(\tfrac{1}{2}\,\mu'' - \tfrac{1}{2}\,\mathcal{L}\mu\big)\,L_-$$

- Gauge trafo $\hat{\mathfrak{a}} = g^{-1}\,(\mathrm{d}+\mathfrak{a})\,g$ with

$$g = \exp\left(xL_+\right)\cdot\exp\left(-\tfrac{1}{2}\mathcal{J}\,L_-\right)$$

where $\partial_v x - \zeta x = \mu$ and $x' - \mathcal{J}\,x = 1$

- Near horizon chemical potential transforms into combination of asymptotic charge and chemical potential!

$$\mu' - \mathcal{J}\,\mu = -\zeta$$

- Asymptotic charges: twisted Sugawara construction with near horizon charges

$$\mathcal{L} = \tfrac{1}{2}\mathcal{J}^2 + \mathcal{J}'$$

- Get Virasoro with non-zero central charge $\delta\mathcal{L} = 2\mathcal{L}\varepsilon' + \mathcal{L}'\varepsilon - \varepsilon'''$

Remarks on asymptotic and near horizon variables

- Asymptotic spin-2 currents fulfill Virasoro algebra, but charges obey still Heisenberg algebra

$$\delta Q = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\varepsilon\,\delta\mathcal{L} = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\eta\,\delta\mathcal{J}$$

Reason: asymptotic "chemical potentials" μ depend on near horizon charges $\mathcal{J}$ and chemical potentials ζ

Remarks on asymptotic and near horizon variables

- Asymptotic spin-2 currents fulfill Virasoro algebra, but charges obey still Heisenberg algebra

$$\delta Q = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\varepsilon\,\delta\mathcal{L} = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\eta\,\delta\mathcal{J}$$

  Reason: asymptotic "chemical potentials" μ depend on near horizon charges $\mathcal{J}$ and chemical potentials ζ
- Our boundary conditions singled out: whole spectrum compatible with regularity

Remarks on asymptotic and near horizon variables

- Asymptotic spin-2 currents fulfill Virasoro algebra, but charges obey still Heisenberg algebra

$$\delta Q = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\varepsilon\,\delta\mathcal{L} = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\eta\,\delta\mathcal{J}$$

Reason: asymptotic "chemical potentials" μ depend on near horizon charges $\mathcal{J}$ and chemical potentials ζ

- Our boundary conditions singled out: whole spectrum compatible with regularity
- For constant chemical potential ζ: regularity = holonomy condition

$$\mu\mu'' - \tfrac{1}{2}\mu'^{\,2} - \mu^2\mathcal{L} = -2\pi^2/\beta^2$$

Solved automatically from map to asymptotic observables; reminder:

$$\mu' - \mathcal{J}\mu = -\zeta \qquad \mathcal{L} = \tfrac{1}{2}\mathcal{J}^2 + \mathcal{J}'$$

Remarks on asymptotic and near horizon variables

- Asymptotic spin-2 currents fulfill Virasoro algebra, but charges obey still Heisenberg algebra

$$\delta Q = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\varepsilon\,\delta\mathcal{L} = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\eta\,\delta\mathcal{J}$$

  Reason: asymptotic "chemical potentials" μ depend on near horizon charges $\mathcal{J}$ and chemical potentials ζ
- Our boundary conditions singled out: whole spectrum compatible with regularity
- For constant chemical potential ζ: regularity = holonomy condition

$$\mu\mu'' - \tfrac{1}{2}\mu'^2 - \mu^2\mathcal{L} = -2\pi^2/\beta^2$$

  Solved automatically from map to asymptotic observables; reminder:

$$\mu' - \mathcal{J}\mu = -\zeta \qquad \mathcal{L} = \tfrac{1}{2}\mathcal{J}^2 + \mathcal{J}'$$

Near horizon boundary conditions natural for near horizon observer

Cardy counting

- Idea: use map to asymptotic observables to do standard Cardy counting
- Twisted Sugawara construction expanded in Fourier modes

$$kL_n = \sum_{p\in\mathbb{Z}} J_{n-p}J_p + iknJ_n$$

Cardy counting

- Idea: use map to asymptotic observables to do standard Cardy counting
- Twisted Sugawara construction expanded in Fourier modes

$$kL_n = \sum_{p\in\mathbb{Z}} J_{n-p}J_p + iknJ_n$$

- Starting from Heisenberg algebra obtain semi-classically Virasoro algebra

$$[L_n,\, L_m] = (n-m)L_{n+m} + \tfrac{1}{2}\,k\,n^3\,\delta_{n+m,\,0}$$

Cardy counting

- Idea: use map to asymptotic observables to do standard Cardy counting
- Twisted Sugawara construction expanded in Fourier modes

$$kL_n = \sum_{p\in\mathbb{Z}} J_{n-p}J_p + iknJ_n$$

- Starting from Heisenberg algebra obtain semi-classically Virasoro algebra

$$[L_n,\, L_m] = (n-m)L_{n+m} + \tfrac{1}{2}\,k\,n^3\,\delta_{n+m,\,0}$$

- Usual Cardy formula yields Bekenstein–Hawking result

$$S_{\text{Cardy}} = 2\pi\sqrt{kL_0^+} + 2\pi\sqrt{kL_0^-} = 2\pi(J_0^+ + J_0^-) = \frac{A}{4G_N} = S_{\text{BH}}$$

Cardy counting

- Idea: use map to asymptotic observables to do standard Cardy counting
- Twisted Sugawara construction expanded in Fourier modes

$$kL_n = \sum_{p\in\mathbb{Z}} J_{n-p}J_p + iknJ_n$$

- Starting from Heisenberg algebra obtain semi-classically Virasoro algebra

$$[L_n,\, L_m] = (n-m)L_{n+m} + \tfrac{1}{2}\,k\,n^3\,\delta_{n+m,\,0}$$

- Usual Cardy formula yields Bekenstein–Hawking result

$$S_{\text{Cardy}} = 2\pi\sqrt{kL_0^+} + 2\pi\sqrt{kL_0^-} = 2\pi(J_0^+ + J_0^-) = \frac{A}{4G_N} = S_{\text{BH}}$$

- Need $J_0^{\text{vac}} = ik/2$ to get correct vacuum value $L_0^{\text{vac}} = -k/4$; with $a = i$ get modular transformed line-element $\mathrm{d}s^2 = \rho^2\,\mathrm{d}t + \mathrm{d}\rho^2 - \mathrm{d}\varphi^2$

Cardy counting

- Idea: use map to asymptotic observables to do standard Cardy counting
- Twisted Sugawara construction expanded in Fourier modes

$$kL_n = \sum_{p\in\mathbb{Z}} J_{n-p}J_p + iknJ_n$$

- Starting from Heisenberg algebra obtain semi-classically Virasoro algebra

$$[L_n,\, L_m] = (n-m)L_{n+m} + \tfrac{1}{2}\, k\, n^3\, \delta_{n+m,\,0}$$

- Usual Cardy formula yields Bekenstein–Hawking result

$$S_{\textrm{Cardy}} = 2\pi\sqrt{kL_0^+} + 2\pi\sqrt{kL_0^-} = 2\pi(J_0^+ + J_0^-) = \frac{A}{4G_N} = S_{\textrm{BH}}$$

- Need $J_0^{\textrm{vac}} = ik/2$ to get correct vacuum value $L_0^{\textrm{vac}} = -k/4$; with $a = i$ get modular transformed line-element $\mathrm{d}s^2 = \rho^2\,\mathrm{d}t + \mathrm{d}\rho^2 - \mathrm{d}\varphi^2$

Precise numerical factor in twist term crucial for correct results

Warped CFT counting

- Map near horizon algebra $J_n^{\pm} = \frac{1}{2}(J_n \pm K_n)$

$$Y_n \sim \sum J_{n-p} K_p \qquad T_n \sim J_n$$

to centerless warped conformal algebra

$$[Y_n,\, Y_m] = (n-m) Y_{n+m}$$
$$[Y_n,\, T_m] = -m T_{n+m}$$
$$[T_n,\, T_m] = 0$$

Warped CFT counting

- Map near horizon algebra $J_n^{\pm} = \frac{1}{2}(J_n \pm K_n)$

$$Y_n \sim \sum J_{n-p} K_p \qquad T_n \sim J_n$$

to centerless warped conformal algebra

$$[Y_n,\, Y_m] = (n-m) Y_{n+m}$$
$$[Y_n,\, T_m] = -m T_{n+m}$$
$$[T_n,\, T_m] = 0$$

- Modular property $Z(\beta, \theta) = \mathrm{Tr}\,(e^{-\beta H + i\theta J}) = Z(2\pi\beta/\theta, -4\pi^2/\theta)$ ($H = Q[\partial_v]$, $J = Q[\partial_\varphi]$) projects partition function to ground state for small imaginary θ (we need $\theta \to 0$)

Warped CFT counting

- Map near horizon algebra $J_n^\pm = \frac{1}{2}(J_n \pm K_n)$

$$Y_n \sim \sum J_{n-p} K_p \qquad T_n \sim J_n$$

to centerless warped conformal algebra

$$[Y_n,\, Y_m] = (n-m) Y_{n+m}$$
$$[Y_n,\, T_m] = -m T_{n+m}$$
$$[T_n,\, T_m] = 0$$

- Modular property $Z(\beta, \theta) = \mathrm{Tr}\,(e^{-\beta H + i\theta J}) = Z(2\pi\beta/\theta, -4\pi^2/\theta)$ ($H = Q[\partial_v]$, $J = Q[\partial_\varphi]$) projects partition function to ground state for small imaginary θ (we need $\theta \to 0$)
- Assuming $J^{\mathrm{vac}} = 0$ yields

$$S = \beta H = S_{\mathrm{BH}}$$

Hamiltonian H is product of BH entropy and Unruh temperature

Outline

Comparison to related approaches

- Brown, Henneaux '86

  Our boundary conditions differ from Brown–Henneaux — their chemical potentials depend on our charges and chemical potentials!

> Virasoro composite in terms of Heisenberg algebra

Comparison to related approaches

- Brown, Henneaux '86
- Donnay, Giribet, González, Pino 1511.08687
  - Observed already $H = TS_{\rm BH}$
  - Changing our bc's to

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho - 2\omega a^{-1}\,\mathrm{d}\varphi\,\mathrm{d}\rho + 4\omega\rho\,\mathrm{d}v\,\mathrm{d}\varphi + \big[\gamma^2 + \tfrac{2\rho}{a}(\gamma^2 - \omega^2)\big]\,\mathrm{d}\varphi^2 + \mathcal{O}(\rho^2)$$

yields AKVs

$$\xi = T(\varphi)\partial_v + Y(\varphi)\partial_\varphi + \mathcal{O}(\rho^3)$$

  - Up to subleading terms same AKVs as DGGP

But: T and Y state-dependent for our boundary conditions!

Comment: map to Brown–Henneaux variables requires second chemical potential, not just Rindler acceleration!

Warped CFT algebra composite in terms of Heisenberg algebra

Comparison to related approaches

- Brown, Henneaux '86
- Donnay, Giribet, González, Pino 1511.08687
- Afshar, Detournay, DG, Oblak 1512.08233

Rindler acceleration state-dependent in that approach

Twisted warped CFT algebra composite in terms of Heisenberg algebra

Comparison to related approaches

- Brown, Henneaux '86
- Donnay, Giribet, González, Pino 1511.08687
- Afshar, Detournay, DG, Oblak 1512.08233
- Hawking, Perry, Strominger 1601.00921
    - We constructed explicitly gravitational soft hair
    - We find no soft hair contribution to black hole entropy
    - BMS_3 follows from Sugawara-like construction from Heisenberg algebra

BMS algebra (supertranslations + superrotation) composite in terms of near horizon Heisenberg algebra

Comparison to related approaches

- Brown, Henneaux '86
- Donnay, Giribet, González, Pino 1511.08687
- Afshar, Detournay, DG, Oblak 1512.08233
- Hawking, Perry, Strominger 1601.00921
- Comment on complementarity:

> - Asymptotic Virasoro algebra composite from near horizon perspective
> - Same physics described naturally in different variables for asymptotic and near horizon observers
> - In particular, asymptotic chemical potentials depend on near horizon charges and chemical potentials

Elaborations and generalizations

- More on dual field theory — to be done
- Flat space
  - Similar story works!
  - Get centerless BMS_3 as composite algebra from Heisenberg algebra!
  - Soft hairy flat space cosmologies
  - Asymptotic chemical potentials again depend on near horizon charges and chemical potentials
  - Obtain again Bekenstein–Hawking entropy with no soft hair contribution

Elaborations and generalizations

- More on dual field theory — to be done
- Flat space
- (Topologically) massive gravity (Deser, Jackiw, Templeton '82) — To be done! Doable!

Elaborations and generalizations

- More on dual field theory — to be done
- Flat space
- (Topologically) massive gravity (Deser, Jackiw, Templeton '82) — To be done! Doable!
- Higher spins — with Stefan Prohazka: similar story works!

Elaborations and generalizations

- More on dual field theory — to be done
- Flat space
- (Topologically) massive gravity (Deser, Jackiw, Templeton '82) — To be done! Doable!
- Higher spins — with Stefan Prohazka: similar story works!
- Lower spins — lowest spin gravity! (see Hofman, Rollier `1411.0672`)

Elaborations and generalizations

- More on dual field theory — to be done
- Flat space
- (Topologically) massive gravity (Deser, Jackiw, Templeton '82) — To be done! Doable!
- Higher spins — with Stefan Prohazka: similar story works!
- Lower spins — lowest spin gravity! (see Hofman, Rollier `1411.0672`)
- 4d — Does it work? Is there soft Heisenberg hair? Is BMS_4 composite? What are near horizon symmetries?

> Near horizon symmetries shed new light on soft hair, microstate counting and complementarity

Thanks for your attention!

H. Afshar, S. Detournay, D. Grumiller, W. Merbis, A. Perez, D. Tempo and R. Troncoso
"Soft Heisenberg hair on black holes in three dimensions,"
Phys.Rev.**D** [R] (2016), in print; 1603.04824.

H. Afshar, S. Detournay, D. Grumiller and B. Oblak
"Near-Horizon Geometry and Warped Conformal Symmetry,"
JHEP **1603** (2016) 187; 1512.08233.

Thanks to Bob McNees for providing the LaTeX beamerclass!

Bonus level: exact metric with generic chemical potentials

Our bc's for the connection $A^{\pm} = b_{\pm}^{-1}(\rho)\left(\mathrm{d}+\mathfrak{a}_{\pm}(x^0,\, x^1)\right)b_{\pm}(\rho)$ with

$$\mathfrak{a}_{\pm} = \left(\mathcal{J}_{\pm}\,\mathrm{d}\varphi + \zeta^{\pm}\,\mathrm{d}v\right)\,L_0$$

and $b_{\pm} = \exp\left(\frac{1}{\zeta^{\pm}}\,L_+\right)\cdot\exp\left(\frac{\rho}{2}\,L_-\right)$ lead to the metric

$$\mathrm{d}s^2 = \frac{1}{2}\left\langle\left(A_\mu^+ - A_\mu^-\right)\left(A_\nu^+ - A_\nu^-\right)\right\rangle\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

$$\begin{aligned}
= &\left(-\frac{(\zeta^{+2}+\partial_v\zeta^+)(\zeta^{-2}+\partial_v\zeta^-)}{\zeta^{+2}\zeta^{-2}}\rho^2 + \frac{\zeta^{+3}\zeta^{-2}+\zeta^{+2}\zeta^{-3}+\partial_v\zeta^+\zeta^{-3}+\zeta^{+3}\partial_v\zeta^-}{\zeta^{+2}\zeta^{-2}}\rho + \frac{1}{4}(\zeta^- - \zeta^+)^2\right)\mathrm{d}v^2 \\
&+\left(\frac{(-\zeta^{+2}-\partial_v\zeta^+)\partial_\varphi\zeta^- + (-\zeta^{-2}-\partial_v\zeta^-)\partial_\varphi\zeta^+ - \mathcal{J}_+\zeta^+\partial_v\zeta^- + \zeta^-(\mathcal{J}_-\zeta^{+2}-\mathcal{J}_+\zeta^+\zeta^- + \mathcal{J}_-\partial_v\zeta^+)}{2\zeta^{+2}\zeta^{-2}}\rho^2\right. \\
&+\frac{\partial_\varphi\zeta^-\zeta^{+3}+\partial_\varphi\zeta^+\zeta^{-3}+\mathcal{J}_+\zeta^{+2}\partial_v\zeta^- - \zeta^-\left(\mathcal{J}_-\partial_v\zeta^+\zeta^- + \zeta^+(\zeta^-+\zeta^+)(\zeta^+\mathcal{J}_- - \zeta^-\mathcal{J}_+)\right)}{2\zeta^{+2}\zeta^{-2}}\rho \\
&\left.-\frac{1}{4}(\zeta^- - \zeta^+)(\mathcal{J}_- + \mathcal{J}_+)\right)\mathrm{d}v\,\mathrm{d}\varphi + \left(1 + \frac{\partial_v\zeta^-\zeta^{+2}+\partial_v\zeta^+\zeta^{-2}}{2\zeta^{+2}\zeta^{-2}}\right)\mathrm{d}v\,\mathrm{d}\rho \\
&+\left(\frac{(\mathcal{J}_+\zeta^+ + \partial_\varphi\zeta_+)(\mathcal{J}_-\zeta^- - \partial_\varphi\zeta^-)}{\zeta^{+2}\zeta^{-2}}\rho^2 + \frac{\mathcal{J}_+\partial_\varphi\zeta^-\zeta^{+2} - \zeta^-\mathcal{J}_-(\zeta^-\partial_\varphi\zeta^+ + \mathcal{J}_+\zeta^+(\zeta^-+\zeta^+))}{\zeta^{+2}\zeta^{-2}}\rho\right. \\
&\left.+\frac{1}{4}(\zeta^- + \zeta^+)^2\right)\mathrm{d}\varphi^2 + \left(\frac{\mathcal{J}_+\zeta^+\zeta^{-2} - \mathcal{J}_-\zeta^{+2}\zeta^- + \partial_\varphi\zeta^+\zeta^{-2} + \partial_\varphi\zeta^-\zeta^{+2}}{2\zeta^{+2}\zeta^{-2}}\right)\mathrm{d}\varphi\,\mathrm{d}r
\end{aligned}$$