Lifshitz anisotropy from boundary conditions

Daniel Grumiller

Institute for Theoretical Physics
TU Wien

~~Applied~~ Newton–Cartan geometry
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Outline

Motivation

Higher spin gravity

Lower spin gravity

Higher lower spin gravity

Einstein gravity

$z \to 0$ and near horizon physics

Outline

Overarching long-term theme: how general is holography?

More specifically: what is the landscape of gravity theories with Lifshitz anisotropy?

Quote from first sentence of workshop description: "Recent studies of non-AdS holography involving Lifshitz spacetimes have led to …"

3[rd] image googling "landscape of theories" (first two: book covers "The Landscape of Qualitative Research")

Anisotropic scaling of Lifshitz type

Asymptotic line-element

$$\mathrm{d}s^2 = -\frac{\mathrm{d}t^2}{r^{2z}} + \frac{1}{r^2}\left(\mathrm{d}r^2 + \mathrm{d}\vec{x}^2\right)$$

with real anisotropy parameter z has anisotropic ("Lifshitz") scaling

$$t \to \lambda^z t \qquad \vec{x} \to \lambda \vec{x} \qquad r \to \lambda r$$

between time t and space $\vec{x}$.

Kachru, Liu, Mulligan '08

Their construction (and many others) use p-form gauge fields; others use massive gauge fields or massive gravitons

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Questions **not** addressed in this talk:

- (How) does this lead to applications in cond-mat or otherwise?
- What are relations to flat space holography?
  work with/by Bagchi et al. '12-'16

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Motivations, applications and relations to Newton–Cartan:
see other talks! Technical simplification: work in 2+1 dimensions!

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Example:

$$\Phi(x \to \infty) = 0$$

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Example: Brown-Henneaux type of bc's (aAdS$_3$):

$$\mathrm{d}s^2_{\text{aAdS}} = \mathrm{d}\rho^2 + \big(e^{2\rho}\eta_{\mu\nu} + \gamma_{\mu\nu} + \mathcal{O}(e^{-2\rho})\big)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

with $\delta\gamma =$ arbitrary

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  1. Trafos that violate bc's (forbidden)
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- Canonical boundary charges (á la Regge–Teitelboim) generate asympotic symmetries
- Consistency means they are finite, integrable, non-trivial and conserved (in time)

Outline

Chern–Simons (CS) theory with gauge group containing $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$

2-minute crash course on higher spins in three dimensions

Simplest example: spin-3 gravity

$$S = I_{\text{CS}}[A^+] - I_{\text{CS}}[A^-]$$

in CS formulation

$$I_{\text{CS}}[A^\pm] = \frac{k}{4\pi} \int \langle A^\pm \wedge \mathrm{d}A^\pm + \tfrac{2}{3}\, A^\pm \wedge A^\pm \wedge A^\pm \rangle$$

with $SL(3,\mathbb{R})$ connections $A^\pm$ and suitable boundary conditions
(more on boundary conditions on next slide!)

Henneaux, Rey '10; Campoleoni, Fredenhagen, Pfenninger, Theisen '10

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- Zuvielbein: $e\sim A^+ - A^-$; higher-spin connection: $\omega\sim A^+ + A^-$
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- Generalization to spin-N: replace $sl(3)$ by $sl(N)$

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- Further generalization: non-principal embeddings of $sl(2) \hookrightarrow sl(N)$

Gravity-like bc's in CS gauge theories

Standard trick: partially fix gauge

$$A^{\pm} = b_{\pm}^{-1}(\rho,\, x^i)\left(\mathrm{d}+\mathfrak{a}_{\pm}(x^i)\right)b_{\pm}(\rho,\, x^i)$$

with some space-time dependent group elements $b_{\pm} \in SL(N)$ with $\delta b_{\pm} = 0$

Drop $\pm$ decorations in most of talk

Manifold topologically a cylinder or torus, with radial coordinate ρ and boundary coordinates x^i

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- Standard AdS$_3$ approach in highest weight gauge

  $$\mathfrak{a} \sim L_1 + \mathcal{L}(x^i)L_{-1} + \mathcal{W}(x^i)W_{-2} \qquad b(\rho) = \exp(\rho L_0)$$

  variations allowed by bc's:

  $$\delta\mathfrak{a} \sim \delta\mathcal{L}(x^i)L_{-1} + \delta\mathcal{W}(x^i)W_{-2} \qquad \delta b = 0$$

  Notation: $sl(2)$: $[L_n,\, L_m] = (n-m)L_{n+m}$
  $sl(3)$: $[L_n, W_m] = (2n-m)W_{n+m}$ and
  $[W_n, W_m] \propto (n-m)(2n^2+2m^2-nm-8)L_{n+m}$

  spin-3 analog of Brown–Henneaux bc's

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- Other embeddings possible for same gauge group

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- $sl(N)$ allows for Lifshitz exponents $z = 1, 2, \dots (N-1)$ and all possible fractions thereof   Gary, DG, Rashkov '12

Simplest example

Gary, DG, Prohazka, Rey '14

... in fact, too simple!

- Spin-3 gravity in principal embedding with almost same bc's as before with additional terms:

$$\mathfrak{a} \sim W_2 + L_1 + \mathcal{L}(x^i)L_{-1} + \mathcal{W}(x^i)W_{-2} + \text{extra terms}$$

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- This is a Lifshitz spacetime with $z = 2$!
Technical origin of possible values of z in spin-N gravity: generators with $sl(2)$-weights $2, 3, \dots N$ in connection $\mathfrak{a}$ lead by BCH-formula (commuting $b = e^{\rho L_0}$ through in $b^{-1}\mathfrak{a}b$) to exponents $e^{\rho}, e^{2\rho}, \dots, e^{(N-1)\rho}$ in zuvielbein

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  On plus side, example above is inequivalent to standard spin-3 black holes with spin-3 chemical potentials, while example in Gutperle, Hijano, Samani '13 is equivalent to them

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- HS theories (without matter) can yield anisotropic scaling!

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What is lower spin gravity? name coined by Hofman, Rollier '14

Working definition of lower spin gravity in 2+1 dimensions:

- CS theory with gauge group **not** containing $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$
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  specific extensions of Bargmann, Newton–Hooke, Schrödinger and supersymmetric Bargmann

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Key aspects:

- Have non-relativistic/anisotropic algebra already as input in action, not only through bc's
- Still, bc's play crucial role for establishing theory with anisotropy

Carroll gravity as example

Bergshoeff, DG, Prohazka, Rosseel '16

Take CS action with connection $(a = 1, 2)$

$$A = \tau\,\mathtt{H} + e^a\,\mathtt{P}_a + \omega\,\mathtt{J} + B^a\,\mathtt{G}_a$$

in the spin-2 Carroll algebra

$$[\mathtt{J},\,\mathtt{P}_a] = \epsilon_{ab}\,\mathtt{P}_b$$
$$[\mathtt{J},\,\mathtt{G}_a] = \epsilon_{ab}\,\mathtt{G}_b$$
$$[\mathtt{P}_a,\,\mathtt{G}_b] = -\epsilon_{ab}\,\mathtt{H}$$

with non-degenerate bi-linear form

$$\langle \mathtt{H}, \mathtt{J} \rangle = -1 \qquad \langle \mathtt{P}_a, \mathtt{G}_b \rangle = \delta_{ab}$$

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Typical question in holographic correspondences on gravity side:

Are there **nice** bc's for this theory?

Asymptotically Carroll geometries and ∞ extension of the Carroll algebra

- Asymptotic Carroll geometry (2d metric plus 1-form) from CS connection:

$$\mathrm{d}s^2_{(2)} = e^a e^b \, \delta_{ab} = \big(\rho^2 + \mathcal{O}(\rho)\big) \, \mathrm{d}\varphi^2 + \mathcal{O}(1) \, \mathrm{d}\rho \, \mathrm{d}\varphi + \mathrm{d}\rho^2$$

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- Our proposed bc's are given by connections of the form

$$A = b^{-1}\big(\,\mathrm{d} + \mathfrak{a}\big)\, b \qquad b = e^{\rho \mathtt{P}_2}$$

with

$$\mathfrak{a}_\varphi = -\mathtt{J} + h(t,\,\varphi)\,\mathtt{H} + p_a(t,\,\varphi)\,\mathtt{P}_a + g_a(t,\,\varphi)\,\mathtt{G}_a$$
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- Leads to line-elements above, i.e., asymptotic Carroll geometries

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- Background independent result for canonical boundary charges:

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- Fourier modes of charges lead to infinite tower of generators $\Rightarrow$ infinite enhancement of global Carroll algebra reminiscent of $\text{AdS}_3/\text{CFT}_2$ $\Rightarrow$ meaningful (and hopefully useful) set of bc's!

Asymptotic symmetry algebra (ASA) of Carroll gravity

- recall: gauge algebra was spin-2 Carroll algebra

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Carroll gravity intriguing theory with numerous possible generalizations

Outline

Can combine higher and lower spin manipulations simultaneously

Spin-N theories with general kinematical algebras Bergshoeff, DG, Prohazka, Rosseel '16

- One example would be a CS theory based on $SL(N) \times U(1)$ (higher spin in one chiral sector and lower spin in another)

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$$[\,\mathfrak{h}, \mathfrak{h}\,] \sim \mathfrak{h} \qquad [\,\mathfrak{h}, \mathfrak{i}\,] \sim \epsilon\,\mathfrak{h} + \mathfrak{i} \to \mathfrak{i} \qquad [\,\mathfrak{i}, \mathfrak{i}\,] \sim \epsilon^2\,\mathfrak{h} + \epsilon\,\mathfrak{i} \to 0$$

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- Obtain zoo of ~~higher lower~~ non-relativstic higher spin theories, e.g. spin-3 versions of Carroll, Galilei and extended Bargmann algebras

Example: spin-3 extended Bargmann   Bergshoeff, DG, Prohazka, Rosseel '16

- Medina–Revoy theorem allows to extend Galilei to extended Bargmann (Galilei + 2 central ext's; non-degenerate bilinear form)

  Special case: if algebra comes from an İnönü–Wigner contraction

$$[\,\mathfrak{h},\mathfrak{h}\,]\sim\mathfrak{h}\qquad[\,\mathfrak{h},\mathfrak{i}\,]\sim\mathfrak{i}\qquad[\,\mathfrak{i},\mathfrak{i}\,]=0$$

  then MR theorem always applicable:
  extends algebra by dual $\mathfrak{h}^*$ and yields commutations relations

$$\begin{aligned}[\,\mathfrak{i},\mathfrak{i}\,]&\sim\mathfrak{h}^* & [\,\mathfrak{h},\mathfrak{h}\,]&\sim\mathfrak{h}^*\\ [\,\mathfrak{h},\mathfrak{i}\,]&\sim\mathfrak{i} & [\,\mathfrak{h},\mathfrak{h}^*\,]&\sim\mathfrak{h}^*\\ [\,\mathfrak{h}^*,\mathfrak{i}\,]&=0 & [\,\mathfrak{h}^*,\mathfrak{h}^*\,]&=0\end{aligned}$$

  and non-degenerate invariant bilinear form

$$\langle\mathfrak{h},\mathfrak{h}^*\rangle=\delta\qquad\langle\mathfrak{i},\mathfrak{i}\rangle=g\qquad\langle\mathfrak{h},\mathfrak{h}\rangle=\text{arbitray (can be 0)}$$

Example: spin-3 extended Bargmann Bergshoeff, DG, Prohazka, Rosseel '16

- Medina–Revoy theorem allows to extend Galilei to extended Bargmann (Galilei + 2 central ext's; non-degenerate bilinear form)
- Applying same methods to spin-3 Galilei yields spin-3 extended Bargmann (2 versions exist, one given below with 2 + 4 ext's)

$[\mathtt{J},\mathtt{G}_a]=\epsilon_{am}\mathtt{G}_m$	$[\mathtt{J},\mathtt{G}_{ab}]=-\epsilon_{m(a}\mathtt{G}_{b)m}$	$[\mathtt{J}^*_a,\mathtt{J}_b]=\epsilon_{ab}\mathtt{J}^*$
$[\mathtt{H},\mathtt{G}_a]=\epsilon_{am}\mathtt{P}_m$	$[\mathtt{J},\mathtt{P}_{ab}]=-\epsilon_{m(a}\mathtt{P}_{b)m}$	$[\mathtt{H}^*_a,\bullet_b]=\epsilon_{ab}\mathtt{H}^*/\mathtt{J}^*$
$[\mathtt{J},\mathtt{P}_a]=\epsilon_{am}\mathtt{P}_m$	$[\mathtt{H},\mathtt{G}_{ab}]=-\epsilon_{m(a}\mathtt{P}_{b)m}$	$[\mathtt{J}_a,\mathtt{J}_b]=\epsilon_{ab}\mathtt{J}$
$[\mathtt{G}_a,\mathtt{G}_b]=\epsilon_{ab}\mathtt{H}^*$	$[\mathtt{G}_a,\bullet_b]=\Delta_{ab}(\mathtt{G}/\mathtt{P})$	$[\mathtt{J}_a,\mathtt{H}_b]=\epsilon_{ab}\mathtt{H}$
$[\mathtt{P}_a,\mathtt{G}_b]=\epsilon_{ab}\mathtt{J}^*$	$[\mathtt{P}_a,\mathtt{J}_a]=\Delta_{ab}(\mathtt{P})$	$[\mathtt{G}_{ab},\bullet_c]=-\delta_{c(a}\epsilon_{b)m}\mathtt{G}_m/\mathtt{P}_m$
$[\mathtt{J},\mathtt{H}^*_a]=\epsilon_{am}\mathtt{H}^*_m$	$[\mathtt{G}_a,\mathtt{G}_{bc}]=\epsilon_{a(b}\mathtt{H}^*_{c)}$	$[\mathtt{P}_{ab},\mathtt{J}_c]=-\delta_{c(a}\epsilon_{b)m}\mathtt{P}_m$
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$[\mathtt{J},\bullet_a]=\epsilon_{am}\bullet_m$	$[\mathtt{J}^*,\mathtt{J}_a]=\epsilon_{am}\mathtt{J}^*_m$	$\Delta_{ab}(\mathtt{P}):=\epsilon_{ma}\mathtt{P}_{bm}+\epsilon_{ba}\mathtt{P}_{mm}$
$[\mathtt{H},\mathtt{J}_a]=\epsilon_{am}\mathtt{H}_m$	$[\mathtt{H}^*,\bullet_a]=\epsilon_{am}\mathtt{H}^*_m/\mathtt{J}^*_m$	$\bullet_a := \mathtt{J}_a/\mathtt{H}_a$ (either/or)

Outline

Motivation

Higher spin gravity

Lower spin gravity

Higher lower spin gravity

Einstein gravity

$z \to 0$ and near horizon physics

AdS_3 bc's in Einstein gravity

Even restricting to Einstein gravity in three dimensions (with negative cosmological constant) different choices exist for bc's and their associated asymptotic symmetry algebras:

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- Troessaert '13: 2 Virasoros plus 2 $u(1)$ current algebras
- Avery–Poojary–Suryanarayana '13: Virasoro plus $sl(2)$ current algebra
- Donnay–Giribet–Gonzalez–Pino '15: centerless warped conformal
- Afshar–Detournay–DG–Oblak '15: twisted warped conformal

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In the following I use neither of these bc's!

Recall AdS_3 Brown–Henneaux bc's in presence of source/chemical potential μ:

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 Perez, Tempo, Troncoso '16

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- $\mu^{\pm} = R_{(k)}^{\pm}$ (Gelfand–Dikii polynomial): k^{th} representative of KdV hierarchy, $\pm\dot{\mathcal{L}}_{\pm} = D^{\pm} R_{(k)}^{\pm}$ with $D^{\pm} = \mathcal{L}_{\pm}' + 2\mathcal{L}_{\pm}\partial_\varphi - 2\partial_\varphi^3$

Reminder: Gelfand–Dikii polynomials defined by recursion relation

$$R_{(k+1)}^{\pm\,\prime} = \frac{k+1}{2k+1}\, D^{\pm} R_{(k)}^{\pm} \qquad \text{with } R_{(0)}^{\pm} = 1$$

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- $\mu^\pm = R_{(k)}^\pm$ (Gelfand–Dikii polynomial): k^{th} representative of KdV hierarchy, $\pm\dot{\mathcal{L}}_\pm = D^\pm R_{(k)}^\pm$ with $D^\pm = \mathcal{L}_\pm' + 2\mathcal{L}_\pm\partial_\varphi - 2\partial_\varphi^3$
- key observation for this talk: EOM invariant under anisotropic scaling

$$z = 2k+1 \qquad t \to \lambda^z t \qquad \varphi \to \lambda\varphi \qquad \mathcal{L}_\pm \to \lambda^{-2}\mathcal{L}_\pm$$

KdV bc's

Perez, Tempo, Troncoso '16

Recall AdS_3 Brown–Henneaux bc's in presence of source/chemical potential μ:

$$A^{\pm} = b_{\pm}^{-1}\big(\mathrm{d}+\mathfrak{a}^{\pm}\big)b_{\pm} \qquad b_{\pm} = e^{\pm\rho L_0}$$

with

$$\mathfrak{a}_\varphi = L_{\pm} + \mathcal{L}_{\pm} L_{\mp}$$

$$\mathfrak{a}_t = \mu^{\pm}\big(L_{\pm} + \mathcal{L}_{\pm} L_{\mp}\big) \mp \mu^{\pm\prime} L_0 + \tfrac{1}{2}\,\mu^{\pm\prime\prime} L_{\mp}$$

If sources depend on charges, $\mu^{\pm} = \delta H^{\pm}/\delta\mathcal{L}_{\pm}$, then get new bc's

- $\mu^{\pm} = 1$: Brown–Henneaux
- $\mu^{\pm} = \mathcal{L}_{\pm}$: KdV, $\pm\dot{\mathcal{L}}_{\pm} = 3\mathcal{L}_{\pm}\mathcal{L}_{\pm}' - 2\mathcal{L}_{\pm}'''$
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Anisotropic scaling of Lifshitz type in Einstein gravity

Outline

Near horizon bc's

Afshar, Detournay, DG, Merbis, Perez, Tempo, Troncoso '16

Non-extremal horizon (Rindler spacetime for $\rho \to 0$) achieved by bc's

$$A^{\pm} = b_{\pm}^{-1}\big(\,\mathrm{d}+\mathfrak{a}_{\pm}\big)b_{\pm} \qquad \mathfrak{a}^{\pm} = \big(\pm\,\mathcal{J}^{\pm}\,\mathrm{d}\varphi + \zeta^{\pm}\,\mathrm{d}t\big)\,L_0 \qquad \delta\zeta^{\pm} = 0$$

Interesting features of our bc's:

- Symmetry algebra: infinite copies of Heisenberg algebras

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Interesting features of our bc's:

- Symmetry algebra: infinite copies of Heisenberg algebras
- Explicit construction of all soft hair descendants
- Explicit proposal for all microstates of BTZ Afshar, DG, Sheikh-Jabbari '16

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Interesting features of our bc's:

- Astonishingly simple and universal* result for entropy

$$S = 2\pi\big(J_0^+ + J_0^-\big)$$

To give an idea how much simpler the formula above is in higher spin theories than usual entropy formulas, here is the same result expressed not in terms of charges $J_0^\pm$ for our bc's, but for Henneaux–Rey–Campoleoni–Fredenhagen–Pfenninger–Theisen bc's (see Guperle, Kraus '11; Ammon, Gutperle, Kraus, Perlmutter '12)

$$S = 2\pi\sqrt{2\pi k}\left(\sqrt{\mathcal{L}_+}\cos\left[\tfrac{1}{3}\arcsin\left(\tfrac{3}{8}\sqrt{\frac{3k}{2\pi\mathcal{L}_+^3}}\mathcal{W}_+\right)\right] + (+\to -)\right)$$

*Applies to AdS, flat space, higher spins, higher derivatives and higher dimensions

Near horizon bc's   Afshar, Detournay, DG, Merbis, Perez, Tempo, Troncoso '16

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- Line-element $\mathrm{d}s^2 = -\zeta^2 r^2\,\mathrm{d}t^2 + \mathrm{d}r^2 + \mathcal{J}^2\,\mathrm{d}\varphi^2 + \dots$ has anisotropic scaling symmetry like Lifshitz with $z \to 0$

$$t \to t \qquad \varphi \to \lambda\varphi \qquad \mathcal{J} \to \lambda^{-1}\mathcal{J}$$

Technical notes: scaling of $\mathcal{J}$ induced by Sugawara construction $\mathcal{L} \sim \mathcal{J}^2 + \mathcal{J}'$ from KdV-type scaling $\mathcal{L} \to \lambda^{-2}\mathcal{L}$
Miura map shows that Rindler acceleration ζ does not scale

Suggests KdV level $k = -1/2$! (recall: $k = 2z + 1$)
If true, Lifshitz entropy formula must reproduce simple result above!

Near horizon bc's   Afshar, Detournay, DG, Merbis, Perez, Tempo, Troncoso '16

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- Result follows indeed from $z \to 0$ limit of entropy formula for theories with Lifshitz scaling in 1+1 dimensions (with $\Delta_{\pm} = J_0^{\pm}$)

$$S = 2\pi(1+z)\sum_{\pm}\Delta_{\pm}{}^{1/(1+z)}\exp\Big(\tfrac{z}{1+z}\,\ln\,\big(\Delta_0^{\pm}\,[1/z]\,/z\big)\Big)$$

note: ground state energies $\Delta_0^{\pm}[z] = \frac{k}{2}\frac{1}{1+z}(-1)^{(z-1)/2}$ also match with gravity side, but not needed in entropy formula for $z \to 0$!

Near horizon bc's   Afshar, Detournay, DG, Merbis, Perez, Tempo, Troncoso '16

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- Interesting math question: why (generalized) Gelfand–Dikii polynomial $R_{(-1/2)}$ and KdV level $k = -1/2$ special?

Summary

Anisotropic spacetimes of Lifshitz or Schrödinger type can be obtained through imposition of suitable bc's in

- Higher spin theories in 2+1

  work with Gary, Rashkov; Afshar, Riegler; Prohazka, Rey; Breunhölder '12-15
- Lower spin theories 2+1

  work with Bergshoeff, Prohazka, Rosseel '16
- ~~Higher lower spin~~ Non-relativistic higher spin theories in 2+1

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Thanks for your attention!