Flat Space Holography

Daniel Grumiller

Institute for Theoretical Physics
TU Wien

Karl Schwarzschild meeting
Frankfurt Institute for Advanced Studies, July 2015

based on work w. Afshar, Bagchi, Basu, Detournay, Fareghbal, Gary, Merbis, Riegler, Rosseel, Salzer, Sarkar, Schöller, Simon, ...
Some of our papers on flat space holography

A. Bagchi, D. Grumiller and W. Merbis,
“Stress tensor correlators in three-dimensional gravity,”
arXiv:1507.05620. [see arXiv today]

A. Bagchi, R. Basu, D. Grumiller and M. Riegler,
“Entanglement entropy in Galilean conformal field theories and flat holography,”

H. Afshar, A. Bagchi, R. Fareghbal, D. Grumiller and J. Rosseel,
“Spin-3 Gravity in Three-Dimensional Flat Space,”

A. Bagchi, S. Detournay, D. Grumiller and J. Simon,
“Cosmic Evolution from Phase Transition of Three-Dimensional Flat Space,”

A. Bagchi, S. Detournay and D. Grumiller,
“Flat-Space Chiral Gravity,”
Outline

Motivations

Assumptions

Flat space holography basics

Recent results

Generalizations & open issues
Outline

Motivations

Assumptions

Flat space holography basics

Recent results

Generalizations & open issues
The 2nd Karl Schwarzschild Meeting on Gravitational Physics ... will focus on the general theme of black holes, gravity and information.
The 2\textsuperscript{nd} Karl Schwarzschild Meeting on Gravitational Physics ... will focus on the general theme of black holes, gravity and information.

This talk focuses on holography.
Quote from the webpage of this meeting

The 2\textsuperscript{nd} Karl Schwarzschild Meeting on Gravitational Physics ... will focus on the general theme of black holes, gravity and information.

This talk focuses on holography.

Main question: how general is holography?
Testing the holographic principle

How general is holography?

- Holographic principle realized in AdS/CFT correspondence
- Special case or generic lesson for quantum gravity?

\( \text{AdS}_{d+1} \to \text{CFT}_d \)

- Use (classical) gravity to learn more about CFTs
- Strong coupling large \( N \) limit: classical gravity
- Useful tool to calculate correlation functions
- Useful tool to calculate entanglement entropy

\( \text{CFT}_d \to \text{AdS}_{d+1} \)

- Use CFTs to learn more about (quantum) gravity
- Gravity in ultra-quantum limit: simple CFT?
- Useful tool to address black hole microstates
- Useful tool for qu-gr puzzles (information paradox)

To what extent do (previous) lessons rely on the particular constructions used to date?

- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?
- Does holography apply only to unitary theories?
- Can we establish a flat space holographic dictionary?
- Generic non-AdS holography/higher spin holography?
Testing the holographic principle

How general is holography?

Historical curiosity:
Karl Schwarzschild (1873-1916)

Simplest of all (classical) black holes: Schwarzschild solution (22.12.1915)

\[
 ds^2 = (1 - \gamma/R) \, dt^2 - \frac{dR^2}{1 - \gamma/R} \\
 - R^2 \left( d\theta^2 + \sin^2\theta \, d\phi^2 \right) 
\]

(Schwarzschild’s conventions in letter to Einstein)
Testing the holographic principle

How general is holography?

Historical curiosity:
Karl Schwarzschild (1873-1916)

Simplest of all (classical) black holes: Schwarzschild solution (22.12.1915)

\[
\begin{align*}
\text{ds}^2 &= (1 - \gamma/R) \, dt^2 - \frac{dR^2}{1 - \gamma/R} \\
&\quad - R^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right)
\end{align*}
\]

(Schwarzschild’s conventions in letter to Einstein)

- Quantum mechanically: among least understood black holes!
- Microstates?
- Log corrections to entropy?
- Dual field theory??
Testing the holographic principle

How general is holography?

- To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?

see numerous talks at KITP workshop “Bits, Branes, Black Holes” 2012
and at ESI workshop “Higher Spin Gravity” 2012
Testing the holographic principle

How general is holography?

- To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?
- Does holography apply only to unitary theories?

originally holography motivated by unitarity
Testing the holographic principle

How general is holography?

- To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?
- Does holography apply only to unitary theories?

- originally holography motivated by unitarity
- plausible AdS/CFT-like correspondence could work non-unitarily
- AdS/log CFT first example of non-unitary holography DG, (Jackiw), Johansson ’08; Skenderis, Taylor, van Rees ’09; Henneaux, Martinez, Troncoso ’09; Maloney, Song, Strominger ’09; DG, Sachs/Hohm ’09; Gaberdiel, DG, Vassilevich ’10; ... DG, Riedler, Rosseel, Zojer ’13
- recent proposal by Vafa ’14
Testing the holographic principle

How general is holography?

▶ To what extent do (previous) lessons rely on the particular constructions used to date?
▶ Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?
▶ Does holography apply only to unitary theories?
▶ Can we establish a flat space holographic dictionary?

The answer appears to be yes — see my current talk and recent papers by Bagchi et al., Barnich et al., Strominger et al., ’12-’15
Testing the holographic principle

How general is holography?

- To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?
- Does holography apply only to unitary theories?
- Can we establish a flat space holographic dictionary?
- Generic non-AdS holography/higher spin holography?

non-trivial hints that it might work at least in 2+1 dimensions
Gary, DG Rashkov ’12; Afshar et al ’12; Gutperle et al ’14–’15; Gary, DG, Prohazka, Rey ’14; ...
Testing the holographic principle

How general is holography?

- To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?
- Does holography apply only to unitary theories?
- Can we establish a flat space holographic dictionary?
- Generic non-AdS holography/higher spin holography?

- Address questions above in simple class of 3D toy models
- Exploit gauge theoretic Chern–Simons formulation
- Restrict to kinematic questions, like (asymptotic) symmetries
Goals of this talk

1. Review general aspects of holography in 3D
Goals of this talk

1. Review general aspects of holography in 3D
2. Discuss flat space holography
Goals of this talk

1. Review general aspects of holography in 3D
2. Discuss flat space holography
3. List selected open issues
Goals of this talk

1. Review general aspects of holography in 3D
2. Discuss flat space holography
3. List selected open issues

Address these issues in 3D!
Outline

Motivations

Assumptions

Flat space holography basics

Recent results

Generalizations & open issues
Assumptions

Working assumptions:
- 3D
Assumptions

Working assumptions:

- 3D
- Restrict to “pure gravity” theories
Assumptions

Working assumptions:

- 3D
- Restrict to “pure gravity” theories
- Define quantum gravity by its dual field theory

Interesting dichotomy:

- Either dual field theory exists \(\rightarrow\) useful toy model for quantum gravity
- Or gravitational theory needs UV completion (within string theory) \(\rightarrow\) indication of inevitability of string theory
Assumptions

Working assumptions:

- 3D
- Restrict to “pure gravity” theories
- Define quantum gravity by its dual field theory

Interesting dichotomy:

- Either dual field theory exists $\rightarrow$ useful toy model for quantum gravity
- Or gravitational theory needs UV completion (within string theory) $\rightarrow$ indication of inevitability of string theory

This talk:

- Remain agnostic about dichotomy
- Focus on generic features of dual field theories that do not require string theory embedding
Gravity in 3D
AdS$_3$ gravity

- Lowest dimension with black holes and (off-shell) gravitons
Gravity in 3D
AdS$_3$ gravity

- Lowest dimension with black holes and (off-shell) gravitons
- Weyl = 0, thus Riemann = Ricci
Gravity in 3D
AdS$_3$ gravity

- Lowest dimension with black holes and (off-shell) gravitons
- Weyl = 0, thus Riemann = Ricci
- Einstein gravity: no on-shell gravitons

Caveat: while there are many string compactifications with AdS$_3$ factor, applying holography just to AdS$_3$ factor does not capture everything!
Gravity in 3D

AdS$_3$ gravity

- Lowest dimension with black holes and (off-shell) gravitons
- Weyl = 0, thus Riemann = Ricci
- Einstein gravity: no on-shell gravitons
- Formulation as topological gauge theory (Chern–Simons)
Gravity in 3D
AdS$_3$ gravity

- Lowest dimension with black holes and (off-shell) gravitons
- Weyl $= 0$, thus Riemann $= $ Ricci
- Einstein gravity: no on-shell gravitons
- Formulation as topological gauge theory (Chern–Simons)
- Dual field theory (if it exists): 2D
Gravity in 3D
AdS$_3$ gravity

- Lowest dimension with black holes and (off-shell) gravitons
- Weyl $= 0$, thus Riemann $= \text{Ricci}$
- Einstein gravity: no on-shell gravitons
- Formulation as topological gauge theory (Chern–Simons)
- Dual field theory (if it exists): 2D
- Infinite dimensional asymptotic symmetries (Brown–Henneaux)
Gravity in 3D

AdS$_3$ gravity

- Lowest dimension with black holes and (off-shell) gravitons
- Weyl = 0, thus Riemann = Ricci
- Einstein gravity: no on-shell gravitons
- Formulation as topological gauge theory (Chern–Simons)
- Dual field theory (if it exists): 2D
- Infinite dimensional asymptotic symmetries (Brown–Henneaux)
- Black holes as orbifolds of AdS$_3$ (BTZ)
Gravity in 3D
AdS$_3$ gravity

- Lowest dimension with black holes and (off-shell) gravitons
- Weyl = 0, thus Riemann = Ricci
- Einstein gravity: no on-shell gravitons
- Formulation as topological gauge theory (Chern–Simons)
- Dual field theory (if it exists): 2D
- Infinite dimensional asymptotic symmetries (Brown–Henneaux)
- Black holes as orbifolds of AdS$_3$ (BTZ)
- Simple microstate counting from AdS$_3$/CFT$_2$
Gravity in 3D
AdS$_3$ gravity

- Lowest dimension with black holes and (off-shell) gravitons
- Weyl = 0, thus Riemann = Ricci
- Einstein gravity: no on-shell gravitons
- Formulation as topological gauge theory (Chern–Simons)
- Dual field theory (if it exists): 2D
- Infinite dimensional asymptotic symmetries (Brown–Henneaux)
- Black holes as orbifolds of AdS$_3$ (BTZ)
- Simple microstate counting from AdS$_3$/CFT$_2$
- Hawking–Page phase transition hot AdS ↔ BTZ

Caveat: while there are many string compactifications with AdS$_3$ factor, applying holography just to AdS$_3$ factor does not capture everything!
Gravity in 3D
AdS$_3$ gravity

- Lowest dimension with black holes and (off-shell) gravitons
- Weyl = 0, thus Riemann = Ricci
- Einstein gravity: no on-shell gravitons
- Formulation as topological gauge theory (Chern–Simons)
- Dual field theory (if it exists): 2D
- Infinite dimensional asymptotic symmetries (Brown–Henneaux)
- Black holes as orbifolds of AdS$_3$ (BTZ)
- Simple microstate counting from AdS$_3$/CFT$_2$
- Hawking–Page phase transition hot AdS ↔ BTZ
- Simple checks of Ryu–Takayanagi proposal
Gravity in 3D

AdS$_3$ gravity

- Lowest dimension with black holes and (off-shell) gravitons
- Weyl = 0, thus Riemann = Ricci
- Einstein gravity: no on-shell gravitons
- Formulation as topological gauge theory (Chern–Simons)
- Dual field theory (if it exists): 2D
- Infinite dimensional asymptotic symmetries (Brown–Henneaux)
- Black holes as orbifolds of AdS$_3$ (BTZ)
- Simple microstate counting from AdS$_3$/CFT$_2$
- Hawking–Page phase transition hot AdS ↔ BTZ
- Simple checks of Ryu–Takayanagi proposal

Caveat: while there are many string compactifications with AdS$_3$ factor, applying holography just to AdS$_3$ factor does not capture everything!
Picturesque analogy: soap films

Both soap films and Chern–Simons theories have

- essentially no bulk dynamics
- highly non-trivial boundary dynamics
- most of the physics determined by boundary conditions
- esthetic appeal (at least for me)
Outline

Motivations

Assumptions

Flat space holography basics

Recent results

Generalizations & open issues
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

▶ Works straightforwardly sometimes, otherwise not

▶ Example where it works nicely: asymptotic symmetry algebra

$\mathcal{L}_n, \bar{\mathcal{L}}_n$

$\mathcal{L}_n = \mathcal{L}_n - \bar{\mathcal{L}}_n + \frac{M_n}{\ell}(\mathcal{L}_n + \bar{\mathcal{L}}_n)$

▶ Make In"on"u–Wigner contraction $\ell \rightarrow \infty$ on ASA

$[\mathcal{L}_n, \mathcal{L}_m] = (n - m)\mathcal{L}_n + m + c\mathcal{L}_{\frac{1}{2}}(n^3 - n)$

$[\mathcal{L}_n, M_m] = (n - m)M_n + m + cM_{\frac{1}{2}}(n^3 - n)$

$[M_n, M_m] = 0$

▶ This is nothing but the BMS$_3$ algebra (or GCA$_2$, URCA$_2$, CCA$_2$)!

▶ Example where it does not work easily: boundary conditions

▶ Example where it does not work: highest weight conditions
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $\mathcal{L}_n, \bar{\mathcal{L}}_n$

\[
L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \quad M_n = \frac{1}{\ell} (\mathcal{L}_n + \bar{\mathcal{L}}_{-n})
\]

- Example where it does not work easily: boundary conditions
- Example where it does not work: highest weight conditions
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
  - Take linear combinations of Virasoro generators $\mathcal{L}_n, \bar{\mathcal{L}}_n$
    \[
    L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \quad M_n = \frac{1}{\ell} \left( \mathcal{L}_n + \bar{\mathcal{L}}_{-n} \right)
    \]
- Make Inönü–Wigner contraction $\ell \rightarrow \infty$ on ASA
  \[
  [L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m, 0}
  \]
  \[
  [L_n, M_m] = (n - m) M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m, 0}
  \]
  \[
  [M_n, M_m] = 0
  \]
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

▶ Works straightforwardly sometimes, otherwise not
▶ Example where it works nicely: asymptotic symmetry algebra
▶ Take linear combinations of Virasoro generators $\mathcal{L}_n, \bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \\
M_n = \frac{1}{\ell} \left( \mathcal{L}_n + \bar{\mathcal{L}}_{-n} \right)$$

▶ Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0} \\
[M_n, L_m] = (n - m) M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m,0} \\
[M_n, M_m] = 0$$

▶ This is nothing but the BMS$_3$ algebra (or GCA$_2$, URCA$_2$, CCA$_2$)!
Ashtekar, Bicak, Schmidt, '96; Barnich, Compere '06

$L_n$: diffeos of circle, $M_n$: supertranslations, $c_{L/M}$: central extensions
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $\mathcal{L}_n, \bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \frac{1}{\ell} (\mathcal{L}_n + \bar{\mathcal{L}}_{-n})$$

- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m, 0}$$

$$[L_n, M_m] = (n - m) M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m, 0}$$

$$[M_n, M_m] = 0$$

- This is nothing but the BMS$_3$ algebra (or GCA$_2$, URCA$_2$, CCA$_2$)!
  If dual field theory exists it must be a 2D Galilean CFT!

Bagchi et al., Barnich et al.
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
  - Take linear combinations of Virasoro generators $\mathcal{L}_n, \bar{\mathcal{L}}_n$
    \[
    L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \quad M_n = \frac{1}{\ell} (\mathcal{L}_n + \bar{\mathcal{L}}_{-n})
    \]
- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA
  \[
  [L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0}
  \]
  \[
  [L_n, M_m] = (n - m) M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m,0}
  \]
  \[
  [M_n, M_m] = 0
  \]
- This is nothing but the BMS$_3$ algebra (or GCA$_2$, URCA$_2$, CCA$_2$)?!
- Example where it does not work easily: boundary conditions
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $L_n$, $\bar{L}_n$
  
  $$L_n = L_n - \bar{L}_{-n} \quad M_n = \frac{1}{\ell} (L_n + \bar{L}_{-n})$$

- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA
  
  $$[L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0}$$
  
  $$[L_n, M_m] = (n - m) M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m,0}$$
  
  $$[M_n, M_m] = 0$$

- This is nothing but the BMS$_3$ algebra (or GCA$_2$, URCA$_2$, CCA$_2$)!
- Example where it does not work easily: boundary conditions
- Example where it does not work: highest weight conditions
Flat space Einstein gravity as $\mathfrak{isl}(2)$ Chern–Simons theory
For details, references and spin-3 generalization see Gary, DG, Riegler, Rosseel ’14

See also talk by Mirah Gary two days ago (Junior plenary session)
Flat space Einstein gravity as $\text{isl}(2)$ Chern–Simons theory
For details, references and spin-3 generalization see Gary, DG, Riegler, Rosseel ’14

- AdS gravity in CS formulation: $\text{sl}(2) \oplus \text{sl}(2)$ gauge algebra

  Achucarro, Townsend ’86; Witten ’88
Flat space Einstein gravity as isl(2) Chern–Simons theory
For details, references and spin-3 generalization see Gary, DG, Riegler, Rosseel '14

- AdS gravity in CS formulation: sl(2)⊕sl(2) gauge algebra
- Flat space: isl(2) gauge algebra

\[ S_{\text{CS}}^{\text{flat}} = \frac{k}{4\pi} \int (\mathcal{A} \wedge d\mathcal{A} + \frac{2}{3} \mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A}) \]

with isl(2) connection \((a = 0, \pm 1)\)

\[ \mathcal{A} = e^a M_a + \omega^a L_a \]

isl(2) algebra (global part of BMS/GCA)

\[ [L_a, L_b] = (a - b) L_{a+b} \]
\[ [L_a, M_b] = (a - b) M_{a+b} \]
\[ [M_a, M_b] = 0 \]

Note: \(e^a\) dreibein, \(\omega^a\) (dualized) spin-connection

Bulk EOM: gauge flatness → Einstein equations

\[ F = d\mathcal{A} + \mathcal{A} \wedge \mathcal{A} = 0 \]
Flat space Einstein gravity as isl(2) Chern–Simons theory
For details, references and spin-3 generalization see Gary, DG, Riegler, Rosseel ’14

- AdS gravity in CS formulation: sl(2)⊕sl(2) gauge algebra
- Flat space: isl(2) gauge algebra

\[ S_{CS}^{\text{flat}} = \frac{k}{4\pi} \int \langle A \wedge dA + \frac{2}{3} A \wedge A \wedge A \rangle \]

with isl(2) connection \((a = 0, \pm 1)\)

\[ A = e^a M_a + \omega^a L_a \]

- Boundary conditions in CS formulation:

\[ A(r, u, \varphi) = b^{-1}(r) \left( d + a(u, \varphi) + o(1) \right) b(r) \]
Flat space Einstein gravity as isl(2) Chern–Simons theory

For details, references and spin-3 generalization see Gary, DG, Riegler, Rosseel ’14

- AdS gravity in CS formulation: \( \text{sl}(2) \oplus \text{sl}(2) \) gauge algebra
- Flat space: isl(2) gauge algebra

\[
S_{\text{CS}}^{\text{flat}} = \frac{k}{4\pi} \int \langle \mathcal{A} \wedge d\mathcal{A} + \frac{2}{3} \mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A} \rangle
\]

with isl(2) connection \((a = 0, \pm 1)\)

\[
\mathcal{A} = e^a M_a + \omega^a L_a
\]

- Boundary conditions in CS formulation:

\[
\mathcal{A}(r, u, \varphi) = b^{-1}(r) \left( d + a(u, \varphi) + o(1) \right) b(r)
\]

- Flat space boundary conditions: \( b(r) = \exp \left( \frac{1}{2} r M_{-1} \right) \) and

\[
a(u, \varphi) = (M_1 - M(\varphi) M_{-1}) \, du + (L_1 - M(\varphi) L_{-1} - N(u, \varphi) M_{-1}) \, d\varphi
\]

with \( N(u, \varphi) = L(\varphi) + \frac{u}{2} M'(\varphi) \)
AdS gravity in CS formulation: $\mathfrak{sl}(2) \oplus \mathfrak{sl}(2)$ gauge algebra

Flat space: $\mathfrak{isl}(2)$ gauge algebra

$$S_{\text{CS}}^{\text{flat}} = \frac{k}{4\pi} \int \langle A \wedge dA + \frac{2}{3} A \wedge A \wedge A \rangle$$

with $\mathfrak{isl}(2)$ connection ($a = 0, \pm 1$)

$$A = e^a M_a + \omega^a L_a$$

Boundary conditions in CS formulation:

$$A(r, u, \varphi) = b^{-1}(r) \left( d + a(u, \varphi) + o(1) \right) b(r)$$

Flat space boundary conditions: $b(r) = \exp \left( \frac{1}{2} r M_{-1} \right)$ and

$$a(u, \varphi) = (M_1 - M(\varphi) M_{-1}) \, du + (L_1 - M(\varphi) L_{-1} - N(u, \varphi) M_{-1}) \, d\varphi$$

with $N(u, \varphi) = L(\varphi) + \frac{u}{2} M'(\varphi)$

metric

$$g_{\mu\nu} \sim \frac{1}{2} \tilde{\text{tr}} \langle A_\mu A_\nu \rangle \quad \rightarrow \quad ds^2 = M \, du^2 - 2 \, du \, dr + 2N \, du \, d\varphi + r^2 \, d\varphi^2$$
Outline

Motivations

Assumptions

Flat space holography basics

Recent results

Generalizations & open issues
Correlation functions in flat space holography

AdS/CFT good tool for calculating correlators
What about flat space/Galilean CFT correspondence?

$\langle T(z_1) T(z_2) \ldots T(z_{22}) \rangle_{\text{CFT}} \sim \delta_{42} \delta_g \Gamma_{\text{EH-AdS}} \bigg|_{\text{EOM}}$

Does it work?

What is the left hand side in a Galilean CFT?

Shortcut to right hand side other than varying EH-action 22 times?

Start slowly with 0-point function
Correlation functions in flat space holography

AdS/CFT good tool for calculating correlators
What about flat space/Galilean CFT correspondence?

What is flat space analogue of

\[ \langle T(z_1)T(z_2)\ldots T(z_{42})\rangle_{\text{CFT}} \sim \frac{\delta^{42}}{\delta g^{42}} \Gamma_{\text{EH-AdS}} \bigg|_{\text{EOM}} \]

?
Correlation functions in flat space holography

AdS/CFT good tool for calculating correlators
What about flat space/Galilean CFT correspondence?

What is flat space analogue of

\[ \langle T(z_1)T(z_2)\ldots T(z_{42}) \rangle_{\text{CFT}} \sim \frac{\delta^{42}}{\delta g^{42}} \Gamma_{\text{EH-AdS}} \bigg|_{\text{EOM}} \]

? 

Does it work?
Correlation functions in flat space holography

AdS/CFT good tool for calculating correlators
What about flat space/Galilean CFT correspondence?

What is flat space analogue of

\[ \langle T(z_1)T(z_2) \ldots T(z_{42}) \rangle_{\text{CFT}} \sim \frac{\delta^{42}}{\delta g^{42}} \Gamma_{\text{EH-AdS}} \bigg|_{\text{EOM}} \]

? 

Does it work?

What is the left hand side in a Galilean CFT?
Correlation functions in flat space holography

AdS/CFT good tool for calculating correlators
What about flat space/Galilean CFT correspondence?

- What is flat space analogue of

\[
\langle T(z_1)T(z_2) \ldots T(z_{42}) \rangle_{\text{CFT}} \sim \frac{\delta^{42}}{\delta g^{42}} \Gamma_{\text{EH-AdS}} \bigg|_{\text{EOM}}
\]

- Does it work?
- What is the left hand side in a Galilean CFT?
- Shortcut to right hand side other than varying EH-action 42 times?
Correlation functions in flat space holography

AdS/CFT good tool for calculating correlators
What about flat space/Galilean CFT correspondence?

▶ What is flat space analogue of

$$\langle T(z_1)T(z_2)\ldots T(z_{42})\rangle_{\text{CFT}} \sim \frac{\delta^{42}}{\delta g^{42}} \Gamma_{\text{EH-AdS}} \bigg|_{\text{EOM}}$$

? 

▶ Does it work?
▶ What is the left hand side in a Galilean CFT?
▶ Shortcut to right hand side other than varying EH-action 42 times?

Start slowly with 0-point function
0-point function (on-shell action)
Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action $\Gamma$

\[ F_{\text{HS}} = -\frac{1}{8} \mathcal{G} \quad F_{\text{FS}} = -r + \frac{8}{\mathcal{G}} \]

- Result of this comparison
  - $r > 1$: FSC dominant saddle
  - $r < 1$: HFS dominant saddle

Critical temperature:
\[ T_c = \frac{1}{2} \pi r_0 = \Omega^2 \pi \]

HFS “melts” into FSC at $T > T_c$

Bagchi, Detournay, DG, Simon '13
0-point function (on-shell action)
Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action $\Gamma$
- Variational principle?

\[
\Gamma = -\frac{1}{16\pi G_N} \int d^3x \sqrt{g} R - \frac{1}{8\pi G_N} \int d^2x \sqrt{\gamma} K - I_{\text{counter-term}}
\]

with $I_{\text{counter-term}}$ chosen such that

\[
\delta \Gamma \big|_{\text{EOM}} = 0
\]

for all $\delta g$ that preserve flat space bc's
0-point function (on-shell action)
Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action $\Gamma$
- Variational principle?

\[\Gamma = -\frac{1}{16\pi G_N} \int d^3 x \sqrt{g} R - \frac{1}{8\pi G_N} \int d^2 x \sqrt{\gamma} K - I_{\text{counter-term}}\]

with $I_{\text{counter-term}}$ chosen such that

\[\delta \Gamma \bigg|_{\text{EOM}} = 0\]

for all $\delta g$ that preserve flat space bc’s

Result (Detournay, DG, Schöller, Simon ’14):

\[\Gamma = -\frac{1}{16\pi G_N} \int d^3 x \sqrt{g} R - \frac{1}{16\pi G_N} \int d^2 x \sqrt{\gamma} K\]

\[\frac{1}{2} \text{GHY!}\]

follows also as limit from AdS using Mora, Olea, Troncoso, Zanelli ’04

independently confirmed by Barnich, Gonzalez, Maloney, Oblak ’15
0-point function (on-shell action)
Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action $\Gamma$
- Variational principle?
- Phase transitions?

Standard procedure (Gibbons, Hawking '77; Hawking, Page '83)

Evaluate Euclidean partition function in semi-classical limit

$$Z(T, \Omega) = \int \mathcal{D}g \, e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T, \Omega)]} \times Z_{\text{fluct.}}$$

Path integral bc’s specified by temperature $T$ and angular velocity $\Omega$

Two Euclidean saddle points in same ensemble if
- same temperature $T = 1/\beta$ and angular velocity $\Omega$
- obey flat space boundary conditions
- solutions without conical singularities

Periodicities fixed:

$$(\tau_E, \varphi) \sim (\tau_E + \beta, \varphi + \beta \Omega) \sim (\tau_E, \varphi + 2\pi)$$
0-point function (on-shell action)
Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action $\Gamma$
- Variational principle?
- Phase transitions?

3D Euclidean Einstein gravity: for each $T$, $\Omega$ two saddle points:
- Hot flat space
  \[ ds^2 = d\tau_E^2 + dr^2 + r^2 \, d\varphi^2 \]
- Flat space cosmology
  \[ ds^2 = r_+^2 \left( 1 - \frac{r_0^2}{r^2} \right) \, d\tau_E^2 + \frac{r^2 \, dr^2}{r_+^2 \left( r^2 - r_0^2 \right)} + r^2 \left( d\varphi - \frac{r + r_0}{r^2} \, d\tau_E \right)^2 \]
  shifted-boost orbifold, see Cornalba, Costa '02
0-point function (on-shell action)
Not check of flat space holography but interesting in its own right

- Calculate the **full** on-shell action $\Gamma$
- Variational principle?
- Phase transitions?
- Plug two Euclidean saddles in on-shell action and compare free energies

$$F_{\text{HFS}} = -\frac{1}{8G_N} \quad F_{\text{FSC}} = -\frac{r_+}{8G_N}$$

Result of this comparison

- $r_+ > 1$: FSC dominant saddle
- $r_+ < 1$: HFS dominant saddle

Critical temperature:

$$T_c = \frac{1}{2}\pi r_0 = \Omega^2 \pi$$

HFS "melts" into FSC at $T > T_c$

Bagchi, Detournay, DG, Simon '13
0-point function (on-shell action)
Not check of flat space holography but interesting in its own right

- Calculate the **full** on-shell action $\Gamma$
- Variational principle?
- Phase transitions?
- Plug two Euclidean saddles in on-shell action and compare free energies

\[
F_{\text{HFS}} = -\frac{1}{8G_N} \quad F_{\text{FSC}} = -\frac{r_+}{8G_N}
\]

- Result of this comparison
  - $r_+ > 1$: FSC dominant saddle
  - $r_+ < 1$: HFS dominant saddle

**Critical temperature:**

\[
T_c = \frac{1}{2\pi r_0} = \frac{\Omega}{2\pi}
\]

HFS “melts” into FSC at $T > T_c$

Bagchi, Detournay, DG, Simon ’13
1-point functions (conserved charges)
First check of entries in holographic dictionary: identification of sources and vevs

In AdS$_3$:

\[
\delta \Gamma \bigg|_{\text{EOM}} \sim \int_{\partial \mathcal{M}} \text{vev} \times \delta \text{source} \sim \int_{\partial \mathcal{M}} T_{\text{BY}}^{\mu \nu} \times \delta g_{\mu \nu}^{\text{NN}}
\]

Note that $T_{\text{BY}}^{\mu \nu}$ follows from canonical analysis as well (conserved charges)
1-point functions (conserved charges)
First check of entries in holographic dictionary: identification of sources and vevs

In AdS$_3$:

$$\delta \Gamma \bigg|_{\text{EOM}} \sim \int_{\partial \mathcal{M}} \text{vev} \times \delta \text{source} \sim \int_{\partial \mathcal{M}} T_{\text{BY}}^{\mu \nu} \times \delta g_{\mu \nu}^{\text{NN}}$$

Note that $T_{\text{BY}}^{\mu \nu}$ follows from canonical analysis as well (conserved charges)

In flat space:

- non-normalizable solutions to linearized EOM?
- analogue of Brown–York stress tensor?
- comparison with canonical results
1-point functions (conserved charges)
First check of entries in holographic dictionary: identification of sources and vevs

In AdS$_3$:

$$\delta \Gamma|_{\text{EOM}} \sim \int_{\partial \mathcal{M}} \text{vev} \times \delta \text{ source} \sim \int_{\partial \mathcal{M}} T^{\mu \nu}_{\text{BY}} \times \delta g^{\text{NN}}_{\mu \nu}$$

Note that $T^{\mu \nu}_{\text{BY}}$ follows from canonical analysis as well (conserved charges)

In flat space:

- non-normalizable solutions to linearized EOM?
- analogue of Brown–York stress tensor?
- comparison with canonical results

everything works (Detournay, DG, Schöller, Simon, ’14)

mass and angular momentum:

$$M = \frac{g_{tt}}{8G}, \quad N = \frac{g_{t\varphi}}{4G}$$

full tower of canonical charges: see Barnich, Compere ’06
2-point functions (anomalous terms)
First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder ($\varphi \sim \varphi + 2\pi$):

\[
\langle M(u_1, \varphi_1) M(u_2, \varphi_2) \rangle = 0
\]
\[
\langle M(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_M}{2s_{12}^4}
\]
\[
\langle N(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_L - 2c_M \tau_{12}}{2s_{12}^4}
\]

with

\[
s_{ij} = 2 \sin[(\varphi_i - \varphi_j)/2], \quad \tau_{ij} = (u_i - u_j) \cot[(\varphi_i - \varphi_j)/2]
\]

Fourier modes of Galilean CFT stress tensor on cylinder:

\[
M := \sum_n M_n e^{-in\varphi} - \frac{c_M}{24}
\]
\[
N := \sum_n (L_n - i nu M_n) e^{-in\varphi} - \frac{c_L}{24}
\]

Conservation equations: $\partial_u M = 0$, $\partial_u N = \partial_\varphi M$
2-point functions (anomalous terms)
First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder ($\varphi \sim \varphi + 2\pi$):

$$\langle M(u_1, \varphi_1) M(u_2, \varphi_2) \rangle = 0$$
$$\langle M(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_M}{2s_{12}^4}$$
$$\langle N(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_L - 2c_M \tau_{12}}{2s_{12}^4}$$

with $s_{ij} = 2 \sin[(\varphi_i - \varphi_j)/2]$, $\tau_{ij} = (u_i - u_j) \cot[(\varphi_i - \varphi_j)/2]$

Short-cut on gravity side:
- Do not calculate second variation of action
2-point functions (anomalous terms)
First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder ($\varphi \sim \varphi + 2\pi$):

$$\langle M(u_1, \varphi_1) M(u_2, \varphi_2) \rangle = 0$$
$$\langle M(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_M}{2s_{12}^4}$$
$$\langle N(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_L - 2c_M \tau_{12}}{2s_{12}^4}$$

with $s_{ij} = 2 \sin[(\varphi_i - \varphi_j)/2]$, $\tau_{ij} = (u_i - u_j) \cot[(\varphi_i - \varphi_j)/2]$

Short-cut on gravity side:

- Do not calculate second variation of action
- Calculate first variation of action on non-trivial background
2-point functions (anomalous terms)
First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder (\( \varphi \sim \varphi + 2\pi \)):

\[
\langle M(u_1, \varphi_1) M(u_2, \varphi_2) \rangle = 0
\]

\[
\langle M(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_M}{2s_{12}^4}
\]

\[
\langle N(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_L - 2c_M \tau_{12}}{2s_{12}^4}
\]

with \( s_{ij} = 2 \sin[(\varphi_i - \varphi_j)/2] \), \( \tau_{ij} = (u_i - u_j) \cot[(\varphi_i - \varphi_j)/2] \)

Short-cut on gravity side:

- Do not calculate second variation of action
- Calculate first variation of action on non-trivial background
- Can iterate this procedure to higher \( n \)-point functions
2-point functions (anomalous terms)
First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder ($\varphi \sim \varphi + 2\pi$):

$$\langle M(u_1, \varphi_1) M(u_2, \varphi_2) \rangle = 0$$

$$\langle M(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_M}{2s_{12}^4}$$

$$\langle N(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_L - 2c_M \tau_{12}}{2s_{12}^4}$$

with $s_{ij} = 2 \sin[(\varphi_i - \varphi_j)/2]$, $\tau_{ij} = (u_i - u_j) \cot[(\varphi_i - \varphi_j)/2]$

Short-cut on gravity side:

- Do not calculate second variation of action
- Calculate first variation of action on non-trivial background
- Can iterate this procedure to higher $n$-point functions

Summarize first how this works in the AdS case
2-point functions (anomalous terms)
Illustrate shortcut in $\text{AdS}_3/\text{CFT}_2$ (restrict to one holomorphic sector)

- On CFT side deform free action $S_0$ by source term $\mu$ for stress tensor

$$S_\mu = S_0 + \int d^2z \, \mu(z, \bar{z}) T(z)$$
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

- On CFT side deform free action $S_0$ by source term $\mu$ for stress tensor

$$S_\mu = S_0 + \int d^2z \mu(z, \bar{z}) T(z)$$

- Localize source

$$\mu(z, \bar{z}) = \epsilon \delta^{(2)}(z - z_2, \bar{z} - \bar{z}_2)$$
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

- On CFT side deform free action $S_0$ by source term $\mu$ for stress tensor
  
  $$S_\mu = S_0 + \int \! d^2z \, \mu(z, \bar{z}) T(z)$$

- Localize source
  
  $$\mu(z, \bar{z}) = \epsilon \delta^{(2)}(z - z_2, \bar{z} - \bar{z}_2)$$

- 1-point function in $\mu$-vacuum $\rightarrow$ 2-point function in 0-vacuum
  
  $$\langle T^1 \rangle_\mu = \langle T^1 \rangle_0 + \epsilon \langle T^1 T^2 \rangle_0 + O(\epsilon^2)$$
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

- On CFT side deform free action $S_0$ by source term $\mu$ for stress tensor
  \[ S_\mu = S_0 + \int d^2z \mu(z, \bar{z}) T(z) \]

- Localize source
  \[ \mu(z, \bar{z}) = \epsilon \delta^{(2)}(z - z_2, \bar{z} - \bar{z}_2) \]

- 1-point function in $\mu$-vacuum $\rightarrow$ 2-point function in 0-vacuum
  \[ \langle T^1 \rangle_\mu = \langle T^1 \rangle_0 + \epsilon \langle T^1 T^2 \rangle_0 + O(\epsilon^2) \]

- On gravity side exploit sl(2) CS formulation with chemical potentials
  \[ A = b^{-1}(d+a)b \]
  \[ b = e^{\rho L_0} \]
  \[ a_z = L_+ - \frac{\mathcal{L}}{k} L_- \]
  \[ a_{\bar{z}} = \mu L_+ + \ldots \]

Drinfeld, Sokolov '84, Polyakov '87, H. Verlinde '90
Bañados, Caro '04
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

- On CFT side deform free action $S_0$ by source term $\mu$ for stress tensor
  \[ S_\mu = S_0 + \int d^2z \mu(z, \bar{z})T(z) \]

- Localize source
  \[ \mu(z, \bar{z}) = \epsilon \delta^{(2)}(z - z_2, \bar{z} - \bar{z}_2) \]

- 1-point function in $\mu$-vacuum $\rightarrow$ 2-point function in 0-vacuum
  \[ \langle T^1 \rangle_\mu = \langle T^1 \rangle_0 + \epsilon \langle T^1 T^2 \rangle_0 + O(\epsilon^2) \]

- On gravity side exploit sl(2) CS formulation with chemical potentials
  \[ A = b^{-1}(d+a)b \quad b = e^{\rho L_0} \]
  \[ a_z = L_+ - \frac{\mathcal{L}}{k} L_- \quad a_{\bar{z}} = \mu L_+ + \ldots \]

- Expand $\mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + O(\epsilon^2)$
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

- On gravity side exploit CS formulation with chemical potentials
  \[
  A = b^{-1}(d + a)b \quad \quad b = e^{\rho L_0}
  \]
  \[
  a_z = L_+ - \frac{\mathcal{L}}{k} L_- \quad \quad a_{\bar{z}} = \mu L_+ + \ldots
  \]

- Expand $\mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2)$
- Write EOM to first subleading order in $\epsilon$
  \[
  \bar{\partial} \mathcal{L}^{(1)}(z) = -\frac{k}{2} \partial^3 \delta^{(2)}(z - z_2)
  \]
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

- On gravity side exploit CS formulation with chemical potentials
  \[ A = b^{-1}(d + a)b \]
  \[ b = e^{\rho L_0} \]
  \[ a_z = L_+ - \frac{\mathcal{L}}{k} L_- \]
  \[ a_{\bar{z}} = \mu L_+ + \ldots \]

- Expand \( \mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2) \)

- Write EOM to first subleading order in \( \epsilon \)
  \[ \bar{\partial} \mathcal{L}^{(1)}(z) = -\frac{k}{2} \bar{\partial}^3 \delta^{(2)}(z - z_2) \]

- Solve them using the Green function on the plane \( G = \ln(z_{12} \bar{z}_{12}) \)
  \[ \mathcal{L}^{(1)}(z) = -\frac{k}{2} \bar{\partial}^4 G(z_{12}) = \frac{3k}{z_{12}^4} \]
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

▶ On gravity side exploit CS formulation with chemical potentials

\[
A = b^{-1}(d + a)b \quad \quad \quad b = e^{\rho L_0}
\]
\[
a_z = L_+ - \frac{\mathcal{L}}{k} L_- \quad \quad \quad a_{\bar{z}} = \mu L_+ + \ldots
\]

▶ Expand \( \mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2) \)
▶ Write EOM to first subleading order in \( \epsilon \)

\[
\bar{\partial} \mathcal{L}^{(1)}(z) = -\frac{k}{2} \partial^3 \delta^{(2)}(z - z_2)
\]
▶ Solve them using the Green function on the plane \( G = \ln(z_{12} \bar{z}_{12}) \)

\[
\mathcal{L}^{(1)}(z) = -\frac{k}{2} \partial^4_{z_1} G(z_{12}) = \frac{3k}{z_{12}^4}
\]
▶ This is the correct CFT 2-point function on the plane with \( c = 6k \)
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

- On gravity side exploit CS formulation with chemical potentials
  \[ A = b^{-1}(d + a)b \quad b = e^{\rho L_0} \]
  \[ a_{\bar{z}} = L_+ - \frac{\mathcal{L}}{k} L_- \quad a_{\bar{z}} = \mu L_+ + \ldots \]

- Expand \( \mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2) \)

- Write EOM to first subleading order in \( \epsilon \)
  \[ \bar{\partial} \mathcal{L}^{(1)}(z) = -\frac{k}{2} \partial^3 \delta^{(2)}(z - z_2) \]

- Solve them using the Green function on the plane \( G = \ln(z_{12}\bar{z}_{12}) \)
  \[ \mathcal{L}^{(1)}(z) = -\frac{k}{2} \partial_{z_1}^4 G(z_{12}) = \frac{3k}{z_{12}^4} \]

- This is the correct CFT 2-point function on the plane with \( c = 6k \)

- Generalize to cylinder
2-point functions (anomalous terms)
Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis ’15)

▶ Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel ’14)
2-point functions (anomalous terms)
Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis '15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel '14)
- Localize chemical potentials $\mu_{M/L} = \epsilon_{M/L} \delta^{(2)}(u - u_2, \varphi - \varphi_2)$
2-point functions (anomalous terms)
Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis ’15)

▶ Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel ’14)
▶ Localize chemical potentials $\mu_{M/L} = \epsilon_{M/L} \delta^{(2)}(u-u_2, \varphi-\varphi_2)$
▶ Expand around global Minkowski space

$$M = -\frac{k}{2} + M^{(1)} \quad N = N^{(1)}$$
2-point functions (anomalous terms)
Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis ’15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel ’14)
- Localize chemical potentials \( \mu_{M/L} = \epsilon_{M/L} \delta(2)(u - u_2, \varphi - \varphi_2) \)
- Expand around global Minkowski space

\[
M = -\frac{k}{2} + M^{(1)} \quad N = N^{(1)}
\]

- Write EOM to first subleading order in \( \epsilon_{M/L} \)

\[
\partial_u M^{(1)} = -k \epsilon_L (\partial^3 \delta + \partial \varphi \delta) \\
\partial_u N^{(1)} = -k \epsilon_M (\partial^3 \delta + \partial \varphi \delta) + \partial \varphi M^{(1)}
\]
2-point functions (anomalous terms)
Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis ’15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel ’14)
- Localize chemical potentials $\mu_{M/L} = \epsilon_{M/L} \delta^{(2)}(u - u_2, \varphi - \varphi_2)$
- Expand around global Minkowski space

\[ M = -k/2 + M^{(1)} \quad N = N^{(1)} \]

- Write EOM to first subleading order in $\epsilon_{M/L}$

\[ \partial_u M^{(1)} = -k \epsilon_{L} \left( \partial^3_\varphi \delta + \partial_\varphi \delta \right) \]

\[ \partial_u N^{(1)} = -k \epsilon_{M} \left( \partial^3_\varphi \delta + \partial_\varphi \delta \right) + \partial_\varphi M^{(1)} \]

- Solve with Green function on cylinder

\[ M^{(1)} = \frac{6k\epsilon_{L}}{s_{12}^4} \quad N^{(1)} = \frac{6k(\epsilon_{M} - 2\epsilon_{L} \tau_{12})}{s_{12}^4} \]
2-point functions (anomalous terms)
Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis ’15)

▶ Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel ’14)
▶ Localize chemical potentials \( \mu_{M/L} = \epsilon_{M/L} \delta^{(2)}(u - u_2, \phi - \phi_2) \)
▶ Expand around global Minkowski space

\[
M = \frac{-k}{2} + M^{(1)} \quad N = N^{(1)}
\]

▶ Write EOM to first subleading order in \( \epsilon_{M/L} \)

\[
\partial_u M^{(1)} = -k \epsilon_L \left( \partial^3 \phi \delta + \partial \phi \delta \right)
\]

\[
\partial_u N^{(1)} = -k \epsilon_M \left( \partial^3 \phi \delta + \partial \phi \delta \right) + \partial \phi M^{(1)}
\]

▶ Solve with Green function on cylinder

\[
M^{(1)} = \frac{6k \epsilon_L}{s_{12}^4} \quad N^{(1)} = \frac{6k (\epsilon_M - 2 \epsilon_L \tau_{12})}{s_{12}^4}
\]

▶ Correct 2-point functions for Einstein gravity with \( c_L = 0, \ c_M = 12k \)
3-point functions (check of symmetries)

First non-trivial check of consistency with symmetries of dual Galilean CFT

Check of 2-point functions works nicely with shortcut; 3-point too?

\[
\begin{align*}
\mu_{M/L}(u_1, \phi_1) &= 3\sum_{i=2}^3 \epsilon_i M/L \delta(2)(u_1 - u_i, \phi_1 - \phi_i) \\
\partial u_M &= -k \partial^3 \phi_{M/L} + \mu_{L} \partial \phi_M + 2M \partial \phi_{M/L} \\
\partial u_N &= -k \partial^3 \phi_{M/L} + (1 + \mu_M) \partial \phi_M + 2M \partial \phi_{M/L} + \mu_L \partial \phi_N + 2N \partial \phi_{M/L}
\end{align*}
\]

Result on gravity side matches precisely Galilean CFT results

\[
\begin{align*}
\langle M_{1N}N_{2N} \rangle &= c_M s_{12} s_{13} s_{23} \\
\langle N_{1N}N_{2N} \rangle &= c_L - c_M \tau_{123} s_{12} s_{13} s_{23}
\end{align*}
\]

provided we choose again the Einstein values

\[
c_L = 0 \quad \text{and} \quad c_M = 12k
\]
3-point functions (check of symmetries)
First non-trivial check of consistency with symmetries of dual Galilean CFT

Check of 2-point functions works nicely with shortcut; 3-point too?

- Yes: same procedure, but localize chemical potentials at two points

\[
\mu_{M/L}(u_1, \varphi_1) = \sum_{i=2}^{3} \epsilon^i_{M/L} \delta^{(2)}(u_1 - u_i, \varphi_1 - \varphi_i)
\]
3-point functions (check of symmetries)
First non-trivial check of consistency with symmetries of dual Galilean CFT

Check of 2-point functions works nicely with shortcut; 3-point too?

▶ Yes: same procedure, but localize chemical potentials at two points

\[ \mu_{M/L}(u_1, \varphi_1) = \sum_{i=2}^{3} \epsilon_{M/L}^i \delta^{(2)}(u_1 - u_i, \varphi_1 - \varphi_i) \]

▶ Iteratively solve EOM

\[ \partial_\mu M = -k \partial^3 \mu_L + \mu_L \partial_\varphi M + 2M \partial_\varphi \mu_L \]
\[ \partial_\mu N = -k \partial^3 \mu_M + (1 + \mu_M) \partial_\varphi M + 2M \partial_\varphi \mu_M + \mu_L \partial_\varphi N + 2N \partial_\varphi \mu_L \]
3-point functions (check of symmetries)
First non-trivial check of consistency with symmetries of dual Galilean CFT

Check of 2-point functions works nicely with shortcut; 3-point too?

- Yes: same procedure, but localize chemical potentials at two points

\[ \mu_{M/L}(u_1, \varphi_1) = \sum_{i=2}^{3} \epsilon_{M/L}^i \delta^{(2)}(u_1 - u_i, \varphi_1 - \varphi_i) \]

- Iteratively solve EOM

\[
\begin{align*}
\partial_u M &= -k \partial^3 \varphi \mu_L + \mu_L \partial \varphi M + 2M \partial \varphi \mu_L \\
\partial_u N &= -k \partial^3 \varphi \mu_M + (1 + \mu_M) \partial \varphi M + 2M \partial \varphi \mu_M + \mu_L \partial \varphi N + 2N \partial \varphi \mu_L
\end{align*}
\]

- Result on gravity side matches precisely Galilean CFT results

\[
\begin{align*}
\langle M^1 N^2 N^3 \rangle &= \frac{c_M}{s_{12}^2 s_{13}^2 s_{23}^2} \\
\langle N^1 N^2 N^3 \rangle &= \frac{c_L - c_M \tau_{123}}{s_{12}^2 s_{13}^2 s_{23}^2}
\end{align*}
\]

provided we choose again the Einstein values \( c_L = 0 \) and \( c_M = 12k \)
4-point functions (enter cross-ratios)
First correlators with non-universal function of cross-ratios

▶ Repeat this algorithm, localizing the sources at three points

\[ \langle M_1 N_2 N_3 N_4 \rangle = 2 c \gamma^4 \left( \sum \right) \]

\[ \langle N_1 N_2 N_3 N_4 \rangle = 2 c_L g_4 (\gamma) + c_M \Delta_4 \]

with the cross-ratio function

\[ g_4 (\gamma) = \frac{\gamma^2}{s_{12} s_{34} s_{13} s_{24}} \]

and

\[ \Delta_4 = 4 g'_4 (\gamma) \eta_{1234} - (\tau_{1234} + \tau_{14} + \tau_{23}) g_4 (\gamma) \eta_{1234} = \sum (-1)^{1+i-j} \frac{u_i - u_j}{s_{213} s_{24}} \sin (\phi_k - \phi_l) \]
4-point functions (enter cross-ratios)
First correlators with non-universal function of cross-ratios

- Repeat this algorithm, localizing the sources at three points
- Derive 4-point functions for Galilean CFTs (Bagchi, DG, Merbis '15)

\[
\langle M^1 N^2 N^3 N^4 \rangle = \frac{2c_M g_4(\gamma)}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}
\]
\[
\langle N^1 N^2 N^3 N^4 \rangle = \frac{2c_L g_4(\gamma) + c_M \Delta_4}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}
\]

with the cross-ratio function

\[
g_4(\gamma) = \frac{\gamma^2 - \gamma + 1}{\gamma}
\]

\[
\gamma = \frac{s_{12} s_{34}}{s_{13} s_{24}}
\]

and

\[
\Delta_4 = 4g'_4(\gamma) \eta_{1234} - (\tau_{1234} + \tau_{14} + \tau_{23})g_4(\gamma)
\]
\[
\eta_{1234} = \sum (-1)^{1+i-j}(u_i - u_j)\sin(\varphi_k - \varphi_l)/(s_{13}^2 s_{24}^2)
\]
4-point functions (enter cross-ratios)
First correlators with non-universal function of cross-ratios

- Repeat this algorithm, localizing the sources at three points
- Derive 4-point functions for Galilean CFTs (Bagchi, DG, Merbis ’15)

\[
\begin{align*}
\langle M^1 N^2 N^3 N^4 \rangle &= \frac{2c_M g_4(\gamma)}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}} \\
\langle N^1 N^2 N^3 N^4 \rangle &= \frac{2c_L g_4(\gamma) + c_M \Delta_4}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}
\end{align*}
\]

with the cross-ratio function

\[
g_4(\gamma) = \frac{\gamma^2 - \gamma + 1}{\gamma} \quad \gamma = \frac{s_{12} s_{34}}{s_{13} s_{24}}
\]

and

\[
\Delta_4 = 4g'_4(\gamma) \eta_{1234} - (\tau_{1234} + \tau_{14} + \tau_{23}) g_4(\gamma)
\]

\[
\eta_{1234} = \sum (-1)^{1+i-j} (u_i - u_j) \sin(\varphi_k - \varphi_l) / (s_{13}^2 s_{24}^2)
\]

- Gravity side yields precisely the same result!
5-point functions (further check of consistency of flat space holography)

Last new explicit correlators I am showing to you today (I promise)

- Repeat this algorithm, localizing the sources at four points

\[
\langle M_{12345} \rangle = 4 c \cdot g_{5}(\gamma, \zeta) \prod_{1 \leq i < j \leq 5} s_{ij}
\]

\[
\langle N_{12345} \rangle = 4 c \cdot L_{g_{5}(\gamma, \zeta)} + 4 c \cdot M_{\Delta 5} \prod_{1 \leq i < j \leq 5} s_{ij}
\]

with the previous definitions and \((\zeta = s_{25} s_{34} s_{35} s_{24})\)

\[
g_{5}(\gamma, \zeta) = \gamma + \zeta - 2(\gamma - \zeta) - (\gamma^{2} - \gamma \zeta + \zeta^{2}) \gamma (\gamma - 1) \zeta (\zeta - 1) (\gamma - \zeta) \times \left[ \gamma (\gamma - 1) + 1 \right] \left[ \zeta (\zeta - 1) + 1 \right] - \gamma \zeta
\]

\[
\Delta 5 = 4 \partial_{\gamma} g_{5}(\gamma, \zeta) \eta_{1234} + 4 \partial_{\zeta} g_{5}(\gamma, \zeta) \eta_{2345} - 2 g_{5}(\gamma, \zeta) \tau_{12345}
\]

Gravity side yields precisely the same result!
5-point functions (further check of consistency of flat space holography)
Last new explicit correlators I am showing to you today (I promise)

- Repeat this algorithm, localizing the sources at four points
- Derive 5-point functions for Galilean CFTs (Bagchi, DG, Merbis '15)

\[
\langle M^1 N^2 N^3 N^4 N^5 \rangle = \frac{4c_M g_5(\gamma, \zeta)}{\prod_{1 \leq i < j \leq 5} s_{ij}}
\]

\[
\langle N^1 N^2 N^3 N^4 N^5 \rangle = \frac{4c_L g_5(\gamma, \zeta) + c_M \Delta_5}{\prod_{1 \leq i < j \leq 5} s_{ij}}
\]

with the previous definitions and \((\dot{\zeta} = \frac{s_{25} s_{34}}{s_{35} s_{24}})\)

\[
g_5(\gamma, \zeta) = \frac{\gamma + \zeta}{2(\gamma - \zeta)} - \frac{(\gamma^2 - \gamma \zeta + \zeta^2)}{\gamma(\gamma - 1)\zeta(\zeta - 1)(\gamma - \zeta)} \times ([\gamma(\gamma - 1) + 1][\zeta(\zeta - 1) + 1] - \gamma \zeta)
\]

\[
\Delta_5 = 4\partial_\gamma g_5(\gamma, \zeta) \eta_{1234} + 4\partial_\zeta g_5(\gamma, \zeta) \eta_{2345} - 2g_5(\gamma, \zeta) \tau_{12345}
\]
5-point functions (further check of consistency of flat space holography)
Last new explicit correlators I am showing to you today (I promise)

- Repeat this algorithm, localizing the sources at four points
- Derive 5-point functions for Galilean CFTs (Bagchi, DG, Merbis ’15)

\[
\langle M^1 N^2 N^3 N^4 N^5 \rangle = \frac{4c_M \, g_5(\gamma, \zeta)}{\prod_{1 \leq i < j \leq 5} s_{ij}}
\]

\[
\langle N^1 N^2 N^3 N^4 N^5 \rangle = \frac{4c_L \, g_5(\gamma, \zeta) + c_M \Delta_5}{\prod_{1 \leq i < j \leq 5} s_{ij}}
\]

with the previous definitions and \((\zeta = \frac{s_{25} \, s_{34}}{s_{35} \, s_{24}})\)

\[
g_5(\gamma, \zeta) = \frac{\gamma + \zeta}{2(\gamma - \zeta)} - \frac{(\gamma^2 - \gamma \zeta + \zeta^2)}{\gamma(\gamma - 1)\zeta(\zeta - 1)(\gamma - \zeta)} \times ([\gamma(\gamma - 1) + 1][\zeta(\zeta - 1) + 1] - \gamma \zeta)
\]

\[
\Delta_5 = 4 \partial_\gamma g_5(\gamma, \zeta) \eta_{1234} + 4 \partial_\zeta g_5(\gamma, \zeta) \eta_{2345} - 2 g_5(\gamma, \zeta) \tau_{12345}
\]

- Gravity side yields precisely the same result!
\( n \)-point functions (holographic Ward identities and recursion relations)
Shortcut to 42 (Bagchi, DG, Merbis '15)

Smart check of all \( n \)-point functions?

- Idea: calculate \( n \)-point function from \((n - 1)\)-point function
Smart check of all $n$-point functions?

- Idea: calculate $n$-point function from $(n-1)$-point function
- Need Galilean CFT analogue of BPZ-recursion relation

\[ \langle T^1 T^2 \ldots T^n \rangle = \sum_{i=2}^{n} \left( \frac{2}{s_{1i}^2} + \frac{c_{1i}}{2} \partial \varphi_i \right) \langle T^2 \ldots T^n \rangle + \text{disconnected} \]
Smart check of all $n$-point functions?

- **Idea:** calculate $n$-point function from $(n-1)$-point function
- **Need** Galilean CFT analogue of BPZ-recursion relation

\[
\langle T^1 T^2 \ldots T^n \rangle = \sum_{i=2}^{n} \left( \frac{2}{s_{1i}^2} + \frac{c_{1i}}{2} \partial \varphi_i \right) \langle T^2 \ldots T^n \rangle + \text{disconnected}
\]

- **After small derivation we find ($c_{ij} := \cot[(\varphi_i - \varphi_j)/2]$)**

\[
\langle M^1 N^2 \ldots N^n \rangle = \sum_{i=2}^{n} \left( \frac{2}{s_{1i}^2} + \frac{c_{1i}}{2} \partial \varphi_i \right) \langle M^2 N^3 \ldots N^n \rangle + \text{disconnected}
\]

\[
\langle N^1 N^2 \ldots N^n \rangle = \frac{c_L}{c_M} \langle M^1 N^2 \ldots N^n \rangle + \sum_{i=1}^{n} u_i \partial \varphi_i \langle M^1 N^2 \ldots N^n \rangle
\]
> **Smart check of all $n$-point functions?**

- **Idea:** calculate $n$-point function from $(n - 1)$-point function
- **Need Galilean CFT analogue of BPZ-recursion relation**

\[
\langle T^1 T^2 \ldots T^n \rangle = \sum_{i=2}^{n} \left( \frac{2}{s_{1i}^2} + \frac{c_{1i}}{2} \partial \varphi_i \right) \langle T^2 \ldots T^n \rangle + \text{disconnected}
\]

- **After small derivation we find ($c_{ij} := \cot [(\varphi_i - \varphi_j)/2]$)**

\[
\langle M^1 N^2 \ldots N^n \rangle = \sum_{i=2}^{n} \left( \frac{2}{s_{1i}^2} + \frac{c_{1i}}{2} \partial \varphi_i \right) \langle M^2 N^3 \ldots N^n \rangle + \text{disconnected}
\]

\[
\langle N^1 N^2 \ldots N^n \rangle = \frac{c_L}{c_M} \langle M^1 N^2 \ldots N^n \rangle + \sum_{i=1}^{n} u_i \partial \varphi_i \langle M^1 N^2 \ldots N^n \rangle
\]

- **We can also derive same recursion relations on gravity side!**
$n$-point functions in flat space holography

Summary

- EH action has variational principle consistent with flat space bc’s (iff we add half the GHY term!)
$n$-point functions in flat space holography

Summary

- EH action has variational principle consistent with flat space bc’s (iff we add half the GHY term!)
- 0-point function shows phase transition exists between hot flat space and flat space cosmologies
Summary

- EH action has variational principle consistent with flat space bc's (iff we add half the GHY term!)
- 0-point function shows phase transition exists between hot flat space and flat space cosmologies
- 1-point functions show consistency with canonical charges and lead to first entries in holographic dictionary
- 2-point functions consistent with Galilean CFT for $c_L = 0$, $c_M = 12$, $k = 3/G_N$
- 42nd variation of EH action leads to 42-point Galilean CFT correlators
- All $n$-point correlators of Galilean CFT reproduced precisely on gravity side (recursion relations!)

Fairly non-trivial check that 3D flat space holography can work!

Daniel Grumiller — Flat Space Holography Recent results 26/30
$n$-point functions in flat space holography

Summary

- EH action has variational principle consistent with flat space bc’s (iff we add half the GHY term!)
- 0-point function shows phase transition exists between hot flat space and flat space cosmologies
- 1-point functions show consistency with canonical charges and lead to first entries in holographic dictionary
- 2-point functions consistent with Galilean CFT for $c_L = 0$, $c_M = 12k = 3/G_N$
$n$-point functions in flat space holography

Summary

- EH action has variational principle consistent with flat space bc’s (iff we add half the GHY term!)
- 0-point function shows phase transition exists between hot flat space and flat space cosmologies
- 1-point functions show consistency with canonical charges and lead to first entries in holographic dictionary
- 2-point functions consistent with Galilean CFT for $c_L = 0$, $c_M = 12k = 3/G_N$
- 42$^{nd}$ variation of EH action leads to 42-point Galilean CFT correlators
$n$-point functions in flat space holography

Summary

- EH action has variational principle consistent with flat space bc’s (iff we add half the GHY term!)
- 0-point function shows phase transition exists between hot flat space and flat space cosmologies
- 1-point functions show consistency with canonical charges and lead to first entries in holographic dictionary
- 2-point functions consistent with Galilean CFT for $c_L = 0$, $c_M = 12k = 3/G_N$
- 42nd variation of EH action leads to 42-point Galilean CFT correlators
- all $n$-point correlators of Galilean CFT reproduced precisely on gravity side (recursion relations!)
$n$-point functions in flat space holography

Summary

- EH action has variational principle consistent with flat space bc’s (iff we add half the GHY term!)
- 0-point function shows phase transition exists between hot flat space and flat space cosmologies
- 1-point functions show consistency with canonical charges and lead to first entries in holographic dictionary
- 2-point functions consistent with Galilean CFT for $c_L = 0$, $c_M = 12k = 3/G_N$
- 42$^{nd}$ variation of EH action leads to 42-point Galilean CFT correlators
- all $n$-point correlators of Galilean CFT reproduced precisely on gravity side (recursion relations!)

Fairly non-trivial check that 3D flat space holography can work!
Other selected recent results

Some further checks that dual field theory is Galilean CFT:

\[ \text{gravity} = \text{S}_{\text{BH}} = \text{Area} \frac{4}{G_N} = 2\pi h \frac{L}{\sqrt{c}} M^2 h \frac{M}{\ell} \]

Also as limit from Cardy formula (Riegler '14, Fareghbal, Naseh '14)

\[ S_{\text{GCFT}}^{\text{EE}} = c L^6 \ln \ell x a \]

like CFT + \[ c M^6 \ell y \ell x \]

like grav anomaly

Calculation on gravity side confirms result above (using Wilson lines in CS formulation)
Other selected recent results

Some further checks that dual field theory is Galilean CFT:
  ▶ Microstate counting?

Works! (Bagchi, Detournay, Fareghbal, Simon '13, Barnich '13)

$S_{\text{gravity}} = S_{\text{BH}} = \frac{\text{Area}}{4G_N} = 2\pi h L \sqrt{\frac{c}{M^2 h}}$

Also as limit from Cardy formula (Riegler '14, Fareghbal, Naseh '14)

▶ (Holographic) entanglement entropy?

Works! (Bagchi, Basu, DG, Riegler '14)

$S_{\text{GCFT}}^\text{EE} = c L^6 \ln \frac{\ell}{a} + c M^6 \ell^y L^x$

Calculation on gravity side confirms result above (using Wilson lines in CS formulation)
Other selected recent results

Some further checks that dual field theory is Galilean CFT:

- Microstate counting?

  Works! (Bagchi, Detournay, Fareghbal, Simon ’13, Barnich ’13)

\[
S_{\text{gravity}} = S_{\text{BH}} = \frac{\text{Area}}{4G_N} = 2\pi h_L \sqrt{\frac{c_M}{2\hbar_M}} = S_{\text{GCFT}}
\]

Also as limit from Cardy formula (Riegler ’14, Fareghbal, Naseh ’14)
Other selected recent results

Some further checks that dual field theory is Galilean CFT:

- Microstate counting?

  Works! (Bagchi, Detournay, Fareghbal, Simon ’13, Barnich ’13)

\[ S_{\text{gravity}} = S_{\text{BH}} = \frac{\text{Area}}{4G_N} = 2\pi h_L \sqrt{\frac{c_M}{2\hbar_M}} = S_{\text{GCFT}} \]

Also as limit from Cardy formula (Riegler ’14, Fareghbal, Naseh ’14)

- (Holographic) entanglement entropy?

Daniel Grumiller — Flat Space Holography Recent results 27/30
Other selected recent results

Some further checks that dual field theory is Galilean CFT:

- Microstate counting?

  Works! (Bagchi, Detournay, Fareghbal, Simon ’13, Barnich ’13)

\[
S_{\text{gravity}} = S_{\text{BH}} = \frac{\text{Area}}{4G_N} = 2\pi h_L \sqrt{\frac{c_M}{2\hbar_M}} = S_{\text{GCFT}}
\]

Also as limit from Cardy formula (Riegler ’14, Fareghbal, Naseh ’14)

- (Holographic) entanglement entropy?

  Works! (Bagchi, Basu, DG, Riegler ’14)

\[
S_{\text{EE}}^{\text{GCFT}} = \frac{c_L}{6} \ln \frac{\ell_y}{a} + \frac{c_M}{6} \frac{\ell_y}{\ell_x}
\]

Calculation on gravity side confirms result above
(using Wilson lines in CS formulation)
Outline

Motivations

Assumptions

Flat space holography basics

Recent results

Generalizations & open issues
Generalizations & open issues

Recent generalizations:

- adding chemical potentials

  Works! (Gary, DG, Riegler, Rosseel ’14)

In CS formulation:

\[ A_0 \rightarrow A_0 + \mu \]

See also talk by Mirah Gary two days ago (Junior plenary session)
Generalizations & open issues

Recent generalizations:

- adding chemical potentials
- 3-derivative theory: flat space chiral gravity *(Bagchi, Detournay, DG ’12)*

Conformal CS gravity at level $k = 1$ with flat space boundary conditions conjectured to be dual to chiral half of monster CFT.

Action (gravity side):

$$I_{CSG} = \frac{k}{4\pi} \int d^3 x \sqrt{-g} \varepsilon^{\lambda \mu \nu} \Gamma^\rho_{\lambda \sigma} \left( \partial_\mu \Gamma^\sigma_{\nu \rho} + \frac{2}{3} \Gamma^\sigma_{\mu \tau} \Gamma^\tau_{\nu \rho} \right)$$

Partition function (field theory side, see *Witten ’07)*:

$$Z(q) = J(q) = \frac{1}{q} + 196884 q + \mathcal{O}(q^2)$$

Note: $\ln 196883 \approx 12.2 = 4\pi + \text{quantum corrections}$
Generalizations & open issues

Recent generalizations:

- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG ’12)
- generalization to supergravity

Works! (Barnich, Donnay, Matulich, Troncoso ’14)

Asymptotic symmetry algebra = super-BMS$_3$
Recent generalizations:

- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG '12)
- generalization to supergravity
- flat space higher spin gravity

Remarkably it exists! (Afshar, Bagchi, Fareghbal, DG, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso ’13)

New type of algebra: W-like BMS ("BMW")

\[
[U_n, U_m] = (n - m)(2n^2 + 2m^2 - nm - 8)L_{n+m} + \frac{192}{c_M}(n - m)\Lambda_{n+m}
\]

\[
- \frac{96(c_L + \frac{44}{5})}{c_M^2}(n - m)\Theta_{n+m} + \frac{c_L}{12} n(n^2 - 1)(n^2 - 4)\delta_{n+m,0}
\]

\[
[U_n, V_m] = (n - m)(2n^2 + 2m^2 - nm - 8)M_{n+m} + \frac{96}{c_M}(n - m)\Theta_{n+m}
\]

\[
+ \frac{c_M}{12} n(n^2 - 1)(n^2 - 4)\delta_{n+m,0}
\]

\[
[L, L], [L, M], [M, M] \text{ as in BMS}_3 \quad [L, U], [L, V], [M, U], [M, V] \text{ as in isl}(3)
\]
Generalizations & open issues

Recent generalizations:
- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG ’12)
- generalization to supergravity
- flat space higher spin gravity

Some open issues:
- Further checks in 3D ($n$-point correlators, partition function, ...)
- Barnich, Gonzalez, Maloney, Oblak ’15: 1-loop partition function matches BMS$_3$ character
Generalizations & open issues

Recent generalizations:
- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG ’12)
- generalization to supergravity
- flat space higher spin gravity

Some open issues:
- Further checks in 3D ($n$-point correlators, partition function, ...)
- Further generalizations in 3D (massive gravity, adding matter, ...)
Generalizations & open issues

Recent generalizations:

▶ adding chemical potentials
▶ 3-derivative theory: flat space chiral gravity (*Bagchi, Detournay, DG ’12*)
▶ generalization to supergravity
▶ flat space higher spin gravity

Some open issues:

▶ Further checks in 3D (*n*-point correlators, partition function, ...)
▶ Further generalizations in 3D (massive gravity, adding matter, ...)
▶ Generalization to 4D? (*Barnich et al, Strominger et al*)
Generalizations & open issues

Recent generalizations:
- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG ’12)
- generalization to supergravity
- flat space higher spin gravity

Some open issues:
- Further checks in 3D ($n$-point correlators, partition function, ...)
- Further generalizations in 3D (massive gravity, adding matter, ...)
- Generalization to 4D? (Barnich et al, Strominger et al)
- Flat space limit of usual AdS$_5$/CFT$_4$ correspondence?
Generalizations & open issues

Recent generalizations:
- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG ’12)
- generalization to supergravity
- flat space higher spin gravity

Some open issues:
- Further checks in 3D (\(n\)-point correlators, partition function, ...)
- Further generalizations in 3D (massive gravity, adding matter, ...)
- Generalization to 4D? (Barnich et al, Strominger et al)
- Flat space limit of usual AdS\(_5\)/CFT\(_4\) correspondence?

- holography seems to work in flat space
Generalizations & open issues

Recent generalizations:
- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG ’12)
- generalization to supergravity
- flat space higher spin gravity

Some open issues:
- Further checks in 3D ($n$-point correlators, partition function, ...)
- Further generalizations in 3D (massive gravity, adding matter, ...)
- Generalization to 4D? (Barnich et al, Strominger et al)
- Flat space limit of usual AdS$_5$/CFT$_4$ correspondence?

- holography seems to work in flat space
- holography more general than AdS/CFT
Generalizations & open issues

Recent generalizations:
- adding chemical potentials
- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG ’12)
- generalization to supergravity
- flat space higher spin gravity

Some open issues:
- Further checks in 3D ($n$-point correlators, partition function, ...)
- Further generalizations in 3D (massive gravity, adding matter, ...)
- Generalization to 4D? (Barnich et al, Strominger et al)
- Flat space limit of usual AdS$_5$/CFT$_4$ correspondence?

- holography seems to work in flat space
- holography more general than AdS/CFT
- (when) does it work even more generally?
Thanks for your attention!

Vladimir Bulatov, M.C. Escher Circle Limit III in a rectangle