Aspects of Holography in 2d Dilaton Gravity

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with Daniel Grumiller, Robert McNees, Carlos Valcárcel, Dmitri Vassilevich
and ongoing work with Hernán Gonzalez
Motivation I - Holography

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MC Escher: Circle Limit III
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- Study asymptotic dynamics of gravity theories in $\text{AdS}_2$
1 Motivation

2 Two-dimensional Dilaton Gravity

3 Dilaton Gravity as a Poisson Sigma model

4 AdS$_2$ Holography
   • Constant dilaton sector
   • Linear dilaton sector

5 Conclusion
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Naïve guess

Two-dimensional EH action

\[
I[g] = \int_{\mathcal{M}} d^2 x \sqrt{|g|} R[g]
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$$I[g + \delta g] = I[g] + \int_{\mathcal{M}} d^2x \sqrt{|g|} \left( R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} \right) \delta g_{\mu\nu}$$

$= 0 \text{ in two dimensions}$

$\Rightarrow$ no restrictions on the metric
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$$\int_{\mathcal{M}} d^2x \sqrt{|g|} R[g] = 2\pi \chi(\mathcal{M})$$

$\Rightarrow$ Topological invariant
\[
I[g, X] = -\frac{1}{16\pi G_2} \int_{\mathcal{M}} d^2x \sqrt{|g|} \left( R \right)
\]
Generalized Dilaton Models- Gravity in two Dimensions

\[ l[g, X] = -\frac{1}{16\pi G_2} \int_{\mathcal{M}} d^2x \sqrt{|g|} \left( X R \right) \]

\[ X \text{...scalar field: “dilaton”} \]
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\[ I[g, X] = -\frac{1}{16\pi G_2} \int_{\mathcal{M}} d^2x \sqrt{|g|} \left( X R - 2V(X) \right) \]

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What is the motivation for studying an action of this specific form?

Motivated by:
- Spherical reduction of Einstein Gravity
- Dilaton Gravity as Gauge Theory
- Dilaton Gravity from Strings
\[ l[g, X] = -\frac{1}{16\pi G_2} \int_{\mathcal{M}} d^2 x \sqrt{|g|} \left( X R - U(X)(\nabla X)^2 - 2V(X) \right) \]

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- Assume that \(\mathcal{M}_d\) is spherically symmetric: \(\mathcal{M}_d \rightarrow \mathcal{M}_2 \times S^{d-2}\)

\[
ds^2 = g_{\mu \nu} \, dx^\mu \, dx^\nu + \lambda^{-2} X^{\frac{2}{d-2}} \, d\Omega^2_{S^{d-2}}
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I = \frac{A_{d-2}}{\lambda^{d-2} 16\pi G_N} \int_{\mathcal{M}} d^2 x \sqrt{|g|} \left( XR + \frac{d - 3}{d - 2} \left( \nabla X \right)^2 \frac{X}{X} + \lambda^2 (d - 2)(d - 3) X^{\frac{d-3}{d-2}} \right)
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\[ \implies \text{Dilaton gravity model!} \]
Generalized Dilaton Models - The action

Bulk action

\[ I = -\frac{1}{16\pi G_2} \left[ \int_{\mathcal{M}} d^2x \sqrt{|g|} \left( X R - U(X)(\nabla X)^2 - 2V(X) \right) \right] \]
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- generalizations, e.g. add a Maxwell-term \( \int_\mathcal{M} d^2x \sqrt{|g|} F(X) f^{\mu\nu} f_{\mu\nu} \) with coupling \( F(X) \)
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Poisson-Sigma models (PSMs)

Poisson sigma model (PSM) formulation simplifies calculations considerably (cf. Chern-Simons in 3d)

Can provide new viewpoints (cf. 3d Chern-Simons ⇔ WZW models)

Poisson-Sigma model [Schaller, Strobl; Ikeda]

Base space: 2d manifold \( M \)

Target space: a Poisson manifold \( \Sigma \) with coordinates \( X^I \)

A Poisson tensor \( P^{IJ}(X^K) = -P^{JI}(X^K) \) obeying the identity

\[
P^{LI} \partial_X P^{JK} + \text{(cycl.)} = 0
\]

Poisson bracket on \( \Sigma \) given by

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Action given by

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- gauge fields $A_I$, one-forms on $\mathcal{M}$ taking values in $\Sigma$
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$$I = -\frac{k}{2\pi} \int_{\mathcal{M}} \left( A_I \, dX^I + \frac{1}{2} P^{IJ}(X^K) A_I \wedge A_J \right).$$
PSM formulation of dilaton gravity

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Relation between PSM and 2d dilaton gravity
PSM formulation of dilaton gravity

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2d dilaton gravity in first order formulation

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- coupling to (non-)abelian gauge fields straightforward
Equations of motion and symmetries

\[ I = -\frac{k}{2\pi} \int_{\mathcal{M}} \left( X^I \, dA_I + \frac{1}{2} P^{IJ} (X^K) A_I \wedge A_J \right). \]

Equations of motion

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\begin{align*}
\text{d}X^I + P^{IJ} A_J &= 0 \\
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Symmetries of the action

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  \delta_{\lambda} X^I &= P^{IJ} \lambda_J \\
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Diffeomorphisms and Lorentz transformations \( \Leftrightarrow \) non-linear gauge symmetry (on-shell), e.g.,

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\[ 0 = -P^{IJ} A_J \partial_I C = dX^I \partial_I C = dC \]
Motivation

Two-dimensional Dilaton Gravity

Dilaton Gravity as a Poisson Sigma model

AdS$_2$ Holography
- Constant dilaton sector
- Linear dilaton sector

Conclusion
Study asymptotic dynamics in the two solution sectors of dilaton gravity
AdS$_2$ holography

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AdS$_2$ CDV holography in PSM formulation: previous results

- AdS$_2$ holography for a particular model of constant dilaton coupled to a $U(1)$ field: chiral CFT dual [Hartman, Strominger (2008); Castro, Grumiller, Larsen, McNees (2008)]
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  Is this true only for this specific model? Maybe non-trivial constant dilaton holography for other models?
Dilaton Holography coupled to $U(1)$ field in the constant dilaton sector on Euclidean AdS$_2$
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- constant dilaton solutions: $X^I = \bar{X}^I = \text{const} \Rightarrow P^{IJ}(\bar{X}^K) = 0$
- $\text{AdS}_2$ solution with $R = -2 \Rightarrow \partial_X Y|_{X^I = \bar{X}^I} = 1$
General Strategy: Boundary Conditions, Gauge Transformations and Symmetries

- Theory not only defined by action but also by **boundary conditions**;
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Choose boundary conditions for fields; should contain some interesting solutions
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Choose boundary conditions for fields; should contain some interesting solutions

Having chosen boundary conditions, one has to classify gauge transformations according to:

- Trivial (proper, pure, genuine) gauge transformations, redundancy in the theory; do not change the physical state

- Non-trivial (improper) gauge transformations that change the physical state

Asymptotic symmetries are the quotient of allowed transformations by trivial transformations

Asymptotic symmetry algebra coincides with symmetry algebra of putative dual theory.
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AdS$_2$ CDV holography in PSM formulation: boundary-conditions

- work at finite temperature $\Rightarrow$ periodicity in Euclidean time: $\phi \sim \phi + \beta$

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work at finite temperature ⇒ periodicity in Euclidean time: \( \phi \sim \phi + \beta \)

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boundary conditions:

\[
\begin{align*}
X^0 &= 0 & A_{\varphi 0} &= \frac{1}{2} e^\rho - \frac{1}{2} e^{-\rho} \mathcal{M}(\varphi) + \mathcal{O}(e^{-3\rho}) & A_{\rho 0} &= 0 \\
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\end{align*}

$\Rightarrow$ line element with unit AdS$_2$ radius:

\[ ds^2 = d\rho^2 + \frac{1}{4} \left( e^{2\rho} + 2M(\varphi) + (M(\varphi))^2 e^{-2\rho} \right) d\varphi^2 \]
AdS$_2$ CDV holography in PSM formulation: asymptotic symmetries

- free function $\mathcal{M}(\varphi)$ in the bc’s; looks like “boundary stress tensor”
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- $\Rightarrow \delta_\lambda \mathcal{M} = -\mathcal{M}' \lambda - 2\lambda' \mathcal{M} - \frac{1}{2} \lambda'''

$\delta_\lambda \mathcal{M}$ transforms under infinitesimal Virasoro symmetry; boundary stress tensor(?)

are these improper symmetry or proper gauge transformations

$\Rightarrow$ calculate the corresponding canonical charge

Use Hamiltonian approach (Regge-Teitelboim), covariant phase space (Lee-Wald, Barnich-Brandt) etc. to calculate canonical boundary current:

$\delta Q_\lambda = k_2 \pi \delta X_I \lambda_I \bigg|_{\rho \to \infty}$

With our bc’s, $\delta X_I = 0$ $\Rightarrow$ $\delta Q_\lambda = 0$

Transformations are pure gauge! Theory is trivial (classically)!

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Quantum correction? \(\Rightarrow\) Calculate one-loop partition function around CDVs
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Study holography in the two solution sectors of dilaton gravity

- Constant dilaton sector (CDV)
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Jackiw–Teitelboim (JT) model

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**Figure:** Penrose diagram for the Jackiw–Teitelboim model
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- Boundary theory of JT model related to SYK model

Figure: Finite temperature black hole
Sachdev-Ye-Kitaev model

Quantum mechanical model of $N$ Majorana fermions $\psi_i$ [Sachdev, Ye; Kitaev; Maldacena, Stanford]

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- reparametrizations $\rightarrow$ Goldstone bosons governed by Schwarzian action

**Schwarzian action**

$$I = \frac{N}{2\beta J} \int_0^\beta d\tau \left( \frac{2\pi^2}{\beta^2} \dot{f}^2 + \{f; \tau\} \right). \quad \{f; \tau\} = \left( \frac{\dddot{f}}{f} \right) - \frac{3}{2} \left( \frac{\ddot{f}}{f} \right)^2$$
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- Schwarzian action governs also asymptotic dynamics of Jackiw–Teitelboim model [Maldacena, Stanford, Yang] $\Rightarrow$ holography! (?)
Schwarzian action– Generalizations?

- SYK model first step towards dual theory to $\text{AdS}_2$; however some problems (e.g., additional matter, random couplings make questions about black holes delicate)
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Understand the appearance of the Schwarzian action in the Jackiw–Teitelboim model in the PSM formalism
Schwarzian action—Generalizations?

- SYK model first step towards dual theory to $\text{AdS}_2$; however some problems (e.g., additional matter, random couplings make questions about black holes delicate)
- Host of other SYK-like models has been proposed; possibly more straightforward to find gravity interpretation
- Understand the appearance of the Schwarzian action in the Jackiw–Teitelboim model in the PSM formalism
- Generalize this from the gravitational perspective, i.e., find a similar action governing the asymptotic dynamics for other models
Linear PSMs

Take as Poisson manifold the dual space $\mathfrak{g}^*$ of a (semi-simple) Lie algebra $\mathfrak{g}$.

$$I = -\frac{k}{2\pi} \int_M \left( X^I \, dA_I + \frac{1}{2} P^{IJ}(X^K) A_I \wedge A_J \right).$$

with

$$\left\{ X^I, X^J \right\} = P^{IJ}(X^K) = f_{IJ}^K X^K \quad f_{IK}^{IJ} \ldots \text{structure constants of } \mathfrak{g}$$
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Using the invariant, bilinear form $\langle \cdot, \cdot \rangle$ on $g$ rewrite the action as

**BF action**

$$I = -\frac{k}{2\pi} \int_M \langle \mathcal{X}, (dA + \frac{1}{2}[A, A]) \rangle = -\frac{k}{2\pi} \int_M \langle \mathcal{X}, \mathcal{F} \rangle.$$

with $\mathcal{X} = X^I L_I, \mathcal{F} = F^I L_I$, where $[L_I, L_J] = f_{IJ}^K L_K$. 
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with $X = X^I L_I$, $F = F^I L_I$, where $[L_I, L_J] = f_{IJ}^K L_K$.

For $g = sl(2, \mathbb{R})$, Jackiw–Teitelboim model
Equation of motion— Symmetries

BF action

\[ I = -\frac{k}{2\pi} \int_{\mathcal{M}} \langle \mathcal{X}, \mathcal{F} \rangle. \]
Equation of motion– Symmetries

**BF action**

\[
I = -\frac{k}{2\pi} \int_\mathcal{M} \langle \mathcal{X}, \mathcal{F} \rangle.
\]

- Equations of motion

\[
d\mathcal{X} + [\mathcal{A}, \mathcal{X}] = 0 \quad \mathcal{F} = d\mathcal{A} + \frac{1}{2} [\mathcal{A}, \mathcal{A}] = 0
\]

Notice: stabilizer equation

\[
\delta \lambda \mathcal{A} = 0
\]

is equal to the dilaton equation of motion.
Equation of motion – Symmetries

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- Gauge symmetries

\[ \delta_\lambda \mathcal{X} = [\lambda, \mathcal{X}] \quad \delta_\lambda A = d\lambda + [A, \lambda] \]
Equation of motion – Symmetries

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  \[ dX + [A, X] = 0 \quad F = dA + \frac{1}{2}[A, A] = 0 \]

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- Observables

**Notice:** stabilizer equation \( \delta_\lambda A = 0 \) is equal to the dilaton equation of motion.
Equation of motion– Symmetries

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- Observables
  - Casimir
  \[ C = -\frac{1}{2} \langle \mathcal{X}, \mathcal{X} \rangle \]
Equation of motion– Symmetries

**BF action**

\[ I = -\frac{k}{2\pi} \int_{\mathcal{M}} \langle \mathcal{F}, \mathcal{F} \rangle. \]

- Equations of motion

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\[ \text{Hol}[A] = \mathcal{P} \exp \int A \]
Equation of motion– Symmetries

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\[ I = -\frac{k}{2\pi} \int_{\mathcal{M}} \langle \mathcal{X}, \mathcal{F} \rangle. \]

- **Equations of motion**

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- **Observables**
  - **Casimir**
    \[ C = -\frac{1}{2} \langle \mathcal{X}, \mathcal{X} \rangle \]
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- **Notice:** stabilizer equation \( \delta_\lambda A = 0 \) is equal to the dilaton equation of motion
Cigar Geometry

- Interested in asymptotic dynamics of the theory
- Define solution space:
  - Solutions should have topology of disc of periodicity $\beta$

\[ X = X_\infty + \cdots \]
\[ \rho \]
\[ \tau \]
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\[ \text{Hol}[A] \neq I \]

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\[ \rho \]

\[ \text{Hol}[\mathcal{A}] = I \]

\[ \tau \]

\[ \mathcal{A} = \mathcal{A}^\infty + \cdots \]

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- Define boundary conditions $X^\infty, A^\infty$ for $X, A$
- Solutions should be smooth $\Rightarrow$ Holonomy must belong to center of group $G$
- Singles out subset $A_S$ of boundary conditions $A^\infty$

$$\rho$$

$\tau$

$$\text{Hol}[A] = I$$

$$\mathcal{A} = \mathcal{A}_S + \cdots$$

$$\mathcal{X} = \mathcal{X}^\infty + \cdots$$
Well-defined action principle—“Holographic renormalization”

- Above solutions are not necessarily saddle-points of

\[
I = -\frac{k}{2\pi} \int_{\mathcal{M}} \langle \mathcal{X}, \mathcal{F} \rangle.
\]
Well-defined action principle—“Holographic renormalization”

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\[ I = -\frac{k}{2\pi} \int_{\mathcal{M}} \langle \mathcal{X}, \mathcal{F} \rangle. \]

\[ \delta I = (\text{equations of motion}) - \frac{k}{2\pi} \int_{\partial \mathcal{M}} \langle \mathcal{X}^\infty, \delta \mathcal{A}^\infty \rangle. \]
Well-defined action principle—“Holographic renormalization”

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- Depending on boundary conditions last term will not be zero \( \Rightarrow \delta I|_{\text{on-shell}} \neq 0 \)
Well-defined action principle—“Holographic renormalization”

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- Depending on boundary conditions last term will not be zero \(\Rightarrow \delta I|_{\text{on-shell}} \neq 0\)
- Add a boundary term \(I_{bdy}\) to \(I\) to remedy this;
Well-defined action principle—“Holographic renormalization”

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\[ I = -\frac{k}{2\pi} \int_{\mathcal{M}} \langle \mathcal{X}, \mathcal{F} \rangle. \]

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- Integrability condition

\[ \mathcal{A}^\infty = df \mathcal{X}^\infty + u^{-1} du \quad u \in G \quad f \ldots \text{arbitrary} \]
Well-defined action principle—“Holographic renormalization”

- Above solutions are not necessarily saddle-points of

\[ l = -\frac{k}{2\pi} \int_{\mathcal{M}} \langle \mathcal{X}, \mathcal{F} \rangle. \]

\[ \delta l = \text{(equations of motion)} - \frac{k}{2\pi} \int_{\partial \mathcal{M}} \langle \mathcal{X}^\infty, \delta \mathcal{A}^\infty \rangle. \]

- Depending on boundary conditions last term will not be zero \( \Rightarrow \delta l \big|_{\text{on-shell}} \neq 0 \)
- Add a boundary term \( l_{\text{bdy}} \) to \( l \) to remedy this;
- integrability condition

\[ \mathcal{A}^\infty = df \mathcal{X}^\infty + u^{-1} du \quad u \in G \quad f \ldots \text{arbitrary} \]

---

**Improved action**

\[ \Gamma = l + l_{\text{bdy}} = -\frac{k}{2\pi} \int_{\mathcal{M}} \langle \mathcal{X}, \mathcal{F} \rangle - \frac{k}{2\pi} \int_{\partial \mathcal{M}} df \ C \]

where \( C = -\frac{1}{2} \langle \mathcal{X}, \mathcal{X} \rangle \).
$G = \text{SL}(2, \mathbb{R})$: Schwarzian action for the JT model

- Imposing the "geometry" equations of motion $\mathcal{F} = 0$

$$\Gamma_{\text{on-shell}} = -\frac{k}{2\pi} \int_{\partial \mathcal{M}} d\tau \dot{f} \ C$$
$G = \text{SL}(2, \mathbb{R})$: Schwarzian action for the JT model

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  \Gamma_{\text{on-shell}} = -\frac{k}{2\pi} \int_{\partial \mathcal{M}} d\tau \dot{f} \ C
  \]

- Choose $G = \text{SL}(2, \mathbb{R})$: $[L_m, L_n] = L_{m+n}$, $m, n = -1, 0, +1$
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\( G = \text{SL}(2, \mathbb{R})\): Schwarzian action for the JT model

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\Gamma_{\text{on-shell}} = -\frac{k}{2\pi} \int_{\partial\mathcal{M}} \dd\tau \dot{f} \ C
\]

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- \( A^\infty = \frac{e^\rho}{2} L_+ + \frac{e^{-\rho}}{2} \dot{\mathcal{M}} L_- \)

- \( \delta \lambda \mathcal{M} = 2 \mathcal{M} \dot{\lambda} + \dot{\mathcal{M}} \lambda - \frac{1}{2} \ddot{\lambda} \)
$G = \text{SL}(2, \mathbb{R}):$ Schwarzian action for the JT model

- Imposing the “geometry” equations of motion $\mathcal{F} = 0$
  \[
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  \]

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- $\delta \lambda \mathcal{M} = 2 \mathcal{M} \dot{\lambda} + \dot{\mathcal{M}} \lambda - \frac{1}{2} \dddot{\lambda}$

- looks like asymptotic symmetries are Virasoro
$G = \text{SL}(2, \mathbb{R})$: Schwarzian action for the JT model

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- Using $A^\infty$ and integrability condition $\Rightarrow$
  $$C = \frac{1}{f^2} \left( \mathcal{M} - \frac{1}{2} \{ f; \tau \} \right)$$
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  \[ C = \frac{1}{\dot{f}^2} \left( \mathcal{M} - \frac{1}{2} \{f, \tau\} \right) \]

- Geometry is smooth for $\mathcal{M} = \frac{\pi^2}{\beta^2}$

---

**Schwarzian action**

\[ \Gamma|_{\text{eom}} = -\frac{k}{4\pi} \int_{\partial \mathcal{M}} d\tau \left( \dot{g}^2 \frac{2\pi^2}{\beta^2} + \{g, \tau\} \right) \quad g := f^{-1} \]
\[ G = \text{SL}(2, \mathbb{R}): \text{Schwarzian action for the JT model} \]

- Imposing the “geometry” equations of motion \( \mathcal{F} = 0 \)
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- But \( \partial_\tau C = \dot{x}^+ \left( 2\mathcal{M} \dot{x}^+ + \dot{\mathcal{M}} x^+ - \frac{1}{2} \dddot{x}^+ \right) = 0, \quad x^+ = \dot{f}^{-1} \) by dilaton e.o.m.
\( G = \text{SL}(2, \mathbb{R}) \): Schwarzian action for the JT model

- Imposing the “geometry” equations of motion \( \mathcal{F} = 0 \)

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\Gamma_{\text{on-shell}} = -\frac{k}{2\pi} \int_{\partial \mathcal{M}} d\tau \dot{f} \ C
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- \( \mathcal{A}^\infty = \frac{e^\rho}{2} L_+ + \frac{e^{-\rho}}{2} \mathcal{M} L_- \)

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### Schwarzian action

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\Gamma|_{\text{eom}} = -\frac{k}{4\pi} \int_{\partial \mathcal{M}} d\tau \left( \dot{g}^2 \frac{2\pi^2}{\beta^2} + \{ g, \tau \} \right) \quad g := f^{-1}
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- But \( \partial_\tau C = \dot{x}^+ \left( 2\mathcal{M} \dot{x}^+ + \dot{\mathcal{M}} x^+ - \frac{1}{2} \dddot{x}^+ \right) = 0 \), \( x^+ = \dot{f}^{-1} \) by dilaton e.o.m.

- Solutions for \( \mathcal{M} = \frac{\pi^2}{\beta^2} \): \( \text{SL}(2) \); breaking Virasoro \( \rightarrow \) \( \text{SL}(2) \)
Generalizations of the Schwarzian action

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- Further generalization to groups not necessarily of the form $\text{SL}(2, \mathbb{R}) \times G$
AdS$_2$ holography subtle
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trivial in constant dilaton sector
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Thank you for your attention!
Conclusion

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Aspects of Holography in 2d Dilaton Gravity

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Holographic Renormalization

Temperature
Euclidean Path Integral

- Important tool for studying holography or black hole thermodynamics: **Euclidean Path integral**

\[
Z = \int Dg DX e^{-I[g,X]},
\]

with \( I = I_{\text{bulk}} + I_{\text{GHY}} \).

Calculate free energy, correlation functions etc. However, yields wrong results for nearly all dilaton models!

E.g., in thermodynamics:
- Free energy is not finite
- First law of black hole mechanics does not hold

What goes wrong?

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- find classical solutions $\bar{g}, \bar{X}$
- expand Euclidean path integral around classical solution

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For thermodynamical stability: one-loop contribution positive definite
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Consider the model: $U = f = 0$, $V(X) = \frac{1}{\ell^2} X$ (again, Jackiw-Teitelboim)

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Consider the model: \( U = f = 0, \ V(X) = \frac{1}{\ell^2} X \) (again, Jackiw-Teitelboim)

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I = \int_M d^2x \sqrt{|g|} X \left( R + \frac{1}{\ell^2} \right) + \int_{\partial M} dx \sqrt{\gamma} X K
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- exact solution: \( X = r \), \( ds^2 = \left( \frac{r^2}{\ell^2} - M \right) d\tau^2 + \left( \frac{r^2}{\ell^2} - M \right)^{-1} dr^2 \)
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- counter-term depends on the specific class of dilaton models
Renormalized Action—Class 1

For a large class of models:
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Thermodynamics well-defined
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Interesting example:

Add a U(1) field with specific coupling to gravity

$$F(X) = X - 1$$: charge $q$ acts as cosmological constant!

Works in dimension $D$ with rank $D$-antisymmetric tensor field (Henneaux, Teitelboim (1984))

First law of BH thermodynamics with variable $\Lambda$:

$$dM = T dS - h(X,h) d\Lambda$$

$h(X,h)$ equals proposed volume of 2D black holes c.c. and $h(X,h)$ form $p-V$ pair

BH thermodynamics in extended phase space (“black hole chemistry”) (Dolan, Kubiznak, Kastor, Mann, Traschen...)
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Saddle-point contribution to Euclidean path integral comes from geometry compatible with periodicity $\beta$ of Euclidean time

Holonomy around time cycle must be trivial

$$\mathcal{P} \exp \oint A = -1$$

equivalently: $\beta$ fixed such that linearly independent solutions $\psi_1, \psi_2$ to Hill's equation

$$\psi'' + \mathcal{T} \psi = 0 \quad \mathcal{T} = \mathcal{T}(\mathcal{L}^+, \mathcal{L}^-, \mathcal{L}^0)$$

have (minus) unit monodromy matrix $M$ up to conjugation

$$\begin{pmatrix} y_1(\varphi + \beta) \\ y_2(\varphi + \beta) \end{pmatrix} = M \begin{pmatrix} y_1(\varphi) \\ y_2(\varphi) \end{pmatrix}$$

Conjugcy classes: $\text{tr} M < 2$ elliptic; $\text{tr} M = 2$ parabolic; $\text{tr} M > 2$ hyperbolic
Correct choice of $\beta$ possible for elliptic case (finite temperature black hole)
Not possible for parabolic case (infinite throat geometry) or hyperbolic case (global AdS $\simeq$ hyperbolic cylinder) unless unwrapped
under infinitesimal reparametrizations of $\varphi$:

$$\delta \epsilon T = \epsilon T' + 2 \epsilon' T + \frac{1}{2} \epsilon''' \quad \delta Y = \epsilon Y' - \epsilon' Y$$

Transformation of $T$ under finite reparametrizations $\varphi \mapsto f(\varphi)$

$$\tilde{T}(f(\varphi)) = \frac{1}{(f'(\varphi))^2} \left[ T(\varphi) - \frac{1}{2} S[f](\varphi) \right] \quad S[f](\varphi) = \frac{1}{2} \left( \frac{f''}{f'} \right)' - \frac{1}{2} \left( \frac{f''}{f'} \right)^2$$

Equation of motion for $Y$

$$Y T' + 2 Y' T' + \frac{1}{2} Y''' = 0.$$
• In generic elliptic case, possible to transform $T$ to \textit{constant representative} $T_0$

\[
T_0 \propto - \left( \oint \frac{1}{Y} \right)^2
\]

is orbit invariant $\Rightarrow$ fixing this quantity fixes orbit and therefore constant representative

• $T_0$ fixes temperature $\rightarrow$ variational

• $\oint \frac{1}{Y}$ was fixed in variational principle $\Rightarrow$ temperature fixed