Massive gravity in three dimensions
The AdS$_3$/LCFT$_2$ correspondence

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Outline

Introduction to 3D gravity

Topologically massive gravity

Logarithmic CFT conjecture

Consequences, Generalizations & Applications
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Consequences, Generalizations & Applications
Motivation

- Quantum gravity
  - Address conceptual issues of quantum gravity
  - Black hole evaporation, information loss, black hole microstate counting, virtual black hole production, ...
  - Technically much simpler than 4D or higher D gravity
  - Integrable models: powerful tools in physics (Coulomb problem, Hydrogen atom, harmonic oscillator, ...)
  - Models should be as simple as possible, but not simpler
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- Gauge/gravity duality
  - Deeper understanding of black hole holography
  - $\text{AdS}_3/\text{CFT}_2$ correspondence best understood
  - Quantum gravity via $\text{AdS}/\text{CFT}$? (Witten ’07, Li, Song, Strominger ’08)
  - Applications to 2D condensed matter systems?
  - Gauge gravity duality beyond standard $\text{AdS}/\text{CFT}$: warped $\text{AdS}$, asymptotic Lifshitz, non-relativistic CFTs, logarithmic CFTs, ...
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- **Physics**
  - Cosmic strings (Deser, Jackiw, 't Hooft '84, '92)
  - Black hole analog systems in condensed matter physics (graphene, BEC, fluids, ...)

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Gravity in lower dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in $D$ dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
- 5D: 50 (35 Weyl and 15 Ricci)
- 4D: 20 (10 Weyl and 10 Ricci)

- 2D: lowest dimension exhibiting black holes (BHs)
- Simplest gravitational theories with BHs in 2D
- 3D: lowest dimension exhibiting BHs and gravitons
- Simplest gravitational theories with BHs and gravitons in 3D
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Pure gravity in 3D

Let us switch off all matter fields and keep only the metric $g$.

$$I_{3DG} = \frac{1}{16\pi G} \int \! \! d^3x \sqrt{-g} \mathcal{L}(g)$$
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- Variation of $\mathcal{L}$ should lead to tensor equations
- Require absence of higher derivatives than fourth (for simplicity)
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The requirements above are fulfilled for

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\mathcal{L} = \mathcal{L}_{MG}(R_{\mu\nu}) + \mathcal{L}_{CS}
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$$\mathcal{L} = \mathcal{L}_{\text{MG}}(R_{\mu\nu}) + \mathcal{L}_{\text{CS}}$$

with the possibility for a gravitational Chern–Simons term

$$\mathcal{L}_{\text{CS}} = \frac{1}{2\mu} \varepsilon^{\lambda\mu\nu} \Gamma^\rho_{\lambda\sigma} \left( \partial_\mu \Gamma^\sigma_{\nu\rho} + \frac{2}{3} \Gamma^\sigma_{\mu\tau} \Gamma^\tau_{\nu\rho} \right)$$
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and the higher derivative Lagrange density

$$\mathcal{L}_{MG}(R_{\mu\nu}) = \sigma R - 2\Lambda + \frac{1}{m^2} \left( R_{\mu\nu} R^{\mu\nu} - \frac{3}{8} R^2 \right) + \mathcal{O}(R^3_{\mu\nu})$$
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Action and equations of motion of topologically massive gravity (TMG)

Consider the action (Deser, Jackiw & Templeton ’82)

\[
I_{\text{TMG}} = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left[ R + \frac{2}{\ell^2} + \frac{1}{2\mu} \varepsilon^{\lambda\mu\nu} \Gamma^\rho_\lambda_\sigma (\partial_\mu \Gamma^\sigma_\nu_\rho + \frac{2}{3} \Gamma^\sigma_\mu_\tau \Gamma^\tau_\nu_\rho) \right]
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Equations of motion:

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{1}{\ell^2} g_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} = 0 \]

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Some properties of TMG

- Massive gravitons and black holes
- AdS solutions and asymptotic AdS solutions
- warped AdS solutions and warped AdS black holes
- Schrödinger solutions and Schrödinger pp-waves
Classical solutions (exact)

Stationarity plus axi-symmetry:
  ► Two commuting Killing vectors
Classical solutions (exact)

Stationarity plus axi-symmetry:
- Two commuting Killing vectors
- Effectively reduce 2+1 dimensions to 1+0 dimensions
Classical solutions (exact)

Stationarity plus axi-symmetry:
- Two commuting Killing vectors
- Effectively reduce 2+1 dimensions to 1+0 dimensions
- Like particle mechanics, but with up to three time derivatives

\[
\mathcal{C} \left[ e, X^i \right] \sim \int d\rho e \left[ \frac{1}{2} e^{-2} \dot{X}^i \dot{X}^j \eta_{ij} - \frac{1}{2} \ell^2 + \frac{1}{2} \mu e^{-3} \epsilon^{ijk} \dot{X}^i \ddot{X}^j \dddot{X}^k \right]
\]

Here \( e \) is the Einbein and \( X^i = (T, X, Y) \) a Lorentzian 3-vector

Classification of solutions:
- Einstein solutions: AdS, BTZ
- warped solutions: warped AdS, warped black holes
- Schrödinger solutions: asymptotic Schrödinger spacetimes, pp-waves
- generic solutions (Ertl, Grumiller & Johansson, '10)
Classical solutions (exact)

Stationarity plus axi-symmetry:
- Two commuting Killing vectors
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- Like particle mechanics, but with up to three time derivatives
- Still surprisingly difficult to get exact solutions!

Reduced action (Clement '94):

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I_C \sim \int d\rho e \left[ \frac{1}{2} e - 2 \dot{X}^i \dot{X}^j \eta_{ij} - \frac{1}{2} \ell^2 + \frac{1}{2} \mu e - 3 \epsilon^{ijk} X^i \ddot{X}^j X^k \right]
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TMG at the chiral point

Definition: TMG at the chiral point is TMG with the tuning

$$\mu \ell = 1$$

between the cosmological constant and the Chern–Simons coupling.
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Why special? (Li, Song & Strominger '08)
**TMG at the chiral point**

**Definition:** TMG at the **chiral** point is TMG with the tuning

\[ \mu \ell = 1 \]

between the cosmological constant and the Chern–Simons coupling.

**Why special?** (Li, Song & Strominger '08)

Calculating the central charges of the dual boundary CFT yields

\[ c_L = \frac{3\ell}{2G} \left(1 - \frac{1}{\mu \ell}\right) \quad c_R = \frac{3\ell}{2G} \left(1 + \frac{1}{\mu \ell}\right) \]

Thus, at the **chiral** point we get

\[ c_L = 0 \quad c_R = \frac{3\ell}{G} \]
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Thus, at the chiral point we get

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- Abbreviate “Cosmological TMG at the chiral point” as CTMG
- CTMG is also known as “chiral gravity”
- Dual CFT: chiral? (conjecture by Li, Song & Strominger ’08)
- More adequate name for CTMG: “logarithmic gravity”
Gravitons around AdS$_3$ in CTMG

Linearization around AdS background.

\[ g_{\mu \nu} = \bar{g}_{\mu \nu} + h_{\mu \nu} \]

Line-element \( \bar{g}_{\mu \nu} \) of pure AdS:

\[ d\bar{s}^2_{\text{AdS}} = \bar{g}_{\mu \nu} \, dx^\mu \, dx^\nu = \ell^2 ( - \cosh^2 \rho \, d\tau^2 + \sinh^2 \rho \, d\phi^2 + d\rho^2 ) \]

Isometry group: \( SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R \)

Useful to introduce light-cone coordinates \( u = \tau + \phi, \, v = \tau - \phi \).

The \( SL(2, \mathbb{R})_L \) generators

\[ L_0 = i \partial_u \]

\[ L_{\pm 1} = ie^{\pm iu} \left[ \frac{\cosh 2\rho}{\sinh 2\rho} \partial_u - \frac{1}{\sinh 2\rho} \partial_v \mp \frac{i}{2} \partial_\rho \right] \]

obey the algebra \( [L_0, L_{\pm 1}] = \mp L_{\pm 1}, \quad [L_1, L_{-1}] = 2L_0 \).

The \( SL(2, \mathbb{R})_R \) generators \( \bar{L}_0, \bar{L}_{\pm 1} \) obey same algebra, but with

\[ u \leftrightarrow v, \quad L \leftrightarrow \bar{L} \]
Gravitons around AdS$_3$ in CTMG

Linearization around AdS background.

\[ g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \]

leads to linearized EOM that are third order PDE

\[ G^{(1)}_{\mu\nu} + \frac{1}{\mu} \mathcal{C}^{(1)}_{\mu\nu} = (\mathcal{D}^R \mathcal{D}^L \mathcal{D}^M h)_{\mu\nu} = 0 \]  \hfill (1)

with three mutually commuting first order operators

\[ (\mathcal{D}^{L/R})_{\mu}^{\ \nu} = \delta_{\mu}^{\nu} \pm \epsilon_{\mu}^{\ \alpha\nu} \bar{\nabla}^{\alpha} \quad (\mathcal{D}^{M})_{\mu}^{\ \nu} = \delta_{\mu}^{\nu} + \frac{1}{\mu} \epsilon_{\mu}^{\ \alpha\nu} \bar{\nabla}^{\alpha} \]
Gravitons around $\text{AdS}_3$ in CTMG

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$$G^{(1)}_{\mu\nu} + \frac{1}{\mu} C^{(1)}_{\mu\nu} = \left(\mathcal{D}^{R} \mathcal{D}^{L} \mathcal{D}^{M} h\right)_{\mu\nu} = 0$$ (1)

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$$\left(\mathcal{D}^{L/R}\right)_{\mu}^{\nu} = \delta_{\mu}^{\nu} \pm \ell \varepsilon_{\mu}^{\alpha\nu} \bar{\nabla}_{\alpha} \quad \left(\mathcal{D}^{M}\right)_{\mu}^{\nu} = \delta_{\mu}^{\nu} + \frac{1}{\mu} \varepsilon_{\mu}^{\alpha\nu} \bar{\nabla}_{\alpha}$$

Three linearly independent solutions to (1):

$$\left(\mathcal{D}^{L}h^{L}\right)_{\mu\nu} = 0 \quad \left(\mathcal{D}^{R}h^{R}\right)_{\mu\nu} = 0 \quad \left(\mathcal{D}^{M}h^{M}\right)_{\mu\nu} = 0$$
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Linearization around AdS background.

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leads to linearized EOM that are third order PDE

\[ G_{\mu\nu}^{(1)} + \frac{1}{\mu} C_{\mu\nu}^{(1)} = (\mathcal{D}^R\mathcal{D}^L\mathcal{D}^M h)_{\mu\nu} = 0 \]  \hspace{1cm} (1)

with three mutually commuting first order operators

\[ (\mathcal{D}^L/R)_{\mu}{}^{\nu} = \delta_{\mu}{}^{\nu} + \ell \varepsilon_{\mu}{}^{\alpha\nu} \bar{\nabla}_{\alpha} \] \hspace{1cm} \[ (\mathcal{D}^M)_{\mu}{}^{\nu} = \delta_{\mu}{}^{\nu} + \frac{1}{\mu} \varepsilon_{\mu}{}^{\alpha\nu} \bar{\nabla}_{\alpha} \]

Three linearly independent solutions to (1):

\[ (\mathcal{D}^L h^L)_{\mu\nu} = 0 \quad (\mathcal{D}^R h^R)_{\mu\nu} = 0 \quad (\mathcal{D}^M h^M)_{\mu\nu} = 0 \]

At chiral point left (L) and massive (M) branches coincide!
Degeneracy at the chiral point
Will be quite important later!

Li, Song & Strominger found all normalizable solutions of linearized EOM.

- **Primaries:** $L_0$, $\bar{L}_0$ eigenstates $\psi^{L/R/M}$ with
  \[ L_1 \psi^{R/L/M} = \bar{L}_1 \psi^{R/L/M} = 0 \]
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- **Descendants**: act with $L_{-1}$ and $\bar{L}_{-1}$ on primaries
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- **General solution:** linear combination of $\psi^{R/L/M}$

Linearized metric is then the real part of the wavefunction
\[ h_{\mu\nu} = \text{Re} \left( \psi_{\mu\nu} \right) \]

At chiral point: $L$ and $M$ branches degenerate. Get log solution (Grumiller & Johansson '08)
\[ \psi^{\text{log}}_{\mu\nu} = \lim_{\mu \ell \to 1} \psi^M_{\mu\nu}(\mu \ell) - \psi^L_{\mu\nu}(\mu \ell - 1) \]
with property
\[
\left( D_L \psi^{\text{log}}_{\mu\nu} \right) = \left( D_M \psi^{\text{log}}_{\mu\nu} \right) \\
\left( (D_L)^2 \psi^{\text{log}}_{\mu\nu} \right) = 0
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with property

$$(\mathcal{D}^L \psi^{\log})_{\mu\nu} = (\mathcal{D}^M \psi^{\log})_{\mu\nu} \neq 0 \ , \quad ((\mathcal{D}^L)^2 \psi^{\log})_{\mu\nu} = 0$$
Sign oder nicht sign?
That is the question. Choosing between Skylla and Charybdis.

- With signs defined as in this talk: BHs positive energy, gravitons negative energy
- With signs as defined in Carlip, Deser, Waldron, Wise '08: BHs negative energy, gravitons positive energy
- Either way need a mechanism to eliminate unwanted negative energy objects — either the gravitons or the BHs
- Even at chiral point the problem persists because of the logarithmic mode. See Figure. (thanks to Niklas Johansson)
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Motivating the conjecture

**Log** mode exhibits interesting property:

\[
H \begin{pmatrix} \psi^{\text{log}} \\ \psi^L \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \psi^{\text{log}} \\ \psi^L \end{pmatrix}
\]

\[
J \begin{pmatrix} \psi^{\text{log}} \\ \psi^L \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \psi^{\text{log}} \\ \psi^L \end{pmatrix}
\]

Here \( H = L_0 + \bar{L}_0 \sim \partial_t \) is the Hamilton operator and \( J = L_0 - \bar{L}_0 \sim \partial_\phi \) the angular momentum operator.
Motivating the conjecture

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Such a **Jordan form** of \( H \) and \( J \) is defining property of a **logarithmic CFT**!
Motivating the conjecture

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\]

\[
J \begin{pmatrix} \psi_{\log} \\ \psi_L \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \psi_{\log} \\ \psi_L \end{pmatrix}
\]

Here \( H = L_0 + \bar{L}_0 \sim \partial_t \) is the Hamilton operator and \( J = L_0 - \bar{L}_0 \sim \partial \phi \) the angular momentum operator.

Such a **Jordan form** of \( H \) and \( J \) is defining property of a logarithmic CFT!

---

**Logarithmic CFT conjecture**

CTMG dual to a logarithmic CFT (Grumiller, Johansson '08)
Early hints for validity of conjecture

Properties of logarithmic mode:

- Perturbative solution of linearized EOM, but not pure gauge

\[
\begin{align*}
E_{\text{log}} &= -\frac{47}{1152} G_{\ell}^3 \\
\text{and negative} &\rightarrow \text{instability! (Grumiller & Johansson '08)}
\end{align*}
\]

- Logarithmic mode is asymptotically AdS

\[
ds^2 = \rho^2 + \left(\gamma^{(0)}_{ij} e^{2\rho/\ell} + \gamma^{(1)}_{ij} \rho + \gamma^{(0)}_{ij} + \gamma^{(2)}_{ij} e^{-2\rho/\ell} + \ldots\right) dx^i dx^j
\]

- but violates Brown–Henneaux boundary conditions! ($\gamma^{(1)}_{ij} \big| \big|_{\text{BH}} = 0$)

- Consistent log boundary conditions replacing Brown–Henneaux (Grumiller & Johansson '08, Martinez, Henneaux & Troncoso '09)

- Brown–York stress tensor is finite and traceless, but not chiral

- Log mode persists non-perturbatively, as shown by Hamilton analysis (Grumiller, Jackiw & Johansson '08, Carlip '08)
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Properties of logarithmic mode:
- Perturbative solution of linearized EOM, but not pure gauge
- Energy of logarithmic mode is finite:
  \[ E^{\log} = - \frac{47}{1152 G \ell^3} \]
  and negative → instability! (Grumiller & Johansson ’08)

\[ d^2 s^2 = d\rho^2 + \left( \gamma^{(0)}_{ij} e^{2\rho/\ell} + \gamma^{(1)}_{ij} \rho + \gamma^{(0)}_{ij} + \gamma^{(2)}_{ij} e^{-2\rho/\ell} + \ldots \right) dx_i dx_j \]

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Correlators in logarithmic CFTs

- Any CFT has a conserved traceless energy momentum tensor.

\[ T_{z\bar{z}} = 0 \quad T_{zz} = \mathcal{O}^L(z) \quad T_{\bar{z}\bar{z}} = \mathcal{O}^R(\bar{z}) \]
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- The 2- and 3-point correlators are fixed by conformal Ward identities.

\[
\begin{align*}
\langle \mathcal{O}^R(z) \mathcal{O}^R(0) \rangle &= \frac{c_R}{2\bar{z}^4} \\
\langle \mathcal{O}^L(z) \mathcal{O}^L(0) \rangle &= \frac{c_L}{2z^4} \\
\langle \mathcal{O}^L(z) \mathcal{O}^R(0) \rangle &= 0 \\
\langle \mathcal{O}^R(z) \mathcal{O}^R(z') \mathcal{O}^R(0) \rangle &= \frac{c_R}{\bar{z}^2\bar{z'}^2(\bar{z} - \bar{z'})^2} \\
\langle \mathcal{O}^L(z) \mathcal{O}^L(z') \mathcal{O}^L(0) \rangle &= \frac{c_L}{z^2z'^2(z - z')^2} \\
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Central charges \(c_L/R\) determine key properties of CFT.
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- Suppose there is an additional operator \( \mathcal{O}^M \) with conformal weights \( h = 2 + \varepsilon, \bar{h} = \varepsilon \)

  \[ \langle \mathcal{O}^M (z, \bar{z}) \mathcal{O}^M (0, 0) \rangle = \frac{\hat{B}}{z^{4+2\varepsilon} \bar{z}^{2\varepsilon}} \]

  which degenerates with \( \mathcal{O}^L \) in limit \( c_L \propto \varepsilon \to 0 \)
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- Then energy momentum tensor acquires logarithmic partner \( O^{log} \)

  \[ O^{log} = b_L \frac{O^L}{c_L} + \frac{b_L}{2} O^M \]

  where

  \[ b_L := \lim_{c_L \to 0} -\frac{c_L}{\varepsilon} \neq 0 \]
Correlators in logarithmic CFTs

- Any CFT has a conserved traceless energy momentum tensor.
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- Then energy momentum tensor acquires logarithmic partner $\mathcal{O}^{\log}$

- Some 2-point correlators:
  \[
  \langle \mathcal{O}^L(z) \mathcal{O}^L(0, 0) \rangle = 0
  \]
  \[
  \langle \mathcal{O}^L(z) \mathcal{O}^{\log}(0, 0) \rangle = \frac{b_L}{2z^4}
  \]
  \[
  \langle \mathcal{O}^{\log}(z, \bar{z}) \mathcal{O}^{\log}(0, 0) \rangle = -\frac{b_L \ln (m_L^2 |z|^2)}{z^4}
  \]

"New anomaly" $b_L$ determines key properties of logarithmic CFT.
Check of logarithmic CFT conjecture for 2- and 3-point correlators

If LCFT conjecture is correct then following procedure must work:

▶ Calculate non-normalizable modes for left, right and logarithmic branches by solving linearized EOM on gravity side
▶ According to AdS$_3$/LCFT$_2$ dictionary these non-normalizable modes are sources for corresponding operators in the dual CFT
▶ Calculate 2- and 3-point correlators on the gravity side, e.g. by plugging non-normalizable modes into second and third variation of the on-shell action
▶ These correlators must coincide with the ones of a logarithmic CFT except for value of new anomaly $b_L$ and freedom in this procedure. Either it works or it does not work.
▶ Works at level of 2-point correlators (Skenderis, Taylor & van Rees '09, Grumiller & Sachs '09)
▶ Works at level of 3-point correlators (Grumiller & Sachs '09)
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Except for value of new anomaly $b_L$ no freedom in this procedure. Either it works or it does not work.
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Alternative calculation of new anomaly $b_L$

As another consistency check perform the following short-cut.

Then weights $h = 2 + \varepsilon$ and $\bar{h} = \varepsilon$ of massive modes differ infinitesimally from weights 2 and 0 of left mode.

The new anomaly is given by the ratio of these two small quantities $b_L = \lim_{\varepsilon \to 0} -c L \varepsilon$.

Result obtained in this way must coincide with result for $b_L$ from the 2- and 3-point correlators.

Recover the result (Grumiller & Hohm '09, Grumiller, Johansson & Zojer, '10) $b_L = -3 \ell_G$. 

D. Grumiller — Massive gravity in three dimensions

Logarithmic CFT conjecture
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As another consistency check perform the following short-cut.

- Consider small but non-vanishing central charge $c_L$

\[ b_L = \lim_{\epsilon \to 0} \frac{c_L}{\epsilon} \]

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1-loop partition function
...yet another non-trivial check (Gaberdiel, Grumiller & Vassilevich ’10)

If LCFT conjecture is true, then the following procedure must work
▶ Calculate 1-loop partition function on gravity side
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Conclusion: all consistency tests show validity of LCFT conjecture!
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...yet another non-trivial check (Gaberdiel, Grumiller & Vassilevich ’10)

If LCFT conjecture is true, then the following procedure must work

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- Check that it is not chiral
- Calculate “minimal part” of partition function (Virasoro descendants of vacuum, descendants of log operator) on CFT side
- Calculate the difference between these partition functions (corresponds to multiple log excitations)
- Check that all multi-log coefficients in this difference are non-negative

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Outline

Introduction to 3D gravity

Topologically massive gravity

Logarithmic CFT conjecture

Consequences, Generalizations & Applications
Summary and comments

TMG at the chiral/logarithmic point $\mu \ell = 1$:

- 3D gravity theory with black holes and massive graviton excitations

Conjectured to be dual to logarithmic CFT

- Conjecture passed several independent consistency tests

- Non-trivial Jordan cell structure on gravity side, like in LCFT

- Operator degenerates with energy-momentum tensor at the point where central charge vanishes
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Central charges:

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- New anomaly: $b_L = -3 \ell / G$

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If conjecture true: first example of AdS$_3$/LCFT$_2$ correspondence!
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Chiral gravity conjectured to exist as consistent quantum theory of gravity by Li, Song & Strominger ’08

Dual CFT would be a chiral CFT with $c_L = 0$ and $c_R = 3\ell/G$

Partition function trivially factorizes holomorphically

Thus avoids problems with original approach by Witten ’07

Chiral gravity defined by truncation of the dual LCFT

Truncation either by requiring periodicity in time or by imposing stricter fall-off conditions than asymptotic AdS (Brown–Henneaux)

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Generalizations to new massive gravity and generalized massive gravity

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A: No!

New massive gravity (Bergshoeff, Hohm & Townsend '09):

\[
INMG = \frac{1}{16\pi G} \int d^3 x \sqrt{-g} \left[ \sigma R + \frac{1}{m^2} \left( R_{\mu\nu} R^{\mu\nu} - \frac{3}{8} R^2 \right) - 2\lambda m^2 \right]
\]

Similar story (Grumiller & Hohm '09, Alishahiha & Naseh '10):

- Linearized EOM around AdS

\[
(D R^R)_{\mu\nu} = 0
\]

- Logarithmic point for \( \lambda = 3/2c L = c_R = 0 \)

- Massive modes degenerate with left and right boundary gravitons

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Extended generalized massive gravity (Paulos ’10)
Reconsider higher curvature theories introduced in the beginning

All actions of type

\[ \mathcal{L} = \mathcal{L}_{MG}(R_{\mu\nu}) + \mathcal{L}_{CS} \]

with gravitational Chern–Simons term

\[ \mathcal{L}_{CS} = \frac{1}{2\mu} \varepsilon^{\lambda\mu\nu} \Gamma^\rho_{\lambda\sigma} \left( \partial_\mu \Gamma^\sigma_{\nu\rho} + \frac{2}{3} \Gamma^\sigma_{\mu\tau} \Gamma^\tau_{\nu\rho} \right) \]

and the specific higher derivative Lagrange density

\[ \mathcal{L}_{MG}(R_{\mu\nu}) = \sigma R - 2\Lambda + \frac{1}{m^2} \left( R_{\mu\nu} R^{\mu\nu} - \frac{3}{8} R^2 \right) + \mathcal{O}(R^3_{\mu\nu}) \]

have an AdS solution (if \( \Lambda_{\text{eff}} < 0 \)) and linearized equations of motion

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Thus, we have infinitely many gravity duals for LCFTs!
Potential applications in condensed matter physics

LCFTs arise in systems with quenched disorder.
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- Quenched disorder: systems with random variable that does not evolve in time
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- **Quenched disorder**: systems with random variable that does not evolve in time
- **Examples**: spin glasses, quenched random magnets, dilute self-avoiding polymers, percolation

\[
\langle O(z) O(0) \rangle = \int D\mathbf{V} \left[ \mathbf{V} \right] \int D\phi \exp \left( -I[\phi] - \int d^2 z' V(z') O(z') O(z) \right) \int D\phi \exp \left( -I[\phi] - \int d^2 z' V(z') O(z') O(z) \right)
\]

- Different ways to deal with denominator (replica trick, SUSY)
- Result: operators degenerate and correlators acquire logarithmic behavior, exactly as in LCFT (Cardy '99)
- Exploit LCFTs to compute correlators of quenched random systems
- Apply AdS$_3$/LCFT$_2$ to describe strongly coupled LCFTs!
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- Apply AdS$_3$/LCFT$_2$ to describe strongly coupled LCFTs!
Next steps

- Quantum gravity
  - Consistency of truncation to chiral gravity?
  - Existence of (log) extremal CFTs for arbitrary level $k$?
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  - Matching of 1-loop partition function in generalized massive gravity? (Bertin, Grumiller & Vassilevich, Zojer, in preparation)
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Thanks for your attention!
Some literature


Thanks to Bob McNees for providing the \LaTeX{} beamerclass!