# Carrollian c-functions

## and flat space holographic RG flows

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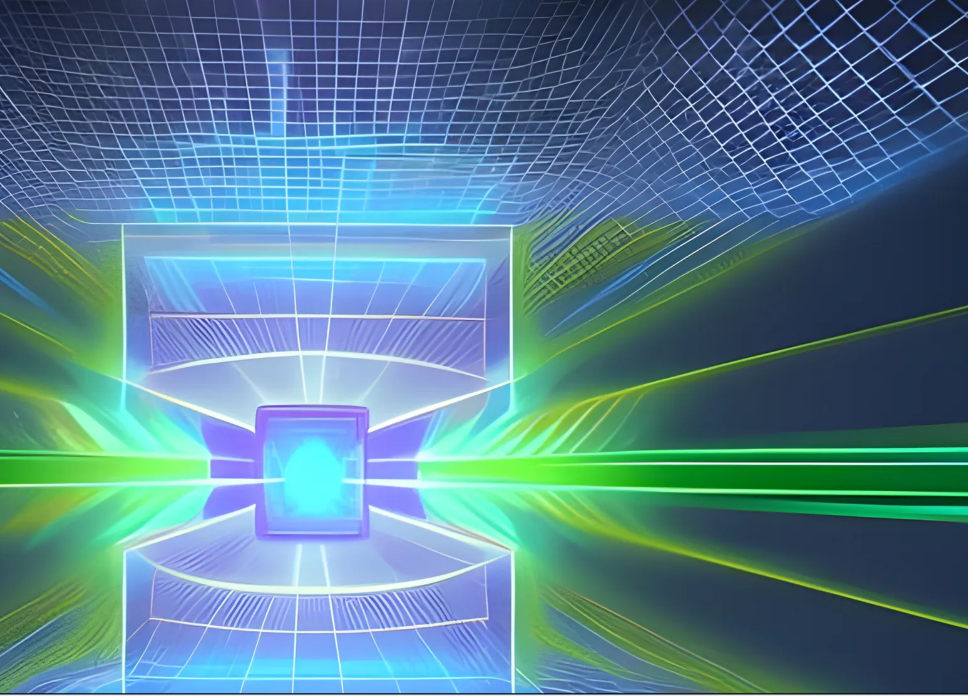
FWF

with Max Riegler, 2309.11539



# Outline

Flat space holography à la Carroll

Holographic $c$-functions in $\mathrm{AdS}_3/\mathrm{CFT}_2$

Flat space domain walls and $\mathrm{BMS}_3$ $c$-functions

Outlook to Casini–Huerta-like $c$-function

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Two main approaches:

- Carrollian (codimension-1 holography)
- Celestial (codimension-2 holography)

Translation between Carrollian and Celestial holography possible

Donnay, Fiorucci, Herfray, Ruzziconi; Bagchi, Banerjee, Basu, Dutta '22

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This talk:

Focus on $BMS_3/CCFT_2$ correspondence

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Review five examples

## Selected checks of $BMS_3/CCFT_2$ correspondence

1. analog of Brown–Henneaux bc's leading to BMS$_3$ Barnich, Compère '06

$$\mathrm{d}s^2 = \left(\mathcal{O}(1)\,\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + \mathcal{O}(1)\,\mathrm{d}u\,\mathrm{d}\varphi + r^2\,\mathrm{d}\varphi^2\right)\left(1 + \mathcal{O}(1/r)\right)$$

preserved by asymptotic Killing vectors

$$M_n = ie^{in\varphi}\partial_u + \dots \qquad L_n = ie^{in\varphi}\left(inu\partial_u - inr\partial_r + (1 + \frac{u}{r}\,n^2)\partial_\varphi\right) + \dots$$

whose Lie bracket algebra generates superrotations & -translations

$$[L_n,\,L_m] = (n-m)\,L_{n+m} + \frac{c_L}{12}\left(n^3 - n\right)\delta_{n+m,\,0}$$

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$$[M_n,\,M_m] = 0$$

canonical realization of asymptotic symmetries can lead to central extensions, depending on the bulk theory

algebra above is Carrollian conformal algebra Duval, Gibbons, Horvathy '14
and isomorphic to Galilean conformal algebra in 2d Bagchi '10

## Selected checks of $\text{BMS}_3/\text{CCFT}_2$ correspondence

1. analog of Brown–Henneaux bc's leading to $\text{BMS}_3$ Barnich, Compère '06
2. match of linearized spectra & gap in spectrum Bagchi, Detournay, DG '12

e.g. in flat space chiral gravity

$$\text{CCFT}_2 : \psi^{(n)} = L_{-n}|0\rangle \qquad\qquad \text{BMS}_3 : \psi^{(n)}_{uu} = -2ne^{-in\varphi}\dots$$

Virsoro descendants of vacuum mapped to "boundary gravitons"

gap: ground state (global Minkowski vacuum) has Virasoro charge $Q_{L_0} = -k$ while non-perturbative states (flat space cosmologies) have Virasoro charge $Q_{L_0} = k\,\alpha^2$

global Minkowski:

$$-\,\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\;\mathrm{d}\varphi^2$$

flat space cosmologies (with horizon radius $r_0$):

$$\alpha^2\left(1-\frac{r_0^2}{r^2}\right)\,\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\left(\mathrm{d}\varphi - \frac{\alpha r_0}{r^2}\;\mathrm{d}u\right)^2$$

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thermal entropy of flat space cosmologies:

$$S_{\text{macro}} = S_{\text{FSC}} = S_{\text{BH}} = \frac{2\pi\, r_0}{4G}$$

thermal entropy in Carrollian $\text{CFT}_2$ using Cardyology:

$$S_{\text{micro}} = S_{\text{CCFT}_2} = 2\pi\, h_L \sqrt{\frac{c_M}{24 h_M}}$$

fineprint: assumed here vanishing Virasoro central charge and non-vanishing BMS central charge, like in Einstein gravity

explicit computation shows $h_L = \frac{\alpha r_0}{4G}$, $h_M = \frac{\alpha^2}{8G}$, $c_M = \frac{3}{G}$ and thus

$$S_{\text{macro}} = S_{\text{micro}}$$

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4. (holographic) entanglement entropy Bagchi, Basu, DG, Riegler '14

Carrollian $CFT_2$ calculation yields EE for vacuum on the plane

$$S_{\text{EE}} = \frac{c_L}{6}\,\ln\frac{\Delta x}{\varepsilon_x} + \frac{c_M}{6}\left(\frac{\Delta u}{\Delta x} - \frac{\varepsilon_u}{\varepsilon_x}\right)$$

get also EE for other states related to vacuum by uniformization DG, Parekh, Riegler '19

another example: EE for vacuum on the cylinder

$$S_{\text{EE}} = \frac{c_L}{6}\,\ln\frac{2\sin\frac{\Delta\varphi}{2}}{\varepsilon_\varphi} + \frac{c_M}{6}\left(\frac{\Delta u}{2}\,\cot\frac{\Delta\varphi}{2} - \frac{\varepsilon_u}{\varepsilon_\varphi}\right)$$

results reproduced on gravity side using Wilson lines Basu, Riegler '15 or swing construction with geodesics Jiang, Song, Wen '17; Apolo, Jiang, Song, Zhong '20

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5. (holographic) stress tensor correlation functions Bagchi, DG, Merbis '15

$\text{CCFT}_2$ conservation equations ($M$: supertranslations, $L$: superrotations)

$$\partial_u\langle M\,\mathcal{O}\rangle = 0 \qquad\qquad \partial_u\langle L\,\mathcal{O}\rangle = \partial_\varphi\langle M\,\mathcal{O}\rangle$$

yield BPZ-like recursion relations ($s_{ij} = 2\sin[(\varphi_1 - \varphi_2)/2]$, $c_{ij} = \cot[(\varphi_1 - \varphi_2)/2]$)

$$\langle M_1 L_2 \dots L_n\rangle = \sum_{i=2}^{n}\Big(\frac{2}{s_{1i}^2} + \frac{c_{1i}}{2}\,\partial_{\varphi_i}\Big)\,\langle M_2 L_3 \dots L_n\rangle$$

$$\langle L_1 L_2 \dots L_n\rangle = \frac{c_L}{c_M}\,\langle M_1 L_2 \dots L_n\rangle + \sum_{i=1}^{n} u_i\partial_{\varphi_i}\,\langle M_1 L_2 \dots L_n\rangle$$

reproduced on gravity side (in Chern–Simons formulation)

## 7-point function of the stress-tensor from 3D holography.

**Wout Merbis**
Institute for Theoretical Physics, Vienna University of Technology
Wiedner Hauptstrasse 8-10/136 A-1040 Wien, AUSTRIA

$$\langle T(z_1)T(z_2)T(z_3)T(z_4)T(z_5)T(z_6)T(z_7)\rangle =$$

Based on work presented in "Stress tensor correlators in three-dimensional gravity" with A. Bagchi and D. Grumiller. Special thanks to Friedrich Schöller for Mathematica wizardry

## $\text{BMS}_3$ in Einstein gravity

Asymptotic symmetries in asymptotically flat space for 3d Einstein gravity:
Ashtekar, Bicak, Schmidt '96; Barnich, Compère '06

$$[L_n,\, L_m] = (n-m)\, L_{n+m}$$
$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\, n(n^2-1)\, \delta_{n+m,\,0}$$
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numerous holographic checks & results based on $\text{BMS}_3$ symmetries

- flat space chiral gravity Bagchi, Detournay, DG '12
- cardyology Bagchi, Detournay, Fareghbal, Simón; Barnich '12
- phase transitions Bagchi, Detournay, DG, Simón '13
- entanglement entropy Bagchi, Basu, DG, Riegler '14; Jiang, Song, Wen '17
- holographic dictionary & 1-point fct.'s Detournay, DG, Schöller, Simón '14
- 1-loop partition fct. & BMS characters Barnich,González,Maloney,Oblak '15
- all stress tensor correlators Bagchi, DG, Merbis '15
- BMS bootstrap Bagchi, Gary, Zodinmawia '16
- BMS blocks Hijano '18
- ...

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- $L_n$: superrotations (diff$(S^1)$)
- $M_n$: supertranslations
- $c_M = 3/G$: $\mathrm{BMS}_3$-central charge

Note: $c_M$ dimensionful ⇒ dimensionless ratios still meaningful

$$\frac{c_M}{h_M}\,, \qquad \frac{c_M^{\mathrm{UV}}}{c_M^{\mathrm{IR}}}\,, \qquad c_M \times \text{length}$$

where

$$M_0|\Psi\rangle = h_M|\Psi\rangle$$

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- BMS$_3$ emerges as UR limit of CFT$_2$ symmetries $\Rightarrow$ Carrollian CFT$_2$

$$L_n := \mathcal{L}^+_n - \mathcal{L}^-_{-n} \qquad\qquad M_n := \frac{1}{\ell}\left(\mathcal{L}^+_n + \mathcal{L}^-_{-n}\right)$$

with Virasoros $[\mathcal{L}^\pm_n,\, \mathcal{L}^\pm_m] = (n-m)\, \mathcal{L}^\pm_{n+m} + \frac{c^\pm}{12}\left(n^3-n\right)\delta_{n+m,\,0}$
yields CCFT$_2 \simeq$ BMS$_3$ with $c_L = c^+ - c^-$ and $c_M = \frac{1}{\ell}\left(c^+ + c^-\right)$

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This talk: bulk theory = Einstein gravity + scalar field

$$I_{\text{bulk}}[g_{\mu\nu},\, \phi] = \frac{1}{16\pi G} \int \mathrm{d}^3x \sqrt{-g}\, \Big(R - \frac{1}{2}\,(\partial\phi)^2 - V(\phi)\Big)$$

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# Zamolodchikov $c$-function

## Consider RG-flow from UV to IR in $\text{QFT}_2$

fineprint: the Euclidean $\text{QFT}_2$ is assumed to be renormalizable, reflection-positive, translation- and rotation-invariant

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- there exists a function $c(g)$ with the properties
  1. Monotonicity

$$\dot{c}(g) := \beta^i(g)\,\frac{\partial c(g)}{\partial g^i} \le 0$$

Equivalently: $c(g)$ non-increasing under dilatations!

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$$\dot{c}(g_*) = 0 \qquad \leftrightarrow \qquad \beta^i(g_*) = 0$$

Note: at fixed points enhancement to $\mathrm{CFT}_2$ Virasoro symmetries

$$[L_n,\, L_m] = (n-m)\,L_{n+m} + \frac{c_{\mathrm{Vir}}(g_*)}{12}\,n(n^2-1)\,\delta_{n+m,\,0}$$

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- Consequence:

$$c_{\text{Vir}}^{\text{UV}} > c_{\text{Vir}}^{\text{IR}} \qquad \Rightarrow \qquad \text{more dof in UV}$$

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- Explicit construction of a $c$-function using stress tensor and its 2-point correlators Zamolodchikov '86

# Domain wall $c$-function in $AdS_3/CFT_2$ (Freedman, Gubser, Pilch, Warner '99)

Two interesting alternative $c$-functions in holography context

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Two interesting alternative $c$-functions in holography context

domain walls in AdS$_3$ (set AdS radius to unity)

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + e^{2A(\rho)}\big(-\mathrm{d}t^2 + \mathrm{d}x^2\big) \qquad \lim_{\rho\to\infty} A(\rho) = \rho + \dots$$

holographic model for RG flow (UV: $\rho \to \infty$)

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- $\rho = \text{const.}$ slices: Poincaré invariant

$$\text{KVs}: \qquad \partial_t \qquad \partial_x \qquad x\partial_t + t\partial_x$$

conformal KVs: infinitely many (CFT$_2$ symmetries)

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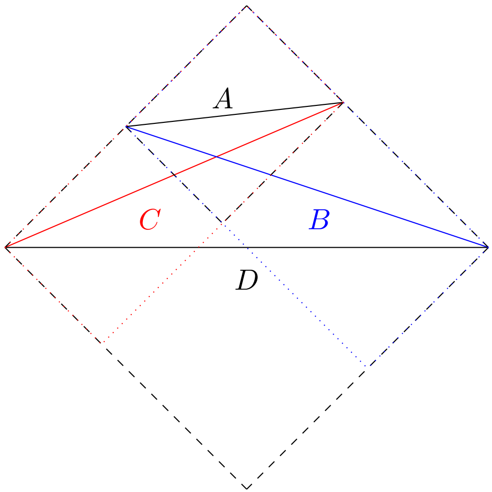
  conformal KVs: infinitely many ($\text{CFT}_2$ symmetries)

- holographic domain wall $c$-function

$$c_{\text{dw}}(\rho) = \frac{c^{\text{UV}}}{A'(\rho)}$$

  monotonicity implied by reality of bulk scalar field [see later!]

## Casini–Huerta (CH) $c$-function (CH '06)




$$AD = BC \;\;\Rightarrow\;\; C = \lambda A,\; D = \lambda B,\; \lambda > 1$$

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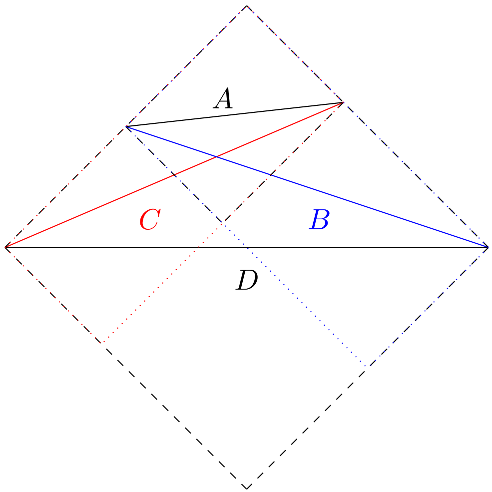


- SSA of EE

$$S(C) + S(B) \geq S(A) + S(D)$$

Lieb, Ruskai '73; Kiefer '59

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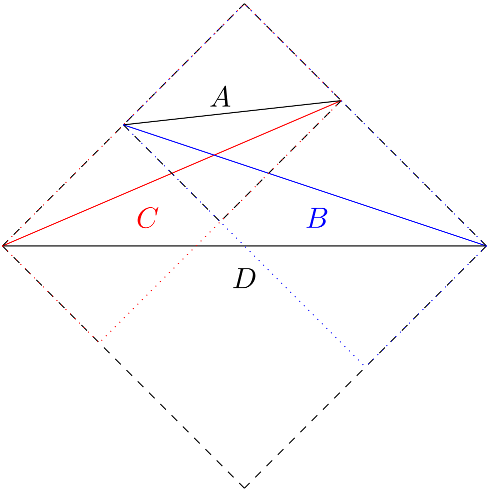


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$$S(C) + S(B) \geq S(A) + S(D)$$

- Use Minkowski diagram

$$S(B) - S(A) \geq S(\lambda B) - S(\lambda A)$$

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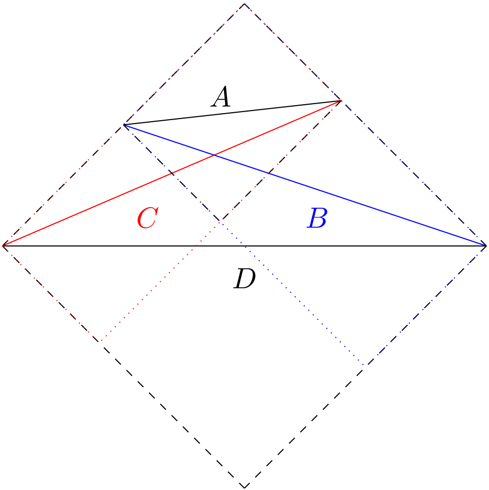


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- Differential instead difference

$$c_{\text{CH}}(L) = \#\, L\, S'(L)$$

non-increasing under dilatations!

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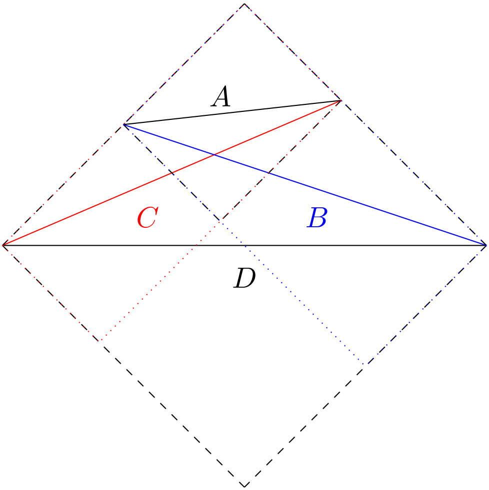


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- Fix normalization

$$c_{\text{CH}}(L) = 3\, L\, S'(L)$$

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Note: at fixed points we have

$$S(L\to 0) = \frac{c^{\rm UV}}{3}\,\ln L \quad\Rightarrow\quad c_{\rm CH}(L\to 0) = \lim_{L\to 0}\big(3L\,S'(L)\big) \to c^{\rm UV}$$

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Holzhey, Larsen, Wilczek '94; Cardy, Calabrese '06

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- Monotonicity of CH $c$-function

$$\frac{c'_{\mathrm{CH}}(L)}{3L} = S''(L) - \frac{S'(L)}{L} + \frac{6}{c_{\mathrm{CH}}(L)}\,(S'(L))^2 \leq 0$$

implies ground state QNEC [see next slide]

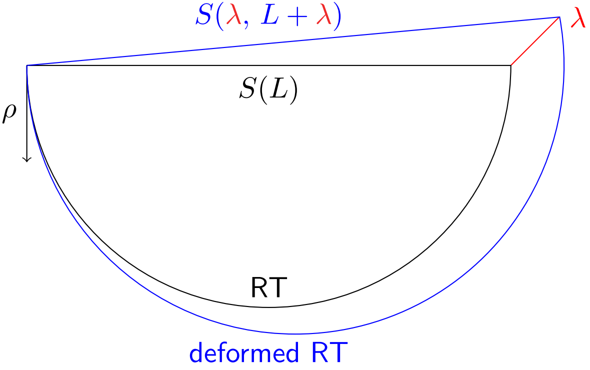
$$S''(L) - \frac{S'(L)}{L} + \frac{6}{c_{\mathrm{UV}}}\,(S'(L))^2 \leq 0$$

Ecker, DG, Soltanpanahi, Stanzer '20

# QNEC in 2d

Quantum Null Energy Condition (QNEC) in 2d:

$$2\pi \langle T_{\pm\pm} \rangle \geq \frac{\mathrm{d}^2 S}{\mathrm{d}\lambda^2}\bigg|_{\lambda=0} + \frac{6}{c^{\mathrm{UV}}} \left( \frac{\mathrm{d}S}{\mathrm{d}\lambda} \right)^2 \bigg|_{\lambda=0}$$




Bousso, Fisher, Leichenauer, Wall '15

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- both sides of QNEC transform with Schwarzian derivative Wall '11

  under bulk diffeos (or boundary conformal trafos):

$$\delta_\xi S = \underbrace{\xi S' - \frac{c}{12}\,\xi'}_{\text{anomalous scalar}}$$

  implies

$$\delta_\xi Q = \underbrace{\xi Q' + 2\xi' Q - \frac{c}{12}\,\xi'''}_{\text{infinitesimal Schwarzian}}$$

  with $Q := S'' + \frac{6}{c}\,(S')^2$

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since $S(\lambda,\, L+\lambda) = S(0, \sqrt{(L+\lambda)^2 - \lambda^2})$
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  Ecker, DG, Sheikh-Jabbari, Stanzer '20
- ground state QNEC necessary for monotonicity of CH $c$-function

# Outline

# Flat space domain walls

$$\mathrm{d}s^2 = -e^{A(r)}\,2\,\mathrm{d}u\,\mathrm{d}r + e^{2A(r)}\;\mathrm{d}x^2$$

as solutions of Einstein–Klein–Gordon bulk theory

$$I_{\text{bulk}}[g_{\mu\nu},\,\phi] = \frac{1}{16\pi G}\int \mathrm{d}^3x\sqrt{-g}\,\Big(R - \frac{1}{2}\,(\partial\phi)^2 - V(\phi)\Big)$$

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- $r = r_0 = const.$ slices have degenerate metric $0\;\mathrm{d}u^2 + 0\;\mathrm{d}u\,\mathrm{d}x + e^{2A(r_0)}\,\mathrm{d}x^2$ whose conformal KVs form $\mathsf{BMS}_3$

$$\text{conformal KVs:}\qquad \big(\xi_M(x) + u\,\xi_L'(x)\big)\,\partial_u + \xi_L(x)\,\partial_x$$

$\xi_L(x)$: generates diff$(S^1)$
$\xi_M(x)$: generates supertranslations

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- can translate all $\mathrm{AdS}_3$ domain walls into flat space domain walls!

## Domain wall $c$-function Fareghbal, Naseh, Rouhani PLB (2017)

Is holographic domain wall function

$$c_{\text{dw}}(r) = \frac{c_M^{\text{UV}}}{A'(r)}$$

$c$-function for Carrollian QFTs with $\text{BMS}_3$ invariant UV & IR fixed points?

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- all desired properties of flat space holographic $c$-function

## Domain wall example

Take flat space domain wall related to AdS domain wall with super-potential $W(\phi) = -2 - \frac{1}{4}\,\phi^2 - \frac{\alpha}{8}\,\phi^4$ (mass $m^2 = -\frac{3}{4}$ above BF)

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$$A(r) = r + A_0 - \frac{j^2}{16}\left(\frac{1}{e^r - \alpha j^2} + \frac{r - \ln\left(e^r - \alpha j^2\right)}{\alpha j^2}\right)$$

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- can read off $\mathrm{BMS}_3$ central charges at fixed points

$$c_M^{\mathrm{IR}} = \frac{c_M^{\mathrm{UV}}}{1 - \frac{1}{16\alpha}} < c_M^{\mathrm{UV}}$$

more info in plot of flat space domain wall $c$-function

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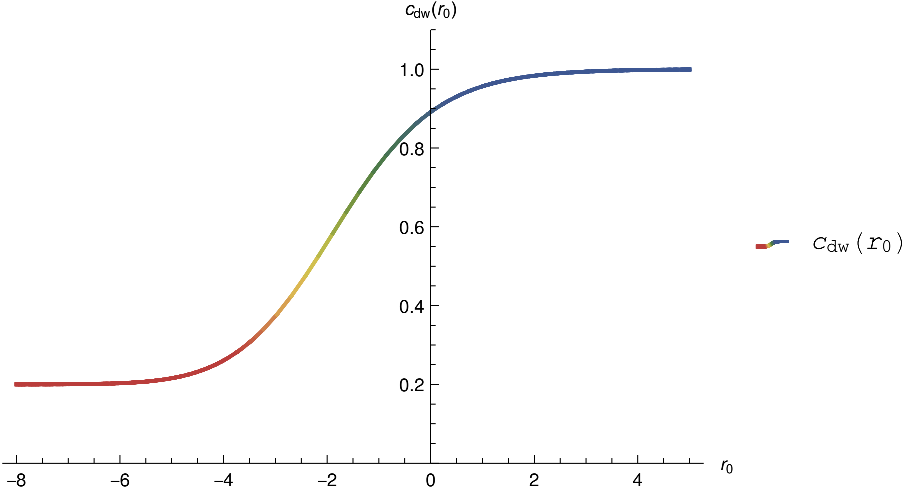

# Outline

Flat space holography à la Carroll

Holographic $c$-functions in $\mathrm{AdS}_3/\mathrm{CFT}_2$

Flat space domain walls and $\mathrm{BMS}_3$ $c$-functions

Outlook to Casini–Huerta-like $c$-function

## Quantum energy conditions in $\text{CCFT}_2$ DG, Parekh, Riegler '19

- split EE into $L$- and $M$-parts:

$$S_{\text{EE}} = S_L + S_M \qquad S_L = \frac{c_L}{6}\,\ln\frac{\Delta x}{\epsilon_x} \qquad S_M = \frac{c_M}{6}\,\Big(\frac{\Delta u}{\Delta x} - \frac{\epsilon_u}{\epsilon_x}\Big)$$

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QECs at our disposal — Casini–Huerta-like $c$-function conceivable!

## Tentative proposal for Casini–Huerta-like $c$-function

- for $c_L \neq 0 = c_M$ CH-like $c$-function postulated as

$$c_L(\Delta x) = 6\Delta x\, S_L'$$

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yields indeed other QEC

Monotonicity of conjectured $c$-functions yields ground state QEC

## Summary

Take-away lessons from this talk:

- Carrollian approach to flat space holography makes sharp predictions (symmetries, spectrum, microstates, entanglement, correlators, ...)

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- flat space domain walls from $AdS_3$ domain walls (exact solutions of 3d Einstein-dilaton gravity, in flat space without potential, in $AdS_3$ with potential)
- proved domain wall $c$-function is a $CCFT_2$ $c$-function with all the right properties (monotonicity from bulk unitarity, correct fixed-point values of $BMS_3$ central charges)

## Summary

Take-away lessons from this talk:

- Carrollian approach to flat space holography makes sharp predictions (symmetries, spectrum, microstates, entanglement, correlators, ...)
- flat space domain walls from $\text{AdS}_3$ domain walls (exact solutions of 3d Einstein-dilaton gravity, in flat space without potential, in $\text{AdS}_3$ with potential)
- proved domain wall $c$-function is a $\text{CCFT}_2$ $c$-function with all the right properties (monotonicity from bulk unitarity, correct fixed-point values of $\text{BMS}_3$ central charges)
- conjectured Casini–Huerta-like $c$-function (only cross-checks so far: compatible with ground state QECs and correct fixed point values)

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All results in 3 bulk dimensions. Do not know how/if all results generalize to other dimensions.

## Conclusions by ChatGPT-3.5 (abridged)

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Ladies and gentlemen, esteemed colleagues, and fellow
enthusiasts of the Carrollian cosmos,
As we prepare to wrap up our journey through the
whimsical wonderland of ``Carrollian c-functions and
flat space holographic RG flows,'' I can't help but feel
a bit like Alice herself, exploring the curious corners
of spacetime where BMS symmetries, Carrollian CFTs, and
c-functions come together in a harmonious dance.
Our adventure today has taken us down the rabbit hole
of flat space holography. But as we prepare to return
from this captivating journey, let us remember that
in science, as in Wonderland, curiosity is our guiding
star. As Carrollian characters once said, ``Begin at
the beginning, and go on till you come to the end: then
stop.''
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# My conclusion

Carrollian holography is a lot of fun — feel free to join!

AI prompt: Carroll and Zamolodchikov meet 't Hooft and Susskind at the rabbit hole to wonderland