Seminar Talk (FSU Jena)

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Dilaton Gravity in 2D: An Overview

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Review: *DG, W. Kummer, D. Vassilevich,*
`hep-th/0204253`

Outline

1. Geometry in 2D (+Motivation)

2. Dilaton gravity in 2D (2^{nd} order)

3. Gravity as gauge theory ($2^{\text{nd}} \to 1^{\text{st}}$ order)

4. All classical solutions (locally+ε)

5. Global structure (Penrose diagrams)

6. Hawking temperature, BH entropy

7. Path integral quantization (with matter)

8. Open problems

1. Geometry in 2D

Hilbert action of Einstein gravity (EH) in D:

$$L = \int\limits_{\mathcal{M}_D} R + \int\limits_{\partial\mathcal{M}_D} K + \int\limits_{\partial\partial\mathcal{M}_D} \alpha_{\text{deficit}} \stackrel{D=2}{=} \text{Euler}$$

Number of propagating physical degrees of freedom (ppdof): $D(D-3)/2$

$D=4$: 2 gravitons
$D=3$: 0 gravitons (cf. e.g. S. Carlip)
$D=2$: "-1 graviton"

In $D=2$ good reasons to add scalar field:

1. EH action yields no EOM
2. Formally brings number of ppdof to 0
3. Scalar-tensor theories "natural" (strings)
4. Scalar field from dimensional reduction

Motivations

- dimensionally reduced models (spherical symmetry)
- strings (2D target space)
- integrable models (PSM)
- models for BH physics (information loss)

study important conceptual problems without encountering insurmountable technical ones
$\rightarrow$ 2D gravity: useful toy model(s) for classical and quantum gravity

most prominent member not just a toy model:
Schwarzschild Black Hole ("Hydrogen atom of General Relativity")

2. Dilaton gravity in 2D, 2^{nd} order

2D scalar tensor theories:

[*J. Russo, A. Tseytlin*, `hep-th/9201021`]

$$S^{(SOG)} = \int_{\mathcal{M}_2} \mathrm{d}^2 x \sqrt{-g} \left(XR - U(X)(\nabla X)^2 + 2V(X) \right)$$

X: "dilaton field" (scalar)

$g_{\mu\nu}$: 2D metric (tensor)

$U(X), V(X)$: potentials defining the model

Note 1: sometimes first term: $Z(X)R$; if Z invertible: form above! if not: singularities at $Z' = 0$! in each regular patch: again form above!

Note 2: conformal transformation to a different model with $U = 0$ possible – but conformal factor singular in general, thus change of global structure!

Note 3: in stringy literature: usually $X = e^{-2\phi}$, where ϕ is the string dilaton

Note 4: exclusively Minkowskian signature for sake of definiteness and because of interest in BHs

Selected list of models

Model	$U(X)$	$V(X)$
Schwarzschild	$-\frac{1}{2X}$	$-\lambda^2$
Jackiw-Teitelboim	0	ΛX
Witten BH/CGHS	$-\frac{1}{X}$	$-2\lambda^2 X$
CT Witten BH	0	$-2\lambda^2$
SRG (generic $D > 3$)	$-\frac{D-3}{(D-2)X}$	$-\lambda^2 X^{(D-4)/(D-2)}$
All above: ab-family	$-\frac{a}{X}$	$-\frac{B}{2}X^{a+b}$
Reissner-Nordström	$-\frac{1}{2X}$	$-\lambda^2 + \frac{Q^2}{X}$
Schwarzschild-(A)dS	$-\frac{1}{2X}$	$-\lambda^2 - \ell X$
Katanaev-Volovich	α	$\beta X^2 - \Lambda$
Achucarro-Ortiz	0	$\Lambda X - \frac{J}{4X^3} + \frac{Q^2}{X}$
Reduced CS	0	$\frac{1}{2}X(c - X^2)$
2D type 0A/0B	$-\frac{1}{X}$	$-2\lambda^2 X + \frac{\lambda^2 q^2}{8\pi}$
exact string BH	?	?

Note: non-standard models possible in 1^{st} order:

$$-U(X)\frac{(\nabla X)^2}{2} + V(X) \to \mathcal{V}((\nabla X)^2, X)$$

Example: dilaton-shift invariant models

$\mathcal{V} = X\,U((\nabla \ln X)^2)$

No principle complications, but rarely in literature!

3. Gravity as gauge theory: $2^{\text{nd}} \to 1^{\text{st}}$

JT model: (A)dS$_2$ ($SO(1,2)$): *C. Teitelboim, PL* **B126** *(1983) 41*, *R. Jackiw, NP* **B252** *(1985) 343*

$$[P_a, P_b] = \Lambda \epsilon_{ab} J \,, \quad [P_a, J] = \epsilon_{ab} P^b$$

1^{st} order form: *K. Isler, C. Trugenberger, PRL* **63** *(1989) 834*, *A. Chamseddine, D. Wyler, PL* **B228** *(1989) 75*

$$L = X_A F^A = X_a (De)^a + X(\mathrm{d}\omega + \frac{1}{2}\Lambda \epsilon_{ab} e^a e^b)$$

$SO(1,2)$ connection: $A = e^a P_a + \omega J$,
$F = \mathrm{d}A + \frac{1}{2}[A, A]$, e^a, ω: "Cartan variables",
X_A: Lagr. mult. (trafo under coadjoint rep.)

CGHS: central extended Poincaré ($ISO(1,1)$): *D. Cangemi, R. Jackiw,* `hep-th/9203056`

$$[P_a, P_b] = \lambda \epsilon_{ab} I \,, \quad [P_a, J] = \epsilon_{ab} P^b \,, \quad [I, J] = 0 = [I, P_a]$$

again first order with $L = X_A F^A$ possible
without central extension: *Verlinde, MG VI*
cf. also *A. Achúcarro,* `hep-th/9207108`

important pre-cursor:
N. Ikeda, `hep-th/9312059`: (non-linear) gauge formulation for $U = 0$ but generic $V(X)$

General first order action

Classical equivalence to:

P. Schaller, T. Strobl, `hep-th/9405110`

$$S^{(FOG)} = \int_{\mathcal{M}_2} \left[X_a T^a + XR + \epsilon \mathcal{V}(X^a X_a, X)\right] \tag{1}$$

$T^a = (De)^a$: torsion 2-form
$R^a{}_b = \epsilon^a{}_b R = \epsilon^a{}_b \, d\omega$: curvature 2-form
$\epsilon = -\frac{1}{2}\epsilon_{ab} e^a \wedge e^b$: volume 2-form
X: "dilaton" (Lagrange mult. f. curvature)
X^a: auxiliary fields (— " — torsion)
$\mathcal{V}$: potential defining the model (as before)

Relation to second order:
dilaton: $X = X$
kinetic term: $(\nabla X)^2 = -X^a X_a$
metric: $g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu$
connection: Levi-Civitá $= \omega-$torsion part

Technical Note: Use light-cone components ($\eta_{+-} = 1 = \eta_{-+}$, $\eta_{++} = 0 = \eta_{--}$); define $\epsilon^\pm{}_\pm = \pm 1$

$T^\pm = (d \pm \omega) \wedge e^\pm, \quad \epsilon = e^+ \wedge e^-, \quad X^a X_a = 2X^+ X^-$

Typically: $\mathcal{V}(X^+X^-, X) = X^+X^-U(X) + V(X)$

Relation to PSM

[*P. Schaller, T. Strobl,* `hep-th/9405110`]

$$\mathcal{S}_{gPSM} = \int_{\mathcal{M}_2} \mathrm{d}X^I \wedge A_I + \frac{1}{2} P^{IJ} A_J \wedge A_I \, .$$

- gauge field 1-forms: $A_I = (\omega, e_a)$, connection, Zweibeine

- target space coordinates: $X^I = (X, X^a)$, dilaton, auxiliary fields

- target space: Poisson manifold

- Poisson tensor: odd dimension $\longrightarrow$ kernel!

- Jacobi: $P^{IL} \partial_L P^{JK} + \mathit{perm}\,(IJK) = 0$

$$P^{IJ} = \begin{pmatrix} 0 & X^+ & -X^- \\ -X^+ & 0 & \mathcal{V} \\ X^- & -\mathcal{V} & 0 \end{pmatrix}$$

Equations of motion (first order):

$$\mathrm{d}X^I + P^{IJ} A_J = 0$$
$$\mathrm{d}A_I + \frac{1}{2}(\partial_I P^{JK}) A_K \wedge A_J = 0$$

Gauge symmetries:

$$\delta X^I = P^{IJ} \varepsilon_J$$
$$\delta A_I = -\mathrm{d}\varepsilon_I - \left(\partial_I P^{JK}\right) \varepsilon_K \, A_J$$

Note 1: if P^{IJ} linear: Lie-Algebra! Otherwise: non-linear gauge symmetries

Note 2: on-shell equivalent to diffeomorphisms+local Lorentz trafos for specific Poisson tensor on previous page

Note 3: comprehensive summary of this approach: *T. Strobl*, `hep-th/0011240`

Note 4: generalization to SUGRA: straightforward

Note 5: inclusion of matter: non-integrable!

4. All classical solutions (locally)

Ansatz: $X^+ \neq 0$ in a patch $\to e^+ = X^+ Z$

Summary of EOM ($\mathcal{V} = X^+X^-U(X) + V(X)$):

$$\begin{aligned} \delta\omega: &\quad \mathrm{d}X + X^- e^+ - X^+ e^- = 0\,, \\ \delta e^\mp: &\quad (\mathrm{d} \pm \omega) X^\pm \mp \mathcal{V} e^\pm = 0\,, \\ \delta X^\mp: &\quad (\mathrm{d} \pm \omega) e^\pm + \epsilon\, X^\pm U(X) = 0\,. \end{aligned}$$

1. use $\delta\omega$ to get $e^- = \mathrm{d}X/X^+ + X^- Z$
2. read off $\epsilon = e^+ \wedge e^- = Z \wedge \mathrm{d}X$
3. use δe^- to get $\omega = -\,\mathrm{d}X^+/X^+ + Z\mathcal{V}$
4. use δX^- to get $\mathrm{d}Z = \mathrm{d}X \wedge ZU(x)$
5. define "integrating factor":

$$I(X) := \exp \int^X U(X')\,\mathrm{d}X'$$

6. obtain $Z =: \hat{Z} I(X)$ with $\mathrm{d}\hat{Z} = 0 \to \hat{Z} = \mathrm{d}u$
7. use $g_{\mu\nu} = e^+_\mu e^-_\nu + e^-_\mu e^+_\nu$

general solution for the line element:

$$\mathrm{d}s^2 = I(X)\left(2\,\mathrm{d}u\,\mathrm{d}X + 2X^+X^- I(X)\,\mathrm{d}u^2\right)$$

$X^+X^- = 0$: trapped surface!

Conservation law:

T. Banks, M. O'Loughlin, NP ***B362*** *(1991) 649;*
V. Frolov, PR ***D46*** *(1992) 5383; R. Mann,* `hep-th/9206044`

PSM: related to Casimir function!

Derivation in absence of matter: EOMs $X^+\delta e^+ + X^-\delta e^-$ using also the EOM $\delta\omega$ establishes

$$\mathrm{d}(X^+X^-) + \mathcal{V}\,\mathrm{d}X = 0$$

for "standard" $\mathcal{V} = X^+X^-U(X) + V(X)$:

$$\mathcal{C} = I(X)X^+X^- + w(X)\,, \quad \mathrm{d}\mathcal{C} = 0$$

with

$$w(X) := \int^X I(X')V(X')\,\mathrm{d}X'$$

"Generalized Birkhoff theorem" (always 1 Killing)

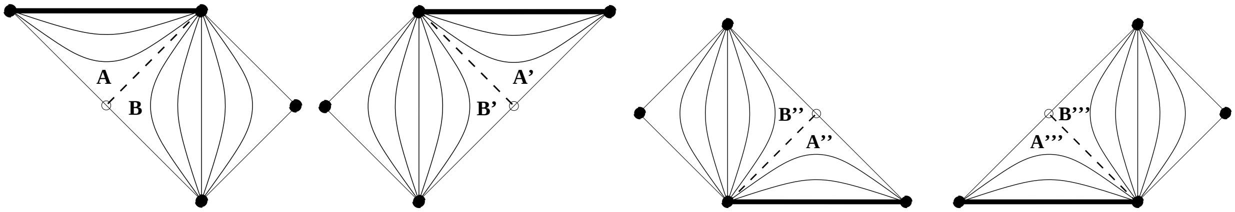
Inserting into line element, $\mathrm{d}r = I(X)\,\mathrm{d}X$:

$$\mathrm{d}s^2 = 2\,\mathrm{d}u\,\mathrm{d}r + 2I(X(r))\,(\mathcal{C} - w(X(r)))\,\mathrm{d}u^2 \tag{2}$$

Eddington-Finkelstein patch!

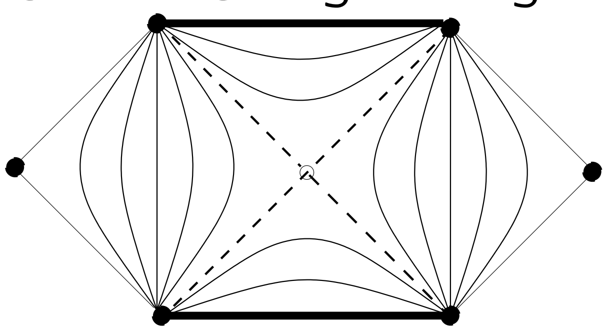
5. Global structure (Penrose diagrams)

Carter-Penrose (CP) diagram of EF patches:



for Schwarzschild-like solutions

Global CP: glue together EF patches:



Only point not covered by EF patches:
bifurcation point: $X^+ = 0 = X^-$

T. Klösch, T. Strobl, `gr-qc/9508020`, `gr-qc/9511081`

open regions with $X^+ = 0 = X^-$: $X = \text{const.}$
"Constant Dilaton Vacua" (very simple)

$$V(X_{CDV}) = 0\,, \qquad R \propto V'(X_{CDV}) = \text{const.}$$

only Minkowski, Rindler, (A)dS

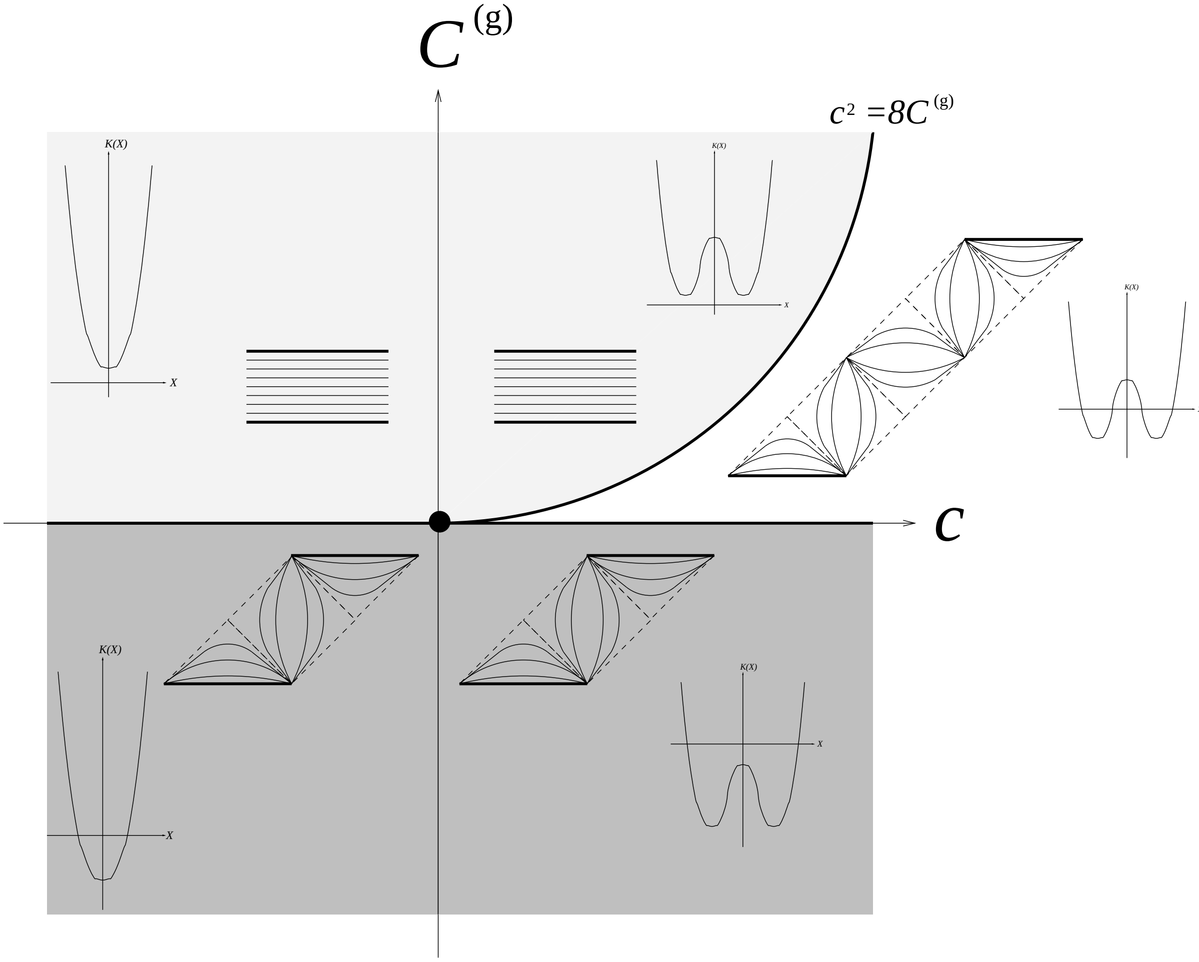
Non-trivial example: KK reduced CS:

G. Guralnik, A. Iorio, R. Jackiw and S.Y. Pi, `hep-th/0305117`,

DG and W. Kummer, `hep-th/0306036`,

L. Bergamin, DG, A. Iorio, C. Nuñez, `hep-th/0409273`

$$U(X) = 0\,, \quad V(X) = \frac{1}{2}X(c - X^2)$$



Kink interpolates between two AdS-CDV

6. Hawking temperature

Naively from surface gravity:

$$T_H = \frac{1}{2\pi}\Big|w'(X)\Big|_{X=X_h}. \qquad (3)$$

Note: independent from $I(X)$

With minimally coupled matter: same result;
e.g. from trace anomaly

S. Christensen, S. Fulling, PR **D15** *(1977) 2088*

$$< T^\mu{}_\mu > \propto R\,, \quad \nabla_\mu T^{\mu\nu} = 0$$

Matter coupled ("non-minimally") to dilaton:
Non-conservation equation!

W. Kummer, D. Vassilevich, gr-qc/9907041

$$\nabla^\mu T_{\mu\nu} = -(\partial_\nu \Phi)\frac{1}{\sqrt{-g}}\frac{\delta W}{\delta \Phi}\,, \quad X = e^{-2\Phi}$$

agrees w. *S. Mukhanov, A. Wipf, A. Zelnikov,* hep-th/9403018

Mass-to-temperature law: need mass!
Sometimes mutually contradicting results for "ADM mass" (e.g. 2D string theory)
clarified in appendix of *DG, D. Mayerhofer,* gr-qc/0404013

BH Entropy

BH: "Bekenstein-Hawking" or "Black Hole"

Simple thermodynamic considerations:

$$\mathrm{d}S = \frac{\mathrm{d}M}{T}$$

J. Gegenberg, G. Kunstatter, D. Louis-Martinez, gr-qc/9408015

$$S = 2\pi \, X|_{\mathcal{C}=w(X)} \equiv \frac{A}{4} \tag{4}$$

confirmed by more elaborate derivations (CFT methods, near horizon conformal symmetry, Cardy-formula, Wald's method)

S. Carlip gr-qc/9906126, gr-qc/0203001, hep-th/0408123, *S. Solodukhin,* hep-th/9812056, *M. Cadoni, S. Mignemi,* hep-th/9810251

open question: counting of microstates is fine, but what are actually the microstates of 2D dilaton gravity?

Another important open problem: end point of BH evaporation!

J. Russo, L. Susskind, L. Thorlacius, hep-th/9206070, *DG,* gr-qc/0307005

7. Path integral quantization (+ matter)

Basic ideas:

W. Kummer, H. Liebl, D. Vassilevich, `hep-th/9809168`

- Matter provides propagating d.o.f.

- Integrate out geometry exactly!
  Possible in first order formulation with EF gauge fixing fermion: gauge fields appear only linearly!

- Treat matter perturbatively

- Reconstruct geometry (Virtual BHs)

- Calculate S-matrix or corrections to classical quantities (like specific heat)

Note: Generalization to SUGRA:

L. Bergamin, DG, W. Kummer, `hep-th/0404004`

Constraint algebra, gauge fixing

Poisson brackets: $\left\{q_i, p^j\right\} = \delta_i^j$
$q_i = (\omega_1, e_1^-, e_1^+)$, $p^i = (X, X^+, X^-)$

Algebra of secondary (first class) constraints:

$$\left\{G^i(x), G^j(x')\right\} = G^k C_k{}^{ij} \delta(x - x')$$

with the same structure functions $C_k{}^{ij}$
crucial: q_i linear in G^i, absent in $C_k{}^{ij}$ $\Rightarrow$
no ordering problems, BRST charge nilpotent without higher order ghost terms

Relation to Virasoro algebra:
$G = G^1$; $H_0 = q_1 G^1 + \varepsilon^a{}_b q_a G^b$; $H_1 = q_i G^i$
$\{G, G'\} = 0$, $\{G, H_0'\} = -G\delta'$, $\{G, H_1'\} = -G\delta'$,
$\{H_0, H_0'\} = (H_1 + H_1')\delta'$,
$\{H_0, H_1'\} = (H_0 + H_0')\delta'$,
$\{H_1, H_1'\} = (H_1 + H_1')\delta'$

Gauge fixing fermion chosen s.t.:

$$\omega_0 = 0\,, \quad e_0^- = 1\,, \quad e_0^+ = 0$$

Matter

Without matter: "pointless" (0 ppdof) $\Rightarrow$
interesting scattering: need matter
focus on massless Klein-Gordon:

$$L^{(m)} = \int F(X) d\phi \wedge (*d\phi)$$

if $F(X) =$ const. minimal else nonminimal
e.g. SRG: $\sqrt{-g^{(4)}} = |X|\sqrt{-g^{(2)}} \Rightarrow F \propto X$

$$\left\{G^+, G^-\right\}_{\text{new}} \propto \ln' F(X) L^{(m)} G^1$$

but: BRST charge essentially unchanged!

integrate out geometry exactly;

$$W = \int \mathcal{D}f \delta\left(f + i\delta/\delta j_{e_1^+}\right) \tilde{W}[f]$$

$$\tilde{W} = \int \mathcal{D}\tilde{\phi} \exp i \int d^2x \left[\text{kinetic} + j_{e_1^+} \hat{E}_1^+ + \text{vertices}\right]$$

DG, Kummer, Vassilevich, hep-th/0204253
relevant: boundary values of X^I (ADM mass);
measure for 1-loop (Polyakov): $\tilde{\phi} = \phi f^{1/2}$;
vertices: nonpolynomial, nonlocal;
backreactions included to each matter order;
geometry reconstructed from matter!

Details of path integral

For simplicity minimal coupling ($F = $ const.):

$$\tilde{W} = \int \mathcal{D}\tilde{\phi}\, \exp i \int (\mathcal{L}^k + \mathcal{L}^s + \mathcal{L}^v) d^2x$$

$$\mathcal{L}^k = \partial_0\phi\partial_1\phi - E_1^-(\partial_0\phi)^2$$

$$\mathcal{L}^s = \sigma\phi + j_{e_1^+}\hat{E}_1^+ + \text{irrelevant}$$

$$\mathcal{L}^v = -w'(\hat{X})\,, \quad w = \int^X I(y)V(y)dy$$

$$\tilde{\phi} = \phi f^{1/2}\,, \quad \hat{E}_1^+ = I(\hat{X})$$

$$\hat{X} = \underbrace{a + bx^0}_{X} + \partial_0^{-2}(\partial_0\phi)^2 + \text{irrelevant}$$

$$a = 0\,, \quad b = 1\,, \quad E_1^- = K_\infty(m_\infty)$$

$$\hat{E}_1^+ = \underbrace{I(X)}_{E_1^+} + I'(X)\partial_0^{-2}(\partial_0\phi)^2 + \dots$$

$$\int \mathcal{D}\tilde{\phi}\, \exp i \int \mathcal{L}^k = \exp \underbrace{\left(i/96\pi \int_x \int_y f R_x \Box^{-1}_{xy} R_y \right)}_{iL^{\text{Pol}}}$$

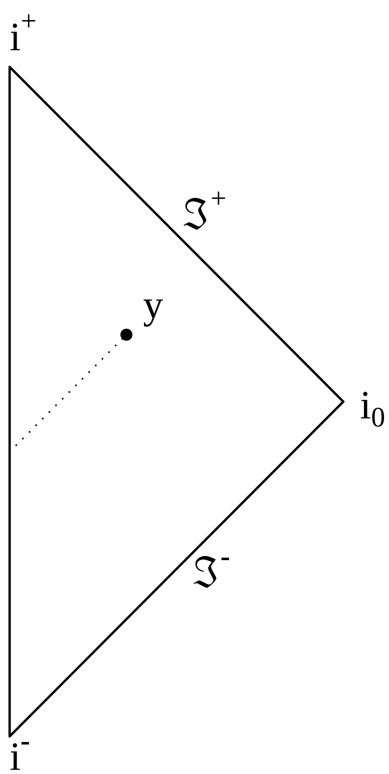
Effective line element

"temporal gauge": $(A_I)_0 = a_0 =$ const. $\Rightarrow$ line element in *outgoing Sachs-Bondi* form:

$$(ds)^2 = 2drdu + K(r,u)(du)^2$$

Killing-norm (SRG, asymp. Minkowski, LO):

$$K(r,u) = \left(1 - \left(\frac{2m}{r} + ar\right)\theta(r_0 - r)\delta(u - u_0)\right),$$



zeros of Killing-norm approximately at $r = 2m$ (*Schwarzschild horizon*) and $r = 1/a$ (*Rindler horizon*) non-vacuum-geometry concentrated on light-like cut; curvature scalar as well; interpretation: virtual black hole (VBH) induced by effective matter interaction; S-matrix: sum over all VBH *DG, W. Kummer, D. Vassilevich,* `gr-qc/0001038`, review: *DG,* `hep-th/0409231`

Scattering amplitude

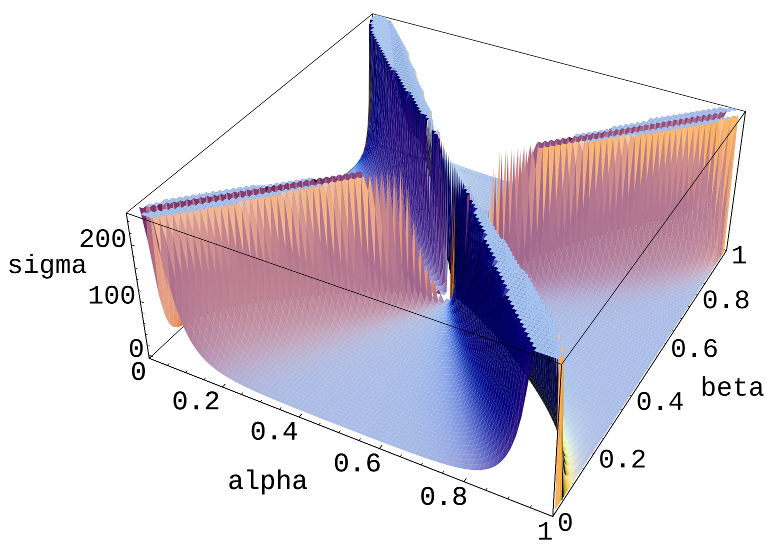
P. Fischer, DG, W. Kummer, D. Vassilevich, `gr-qc/0105034`, *DG,* `gr-qc/0105078`, `gr-qc/0111097`, `hep-th/0409231`

S-matrix for s-wave scattering: ingoing modes q, q'; outgoing ones k, k'; $E := q + q'$

$$T \propto \tilde{T}\delta(k + k' - q - q')E^3/|kk'qq'|^{3/2}$$

interesting part: *scale independent* $\tilde{T}$, momentum transfer $\Pi := (k + k')(k - q)(k' - q)$

$$\tilde{T}(q, q'; k, k') := \frac{1}{E^3}\Bigg[\Pi \ln \frac{\Pi^2}{E^6} + \frac{1}{\Pi} \sum_{p \in \{k,k',q,q'\}} p^2$$
$$\ln \frac{p^2}{E^2}\left(3kk'qq' - \frac{1}{2}\sum_{r \neq p}\sum_{s \neq r,p}\left(r^2 s^2\right)\right)\Bigg],$$



$k = \alpha E$

$k' = (1 - \alpha)E$

$q = \beta E$

$q' = (1 - \beta)E$

8. Open problems

- Simple derivation of S-matrix?

- Understanding of microstates within 2D?

- End point of BH evaporation?

- Generic NC dilaton gravity? (NC JT model: *S. Cacciatori et al.,* hep-th/0203038; NC Witten BH: *D. Vassilevich* hep-th/0502120)

- Target space action for exact string BH? (solved: *DG, hep-th/0501208*)

Conclusion: Still a lot to do in “Lineland”!