# Soft Heisenberg Hair


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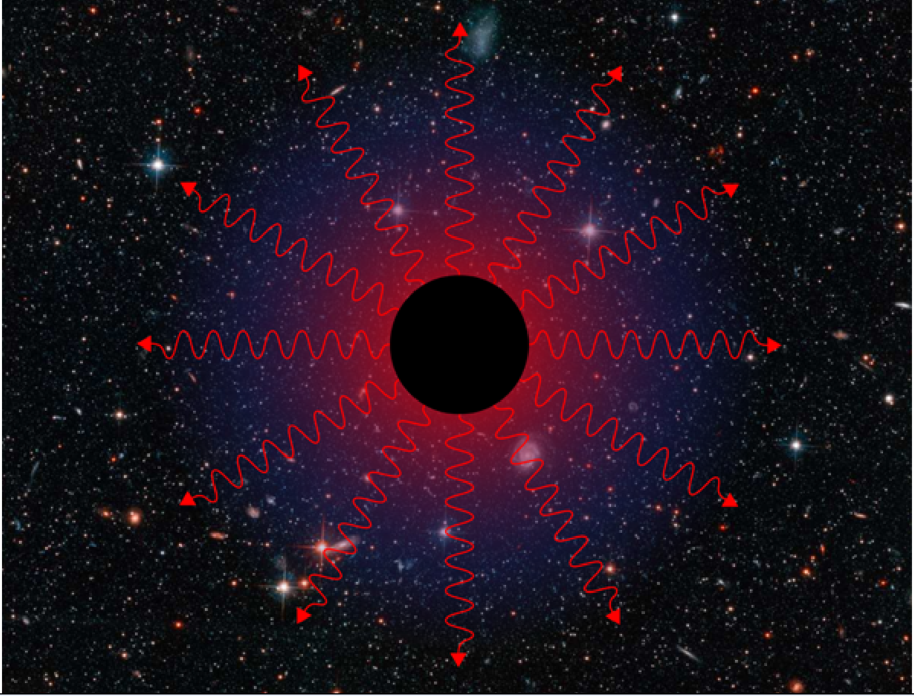

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1. Heisenberg algebra

$$[X_n,\, P_m] = i\,\delta_{n,\,m}$$

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2. Black hole microstates identified as specific "soft hair" descendants

based on work with

- Hamid Afshar [IPM Teheran]
- Stephane Detournay [ULB]
- Wout Merbis [TU Wien]
- Blagoje Oblak [ULB / ETH]
- Alfredo Perez [CECS Valdivia]
- Stefan Prohazka [TU Wien]
- Shahin Sheikh-Jabbari [IPM Teheran]
- David Tempo [CECS Valdivia]
- Ricardo Troncoso [CECS Valdivia]

# Outline

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Motivation

Near horizon boundary conditions

Explicit construction of BTZ microstates

Discussion

## Black hole microstates

Bekenstein–Hawking

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- Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...)

warped CFT: Detournay, Hartman, Hofman '12
Galilean CFT: Bagchi, Detournay, Fareghbal, Simon '13; Barnich '13

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- Main idea: consider near horizon symmetries for non-extremal horizons

  Related ideas pursued e.g. by
  - Donnay, Giribet, Gonzalez, Pino '15
  - Hawking, Perry, Strominger '16

  Postpone comparison with related approaches after discussing our approach

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- Main idea: consider near horizon symmetries for non-extremal horizons
- Near horizon line-element with Rindler acceleration $a$:

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

Meaning of coordinates:

- $\rho$: radial direction ($\rho = 0$ is horizon)
- $\varphi \sim \varphi + 2\pi$: angular direction
- $v$: (advanced) time

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  suggestion in `1511.08687`

  We make this choice in this talk!

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- Work in 3d Einstein gravity in Chern–Simons formulation

$$I_{\text{CS}} = \pm \sum_{\pm} \frac{k}{4\pi} \int \langle A^\pm \wedge \mathrm{d}A^\pm + \tfrac{2}{3} A^\pm \wedge A^\pm \wedge A^\pm \rangle$$

with $sl(2)$ connections $A^\pm$ and $k = \ell/(4G_N)$ with AdS radius $\ell = 1$

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Example:

$$\Phi(x \to \infty) = 0$$

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Example: Brown-Henneaux type of bc's ($\mathrm{aAdS}_3$):

$$\mathrm{d}s^2_{\mathrm{aAdS}} = \mathrm{d}\rho^2 + \big(e^{2\rho}\eta_{\mu\nu} + \gamma_{\mu\nu} + \mathcal{O}(e^{-2\rho})\big)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

with $\delta\gamma =$ arbitrary

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- Local diffeos and gauge trafos fall into three classes:
  1. Trafos that violate bc's (forbidden)
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- Canonical boundary charges (á la Regge–Teitelboim) generate asympotic symmetries of "edge states"
- Consistency means they are finite, integrable, non-trivial and conserved (in time)

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- Troessaert (2013): 2 Virasoros plus 2 $u(1)$ current algebras
- Avery–Poojary–Suryanarayana (2013): Virasoro plus $sl(2)$ current algebra
- Donnay–Giribet–Gonzalez–Pino (2015): centerless warped conformal
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Our near horizon bc's simpler than any of the above!

## Explicit specification of our bc's in diagonal gauge

Standard trick: partially fix gauge

$$A^{\pm} = b_{\pm}^{-1}(\rho)\,\big(\mathrm{d}+\mathfrak{a}_{\pm}(x^0,\,x^1)\big)b_{\pm}(\rho)$$

with some group element $b \in SL(2)$ depending on radius $\rho$ with $\delta b = 0$

Drop $\pm$ decorations in most of talk

Manifold topologically a cylinder or torus, with radial coordinate $\rho$ and boundary coordinates $(x^0, x^1) \sim (v, \varphi)$

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- Standard AdS$_3$ approach: highest weight gauge

$$\mathfrak{a} \sim L_+ + \mathcal{L}(x^0,\, x^1)L_- \qquad b(\rho) = \exp\left(\rho L_0\right)$$

$sl(2)$: $[L_n,\, L_m] = (n-m)L_{n+m}, \quad n, m = -1, 0, 1$

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  $sl(2)$: $[L_n,\, L_m] = (n-m) L_{n+m}, \quad n, m = -1, 0, 1$
- For near horizon purposes diagonal gauge useful:

  $$\mathfrak{a} \sim \mathcal{J}(x^0,\, x^1)\, L_0$$

- Precise boundary conditions ($\zeta$: chemical potential):

  $$\mathfrak{a} = (\mathcal{J}\ \mathrm{d}\varphi + \zeta\ \mathrm{d}v)\ L_0 \qquad \delta\mathfrak{a} = \delta\mathcal{J}\ \mathrm{d}\varphi\, L_0$$

  and $b = \exp{(\frac{1}{\zeta}\, L_+)} \cdot \exp{(\frac{\rho}{2}\, L_-)}$. (assume constant $\zeta$ for simplicity)

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state-dependent functions $\mathcal{J}^\pm = \gamma \pm \omega$, chemical potentials $\zeta^\pm = -a \pm \Omega$

For simplicity set $\Omega = 0$ and $a = \text{const.}$ in metric above

EOM imply $\partial_v \mathcal{J}^\pm = \pm\partial_\varphi \zeta^\pm$; in this case $\partial_v \mathcal{J}^\pm = 0$

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Comments:

- Recover desired near horizon metric

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- $\gamma = \gamma(\varphi)$: "black flower"

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Background independent result for Chern–Simons yields

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Meaningful near horizon boundary conditions and non-trivial theory!

## Near horizon symmetry algebra

- Near horizon symmetry algebra = all near horizon boundary conditions preserving trafos, modulo trivial gauge trafos

  Most general trafo

  $$\delta_\epsilon \mathfrak{a} = \mathrm{d}\epsilon + [\mathfrak{a},\, \epsilon] = \mathcal{O}(\delta\mathfrak{a})$$

  that preserves our boundary conditions for constant $\zeta$ given by

  $$\epsilon = \epsilon^+ L_+ + \eta L_0 + \epsilon^- L_-$$

  with

  $$\partial_v \eta = 0$$

  implying

  $$\delta_\epsilon \mathcal{J} = \partial_\varphi \eta$$

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- Expand charges in Fourier modes

$$J_n^{\pm} = \frac{k}{4\pi} \oint \mathrm{d}\varphi \, e^{in\varphi} \mathcal{J}^{\pm}(\varphi)$$

What should we expect?

  - Virasoro? (spacetime is locally $\mathrm{AdS}_3$)
  - $\mathrm{BMS}_3$? (Rindler boundary similar to scri)
  - warped conformal algebra? (this is what we found for Rindleresque holography and what Donnay, Giribet, Gonzalez, Pino found in their near horizon analysis)

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$$[J_n^\pm, \, J_m^\pm] = \pm \tfrac{1}{2} k n \delta_{n+m,\,0} \qquad [J_n^+, \, J_m^-] = 0$$

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- Map

$$P_0 = J_0^+ + J_0^- \qquad P_n = \tfrac{i}{kn}\,(J_{-n}^+ + J_{-n}^-) \text{ if } n \neq 0 \qquad X_n = J_n^+ - J_n^-$$

  yields Heisenberg algebra (with Casimirs $X_0$, $P_0$)

$$[X_n, \, X_m] = [P_n, \, P_m] = [X_0, \, P_n] = [P_0, \, X_n] = 0$$
$$[X_n, \, P_m] = i\delta_{n,m} \quad \text{if } n \neq 0$$

## Brief list of generalizations

Heisenberg algebras as near horizon symmetries arise not only in $\mathrm{AdS}_3$ Einstein gravity, but also in ...

- ... flat space Einstein gravity in three dimensions
  Afshar, DG, Merbis, Perez, Tempo, Troncoso '16
- ... higher spin gravity in three dimensions
  DG, Perez, Prohazka, Tempo, Troncoso '16
- ... higher derivative gravity in three dimensions
  Setare, Adami '16
- ... general relativity (in four dimensions)
  Afshar, DG, Sheikh-Jabbari '16
- ... flat space higher spin gravity in three dimensions
  Ammon, Grumiller, Prohazka, Riegler, Wutte '17

Conclusions about near horizon symmetry algebra fairly general!

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- Near horizon Hilbert space: define vacuum by highest weight conditions

$$\mathcal{J}_n^\pm |0\rangle = 0 \qquad \text{for all} \quad n \geq 0.$$

- Construct near horizon Virasoro through standard Sugawara construction

$$\mathcal{L}_n^\pm \equiv \sum_{p\in\mathbb{Z}} :\mathcal{J}_{n-p}^\pm\, \mathcal{J}_p^\pm:$$

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- Call this "near horizon symmetry algebra" (note: independent from $\ell$)

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- Will exploit this property to provide cut-off on soft hair spectrum!

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- Microstates = all states in near horizon Hilbert space obeying equations above

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- Vector space $\mathcal{V}_{\mathcal{B}}$ of BTZ microstates defined by

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- Useful observation:

$$\Delta^{\pm} = \langle\mathcal{B}|L_0^{\pm}|\mathcal{B}\rangle \approx \frac{1}{c}\langle\mathcal{B}|\mathcal{L}_0^{\pm}|\mathcal{B}\rangle = \frac{1}{c}\sum_i n_i^{\pm} = \frac{1}{c}\,\mathcal{E}^{\pm}_{\mathcal{B}}$$

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- Agrees with Bekenstein–Hawking and Cardy formula

# Outline

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# Generalization to four dimensions

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### Compare with near horizon construction of Donnay, Giribet, Gonzalez, Pino '15

- Near horizon algebra similar to but different from BT-BMS$_4$:

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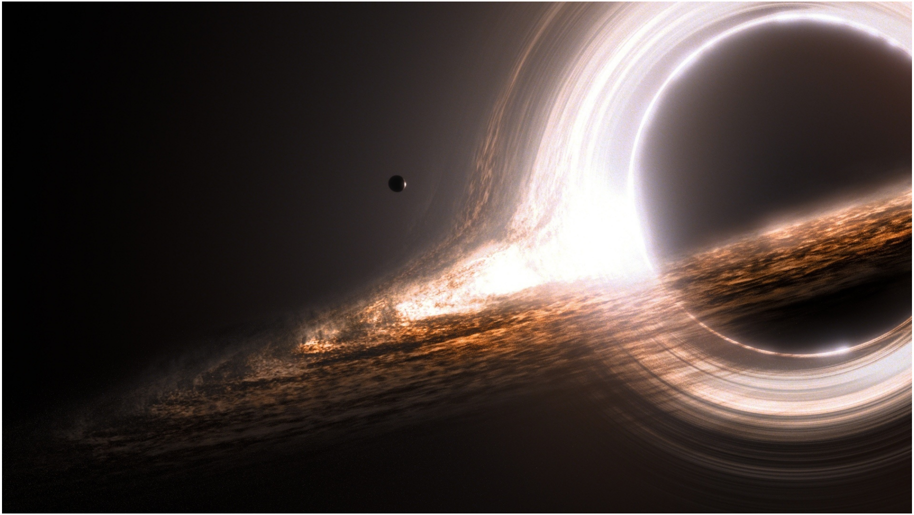
- Indicates existence of soft Heisenberg hair in 4d

# Microstates of non-extremal Kerr?



Main challenge: how to provide (controlled) cut-off on soft hair spectrum in four dimensions?

# Thanks for your attention!

- H. Afshar, D. Grumiller and M.M. Sheikh-Jabbari "Near Horizon Soft Hairs as Microstates of Three Dimensional Black Holes," 1607.00009.
- H. Afshar, S. Detournay, D. Grumiller, W. Merbis, A. Perez, D. Tempo and R. Troncoso "Soft Heisenberg hair on black holes in three dimensions," Phys.Rev. **D93** (2016) 101503(R); 1603.04824.

Thanks to Bob McNees for providing the LaTeX beamerclass!

## Map to asymptotic variables

- Usual asymptotic AdS$_3$ connection with chemical potential $\mu$:

$$\hat{A} = \hat{b}^{-1}\big(\mathrm{d}+\hat{\mathfrak{a}}\big)\hat{b} \qquad \hat{\mathfrak{a}}_\varphi = L_+ - \tfrac{1}{2}\,\mathcal{L}\,L_-$$

$$\hat{b} = e^{\rho L_0} \quad \hat{\mathfrak{a}}_t = \mu L_+ - \mu' L_0 + \big(\tfrac{1}{2}\,\mu'' - \tfrac{1}{2}\,\mathcal{L}\mu\big)\,L_-$$

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- Get Virasoro with non-zero central charge $\delta\mathcal{L} = 2\mathcal{L}\varepsilon' + \mathcal{L}'\varepsilon - \varepsilon'''$

## Remarks on asymptotic and near horizon variables

- Asymptotic spin-2 currents fulfill Virasoro algebra, but charges obey still Heisenberg algebra

$$\delta Q = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\varepsilon\,\delta\mathcal{L} = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\eta\,\delta\mathcal{J}$$

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Near horizon boundary conditions natural for near horizon observer

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- Example ([Lunin], Maldacena, Maoz '02): states in $\mathcal{H}_{\mathrm{CFT}}$ corresponding to conic spaces discrete family $L_0^\pm = -rk/4$ ($r=0$: massless BTZ, $r=k$: global AdS, $0<r<k$: spectral flow)

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- Spectral flow and discrete conic spaces generated by $\mathcal{J}_r^\pm$ ($r = 1, 2, \dots c-1$), the "horizon fluffs"

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- Mismatch in coefficients; not sure yet if bug or feature