Killing horizons kill horizon degrees

hep-th/0512230

Phase space reduction through horizon constraints

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11th Marcel Grossmann meeting, Berlin 2006
Outline

1. Motivation
   - Universality of black hole entropy
   - Carlip’s approach

2. Horizons and generic boundaries
   - First order action
   - Constraints, symmetries and classical phase space
   - Killing horizons kill horizon degrees
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Bekenstein-Hawking entropy
The closest thing to “Experimental Data” in Quantum Gravity...

Many independent derivations of

\[ S_{BH} = \frac{1}{4} A \]  

Microstates from

- D-branes
- spin network states
- ...

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Many competing “quantum gravity” models
Why do they all agree?

- possible explanation: symmetry!
- near horizon dynamics: effectively 2D
- stationary black hole: conformal Killing vector
- Cardy formula: entropy from central charge

Entropy from “Goldstone degrees of freedom”

Gauge-to-physics conversion
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2D dilaton gravity
Universality: Black Holes “essentially” 2D...

Second order action:

\[ S_{2DG} = \int d^2 x \sqrt{-g} \left[ XR + U(X)(\nabla X)^2 - 2V(X) \right] \] (2)

- 2D scalar-tensor theory (dilaton $X$, metric $g$)
- Spherical reduction: Schwarzschild BH
- Strings in 2D: Witten BH
- Many intrinsically 2D toy models
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Near horizon constraints

S. Carlip, gr-qc/0601041

- Structure of phase space changed due to constraints
  - Carlip: “stretched horizon”
  - Virasoro algebra with (classical) central charge
  - recovers Bekenstein-Hawking

Problems:
- result valid for generic boundary, not just black holes
- technically challenging to impose sharp horizon constraints
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Vienna School Approach
Gravity as non-linear gauge theory

Classically equivalent reformulation:

\[ S_1 = - \int \left[ X_a T^a + XR + \epsilon \left( X^+ X^- U(X) + V(X) \right) \right] + S_B \] (3)

- Classically integrable
- Semi-classical analysis + Thermodynamics: OK
- Path integral quantization with matter: possible

Talk by R. Meyer, session QG2 (Tuesday)
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\[ T^\pm = (De)^\pm = (d \pm \omega) \wedge e^\pm, \quad R = d\omega, \quad \epsilon = e^+ \wedge e^- \]

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Boundary action
York-Gibbons-Hawking term in first order formulation

\[ S_B = \int_{\partial M} [X \omega + X d\gamma] \]

Variational principle:
- Dilaton \( X = \text{const.} \) at boundary
- \( X^\pm \delta e^\mp = 0 \)

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Constraint analysis
Bulk and boundary constraints

- Hamiltonian: sum over (bulk) constraints
- Impose additionally boundary constraints

**generic boundary**
- $B_1 = X - X_b$
- $B_2 = e^- - E^-$
- $B_3 = e^+ - E^+$
- All boundary constraints second class
- (Almost) all bulk constraints second class at boundary

**horizon**
- $B_1 = X - X_h$
- $B_2 = e^-$
- $B_3 = X^-$
- Two boundary constraints remain first class
- Most of the bulk constraints first class at boundary
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Boundary conditions on Symmetry Parameters

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Boundary conditions on Symmetry Parameters

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Classical phase space
Solve all classical equations of motion globally

generic boundary
- One constant of motion remains (ADM mass)

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- No free constant remains
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Symmetry enhancement at horizon:
Construct reduced phase space!
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Physical degrees at generic boundary converted into gauge degrees at horizon

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- Include matter (in collaboration with R. Meyer)
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Talk by L. Bergamin, session BHT4 (Friday)!
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Results of the present talk have been obtained in: