Lower-dimensional holography

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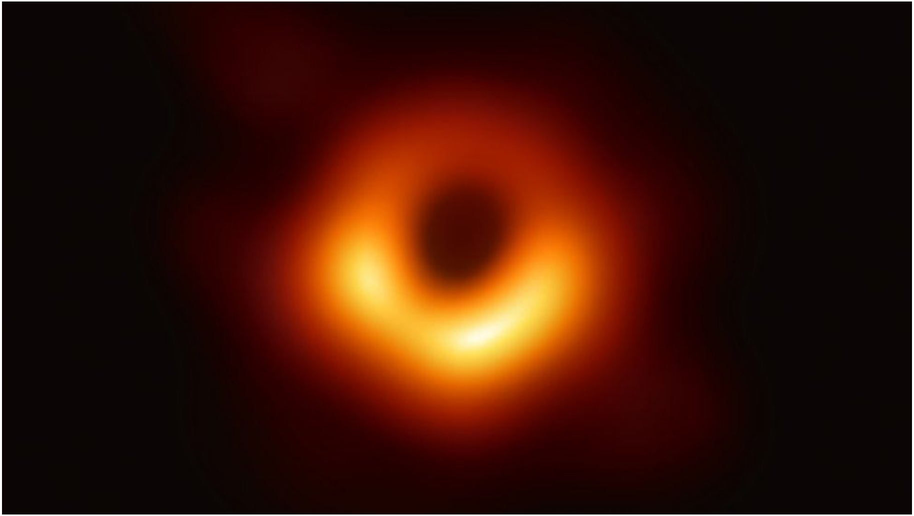
Outline

Black holes hide key secrets to Nature



Outline

Some open issues in gravity

- IR (classical gravity)

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  - asymptotic symmetries
  - soft physics
  - near horizon symmetries

Take-away slogan

Equivalence principle needs modification

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  - **black hole** microstates

Take-away homework

Find 'hydrogen-atom' of quantum gravity

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- UV/IR (holography)

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  - AdS/CFT and applications
  - precision holography
  - generality of holography

Take-away question(s)

(When) is quantum gravity in $D+1$ dimensions equivalent to (which) quantum field theory in D dimensions?

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  - soft physics
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- all issues above can be addressed in lower dimensions
- lower dimensions technically simpler
- hope to resolve conceptual problems

Gravity in various dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
- 5D: 50 (35 Weyl and 15 Ricci)
- 4D: 20 (10 Weyl and 10 Ricci)

Caveat: just counting tensor components can be misleading as measure of complexity

Example: large D limit actually simple for some problems (Emparan et al.)

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- 2D: 1 (Ricci scalar)
- 1D: 0 (space or time but not both $\Rightarrow$ no lightcones)

Apply as mantra the slogan "as simple as possible, but not simpler"

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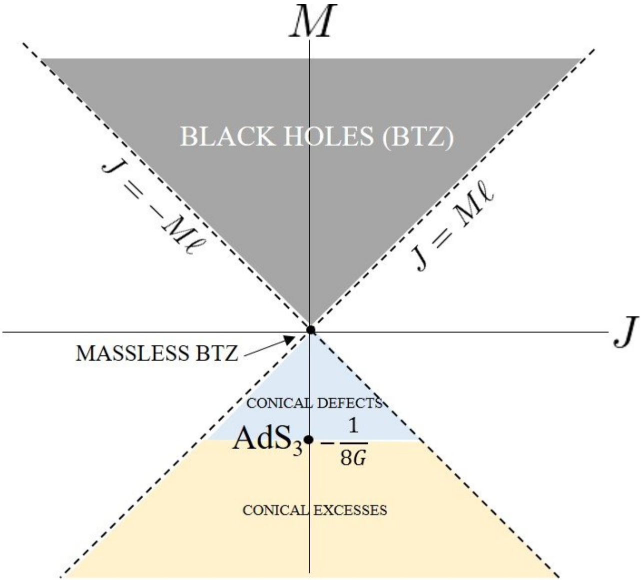
- 3D: lowest dimension exhibiting **BHs** and gravitons
- Simplest gravitational theories with **BHs** and gravitons in 3D
- Lowest dimension for Einstein gravity (**BHs** but no gravitons)

Outline

Spectrum of BTZ **black holes** and related physical states

Could this **black hole** be the 'hydrogen atom' for quantum gravity?



Choice of theory

- **Choice of bulk action**

  Pick Einstein–Hilbert action with negative cc ($\Lambda = -1/\ell^2$)

$$I_{\textrm{EH}}[g] = -\frac{1}{16\pi G}\int_{\mathcal{M}} \mathrm{d}^3x\sqrt{-g}\left(R + \frac{2}{\ell^2}\right)$$

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t: time, $\varphi \sim \varphi + 2\pi$: angular coordinate, r: radial coordinate
$r \to \infty$: asymptotic region
$r \to r_+ \geq r_-$: **black hole** horizon
$r \to r_- \geq 0$: inner horizon
$r_+ \to r_- > 0$: extremal BTZ
$r_- \to 0$: non-rotating BTZ

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  - Bekenstein–Hawking entropy

$$S_{\mathrm{BH}} = \frac{A}{4G} = \frac{\pi r_+}{2G}$$

    Hawking–Unruh temperature: $T = (r_+^2 - r_-^2)/(2\pi r_+ \ell^2)$

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  Pick whatever suits best to describe relevant physics

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> - Goal: understand micoscopic structure of BTZ black holes
> - Tool: near horizon symmetries and edge states
> - Task: recall first in general how edge states emerge

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Definition of asymptotic symmetries

All boundary condition preserving gauge transformations (bcpgt's) modulo trivial gauge transformations

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- Impose some bc's at (asymptotic or actual) boundary:

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$\xi^\mu_{(0)}(r_b,\, x^i)$: generates asymptotic symmetries/changes physical state
subleading terms: generate trivial diffeos

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Definition of asymptotic symmetry algebra

Lie bracket quotient algebra of asymptotic Killing vectors modulo trivial diffeos

Canonical boundary charges

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  - Robin bc's

    $$\left(\psi + \alpha\psi'\right)\big|_{x=0^+} = 0 \qquad \alpha \in \mathbb{R}^+$$

    lead to one bound state ("edge state")

    $$\psi(x)\big|_{x\geq 0} = \sqrt{\frac{2}{\alpha}}\, e^{-x/\alpha}$$

    with energy $E = -1/\alpha^2$, localized exponentially near $x = 0$

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- changing boundary conditions can change physical spectrum
- to distinguish asymptotic symmetries from trivial gauge trafos: either use Noether's second theorem and covariant phase space analysis or perform Hamiltonian analysis in presence of boundaries

  Some references:
  - covariant phase space: Lee, Wald '90, Iyer, Wald '94 and Barnich, Brandt '02
  - review: see Compère, Fiorucci '18 and refs. therein
  - canonical analysis: Arnowitt, Deser, Misner '59, Regge, Teitelboim '74 and Brown, Henneaux '86
  - review: see Bañados, Reyes '16 and refs. therein

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- in Hamiltonian language: gauge generator $G[\epsilon]$ varies as

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not functionally differentiable in general (Σ: constant time slice)

Φ: shorthand for phase space variables
ϵ: smearing function/parameter of gauge trafos
δ: arbitrary field variation

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Trivial gauge transformations generated by some ϵ with $Q[\epsilon] = 0$

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Motivation for near horizon boundary conditions

Old idea by Strominger '97 and Carlip '98

Main idea

Impose existence of non-extremal horizon as boundary condition on state space

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1. Why only semi-classical input for entropy?
2. What are microstates?
3. Semi-classical construction of microstates?
4. Does counting of microstates reproduce S_{BH}?

Explicit form of near horizon boundary conditions

See Donnay, Giribet, Gonzalez, Pino '15 and Afshar et al '16

Postulates of near horizon boundary conditions:

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Postulates of near horizon boundary conditions:

1. Rindler approximation

$$\mathrm{d}s^2 = -\kappa^2 r^2\, \mathrm{d}t^2 + \mathrm{d}r^2 + \Omega_{ab}(t,\, x^c)\, \mathrm{d}x^a\, \mathrm{d}x^b + \dots$$

   $r \to 0$: Rindler horizon

   κ: surface gravity

   Ω_{ab}: metric transversal to horizon

   $\dots$: terms of higher order in r or rotation terms

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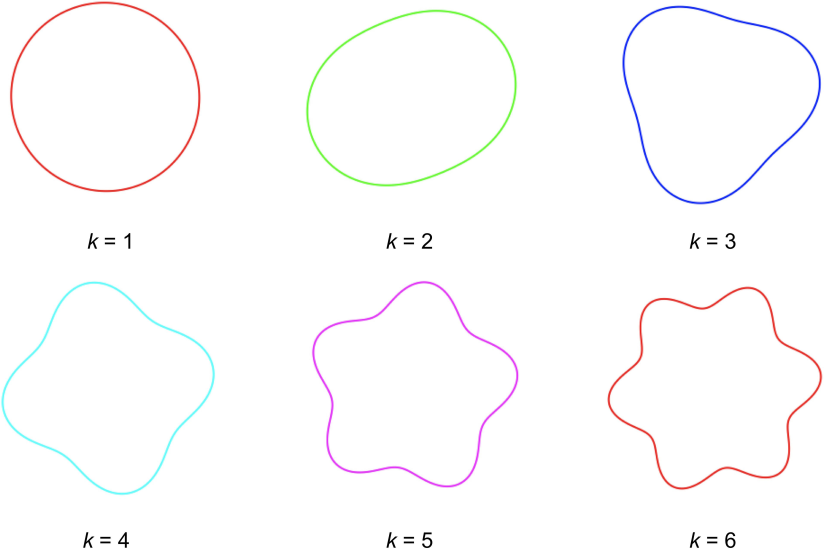
4. Remaining terms fixed by consistency of canonical boundary charges

Black holes can be deformed into black flowers Afshar et al. 16

Horizon can get excited by area preserving shear-deformations



Near horizon symmetries = "asymptotic symmetries" for near horizon bc's

Restrict for the time being to AdS_3 black holes (BTZ)

Simplification in 3d:

$$\mathrm{d}s^2 = \Big[-\kappa^2 r^2 \, \mathrm{d}t^2 + \mathrm{d}r^2 + \gamma^2(\varphi)\, \mathrm{d}\varphi^2 + 2\kappa\, \omega(\varphi)\, r^2 \, \mathrm{d}t\, \mathrm{d}\varphi \Big] \big(1 + \mathcal{O}(r^2)\big)$$

- Map from round S^1 to Fourier-excited S^1: diffeo $\gamma(\varphi)\, \mathrm{d}\varphi = \mathrm{d}\tilde{\varphi}$

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$$\mathrm{d}s^2 = \Big[-\kappa^2 r^2\, \mathrm{d}t^2 + \mathrm{d}r^2 + \gamma^2(\varphi)\, \mathrm{d}\varphi^2 + 2\kappa\, \omega(\varphi)\, r^2\, \mathrm{d}t\, \mathrm{d}\varphi \Big] \big(1 + \mathcal{O}(r^2)\big)$$

- Map from round S^1 to Fourier-excited S^1: diffeo $\gamma(\varphi)\, \mathrm{d}\varphi = \mathrm{d}\tilde{\varphi}$
- Trivial or non-trivial?
  Answer provided by boundary charges!

Near horizon symmetries = "asymptotic symmetries" for near horizon bc's
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- Canonical analysis yields

$$Q^\pm[\epsilon^\pm] \sim \oint \mathrm{d}\varphi\,\epsilon^\pm(\varphi)\,\big(\gamma(\varphi) \pm \omega(\varphi)\big)$$

where $\epsilon^\pm$ are functions appearing in asymptotic Killing vectors

charge conservation follows from on-shell relations $\partial_t\gamma = 0 = \partial_t\omega$

hairy black holes: γ and ω are hair of black hole

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- Near horizon symmetry algebra Fourier modes $\mathcal{J}_n^\pm = Q^\pm[\epsilon^\pm = e^{in\varphi}]$

$$[\mathcal{J}_n^\pm,\, \mathcal{J}_m^\pm] = \tfrac{1}{2}\,n\,\delta_{n+m,\,0}$$

Two $u(1)$ current algebras! Afshar et al. 16

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$$[\mathcal{J}_n^\pm,\,\mathcal{J}_m^\pm] = \tfrac{1}{2}\,n\,\delta_{n+m,\,0}$$

- Isomorphic to Heisenberg algebras plus center

$$[X_n,\,P_m] = i\,\delta_{n,m} \qquad\qquad [P_0,\,X_n] = 0 = [X_0,\,P_n]$$

$P_0 = \mathcal{J}_0^+ + \mathcal{J}_0^-$, $X_n = \mathcal{J}_n^+ - \mathcal{J}_{-n}^-$, $P_n = 2i/n(\mathcal{J}_{-n}^+ + \mathcal{J}_n^-)$ for $n \neq 0$

Unique features of near horizon boundary conditions

1. All states allowed by bc's have same temperature

   By contrast: asymptotically AdS or flat space bc's allow for black hole states at different masses and hence different temperatures

Unique features of near horizon boundary conditions

1. All states allowed by bc's have same temperature
2. All states allowed by bc's are regular

   (in particular, they have no conical singularities at the horizon in the Euclidean formulation)

   By contrast: for given temperature not all states in theories with asymptotically AdS or flat space bc's are free from conical singularities; usually a unique black hole state is picked

Unique features of near horizon boundary conditions

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2. All states allowed by bc's are regular

   (in particular, they have no conical singularities at the horizon in the Euclidean formulation)
3. There is a non-trivial reducibility parameter ($=$ Killing vector)

   By contrast: for any other known (non-trivial) bc's there is no vector field that is Killing for all geometries allowed by bc's

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4. Technical feature: in Chern–Simons formulation of 3d gravity simple expressions in diagonal gauge

$$A^{\pm} = b^{\mp 1}\big(\,\mathrm{d}+a^{\pm}\big)\,b^{\pm 1}$$

$$a^{\pm} = L_0\,\Big(\big(\gamma(\varphi) \pm \omega(\varphi)\big)\,\mathrm{d}\varphi + \kappa\,\mathrm{d}t\Big)$$

$$b = \exp\big[\big(L_+ - L_-\big)\,r/2\big]$$

$L_{\pm}$ are $sl(2,\,\mathbb{R})$ raising/lowering generators
L_0 is $sl(2,\,\mathbb{R})$ Cartan subalgebra generator

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5. Leads to soft Heisenberg hair (see next slides!)

Soft Heisenberg hair for BTZ

- Black flower excitations = hair of black holes
  Algebraically, excitations from descendants

$$|\text{black flower}\rangle \sim \prod_{n_i^\pm > 0} \mathcal{J}^+_{-n_i^+} \mathcal{J}^-_{-n_i^-} |\text{black hole}\rangle$$

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$$H = Q[\partial_t] = \kappa \, P_0$$

  commutes with all generators $\mathcal{J}^\pm_n$

* units defined by specifying κ

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Call it "soft Heisenberg hair"

New entropy formula

Express entropy in terms of near horizon charges:

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$$\delta S = \frac{2\pi}{\kappa}\, \delta\big(\kappa P_0\big) \qquad \Rightarrow \qquad T\, \delta S = \delta H$$

with Hawking–Unruh-temperature

$$T = \frac{\kappa}{2\pi}$$

δ refers to any variation of phase space variables allowed by the boundary conditions

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Can we understand entropy law microscopically?

Semi-classical microstates?

Given our soft Heisenberg hair, attack now entropy questions

1. Why only semi-classical input for entropy?
2. What are microstates?
3. Semi-classical construction of microstates?
4. Does counting of microstates reproduce $S_{\rm BH}$?

Regarding 1. and 3.: may expect decoupling of scales so that description of microstates does not need info about UV completion, but rather only some semi-classical "Bohr-like" input

Evidence for this: universality of BH entropy for large black holes

$$S_{\rm BH} = \frac{A}{4G} + \dots$$

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- possible resolution: do not consider asymptotic but near horizon observer (i.e., employ near horizon bc's and symmetry algebra)

Fluff proposal (with Afshar, Sheikh-Jabbari '16 and also with Yavartanoo '17)

Semi-classical BTZ black hole microstates as near horizon descendants of vacuum

Highest weight vacuum $|0\rangle$

$$\mathcal{J}_n^{\pm}|0\rangle = 0 \quad \forall n \geq 0$$

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derived from Bohr-type quantization conditions

- quantization of central charge $c = 3\ell/(2G)$ in integers
- quantization of conical deficit angles in integers over c
- black hole/particle correspondence
  (black hole = gas of coherent states of particles on AdS_3)

Check of fluff proposal

Microstates for BTZ black hole with mass M and angular momentum J:

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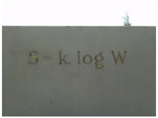

(we set $k = 1$ and $W = p$)

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- leading + subleading order yields BH entropy plus log corrections

$$S = \frac{A}{4G} - 2\ln\left(A/(4G)\right) + \dots$$

Generalizations

- Near horizon boundary conditions

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  works in any dimension, for any local geometry, for any reasonable theory* (with metric) and for any type of non-extremal horizon

* theories checked so far:
Einstein gravity with negative cosmological constant ($d \geq 3$)
Einstein gravity with vanishing cosmological constant ($d \geq 3$)
higher spin gravity ($d = 3$, principal embedding of $sl(2)$)
various massive gravity theories ($d = 3$)

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* for instance, for Schwarzschild

$$\{Q_{lm},\, P_{l'm'}\} = \frac{1}{8\pi G}\,\delta_{ll'}\,\delta_{mm'} \qquad l > 0 \qquad\qquad \{P_{00},\, \bullet\} = 0$$

Q_{lm}: spherical harmonics of area preserving shear deformations
P_{lm}: spherical harmonics of near horizon supertranslations
Entropy given by $S = 2\pi\, P_{00}$

Kerr has additional generators: area preserving twist deformations

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  might work more generally, but so far only checked BTZ black hole; needed Bohr-type rules to succeed

Outline

Motivation

Gravity in three dimensions

Near horizon soft hair

Gravity in two dimensions

Flat space holography and complex SYK

What about non-AdS holography?

Key question

(When) is quantum gravity in $D+1$ dimensions equivalent to (which) quantum field theory in D dimensions?

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Let us be modest and refine this question:

More modest question

(How) does holography work in flat space?

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See work by Bagchi et al.

What about flat space holography?

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(When) is quantum gravity in $D+1$ dimensions equivalent to (which) quantum field theory in D dimensions?

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(How) does holography work in flat space?

See work by Bagchi et al.

Would like concrete model for flat space holography

Selected list of models (see review hep-th/0604049 with Meyer)

Black holes in (A)dS$_2$, asymptotically flat or arbitrary spaces (Wheeler property)

Model	$U(X)$	$V(X)$
1. Schwarzschild (1916)	$-\frac{1}{2X}$	$-\lambda^2$
2. Jackiw-Teitelboim (1984)	0	ΛX
3. Witten **Black Hole** (1991)	$-\frac{1}{X}$	$-2b^2X$
4. CGHS (1992)	0	-2Λ
5. $(A)dS_2$ ground state (1994)	$-\frac{a}{X}$	BX
6. Rindler ground state (1996)	$-\frac{a}{X}$	BX^a
7. **Black Hole** attractor (2003)	0	BX^{-1}
8. Spherically reduced gravity ($N > 3$)	$-\frac{N-3}{(N-2)X}$	$-\lambda^2 X^{(N-4)/(N-2)}$
9. All above: ab-family (1997)	$-\frac{a}{X}$	BX^{a+b}
10. Liouville gravity	a	$be^{\alpha X}$
11. Reissner-Nordström (1916)	$-\frac{1}{2X}$	$-\lambda^2 + \frac{Q^2}{X}$
12. Schwarzschild-$(A)dS$	$-\frac{1}{2X}$	$-\lambda^2 - \ell X$
13. Katanaev-Volovich (1986)	α	$\beta X^2 - \Lambda$
14. BTZ/Achucarro-Ortiz (1993)	0	$\frac{Q^2}{X} - \frac{J}{4X^3} - \Lambda X$
15. KK reduced CS (2003)	0	$\frac{1}{2}X(c - X^2)$
16. KK red. conf. flat (2006)	$-\frac{1}{2}\tanh(X/2)$	$A\sinh X$
17. 2D type 0A string **Black Hole**	$-\frac{1}{X}$	$-2b^2X + \frac{b^2q^2}{8\pi}$
18. exact string **Black Hole** (2005)	lengthy	lengthy

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- **Choice of bulk action**
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  Dilaton gravity in two dimensions ($X =$ dilaton):

$$I[X,\, g_{\mu\nu}] = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \big[XR - U(X)(\nabla X)^2 - 2V(X)\big]$$

  - kinetic potential $U(X)$ and dilaton potential $V(X)$
  - constant dilaton and linear dilaton solutions
  - all solutions known in closed form globally for all choices of potentials
  - gauge theoretic reformulation as (deformed) BF-theory with non-linear gauge symmetries (Ikeda '93; Schaller, Strobl '94)
  - simple choice (Jackiw–Teitelboim):

$$U(X) = 0 \qquad\qquad V(X) = \Lambda X$$

Choice of theory (review: see hep-th/0204253)

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  JT model:

$$I_{\text{JT}}[X,\, g_{\mu\nu}] = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\; [XR - 2\Lambda X]$$

  Leads to (A)dS$_2$ solutions

$$R = 2\Lambda$$

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- Flat space choice of bulk action
  CGHS model

$$I_{\mathrm{CGHS}}[X,\, g_{\mu\nu}] = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \left[X R - 2\Lambda\right]$$

  Leads to flat solutions

$$R = 0$$

  Flat space holography proposal: Afshar, González, DG, Vassilevich '19

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Interlude: SYK in one slide (Kitaev '15; Maldacena, Stanford '16)

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$$\Gamma[h] \sim -\frac{N}{J}\int\limits_0^\beta \mathrm{d}\tau \left[\dot{h}^2 + \tfrac{1}{2}\{h;\,\tau\}\right] \qquad \{h;\,\tau\} = \frac{\dddot{h}}{\dot{h}} - \frac{3}{2}\frac{\ddot{h}^2}{\dot{h}^2}$$

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- Schwarzian action also follows from JT gravity

Flat space holography and complex SYK 1911.05739

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- A2: The twisted warped action

$$\Gamma[h,\, g] = \kappa \int\limits_0^\beta \mathrm{d}\tau \left(\dot{h}^2 - \dot{g}\left(\frac{2\pi i}{\beta}\,\dot{h} + \frac{\ddot{h}}{\dot{h}}\right) + \ddot{g}\right)$$

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$$[L_n,\, L_m] = (n-m)\, L_{n+m}$$
$$[L_n,\, J_m] = -m\, J_{n+m} - i\kappa\,(n^2 - n)\,\delta_{n+m,\,0}$$
$$[J_n,\, J_m] = 0$$

and the two-dimensional Maxwell symmetries (L_1, L_0, J_{-1}, J_0)

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Concrete model for flat space holography

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- Numerous open questions in gravity and holography
- Many can be addressed in lower dimensions
- If you are stuck in higher D try $D = 3$ or $D = 2$

Thank you for your attention!