Wolfgang Kummer's pioneering approach to 2d dilaton gravity

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word cloud generated using INSPIRE data of Wolfgang Kummer's scientific publications (titles + coauthors)

Outline

Motivations (2d or not 2d?)

2d geometry

2d quantum gravity

2d holography

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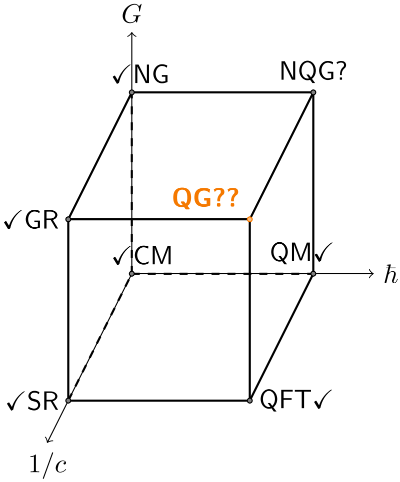

Motivations for studying gravity in 2d

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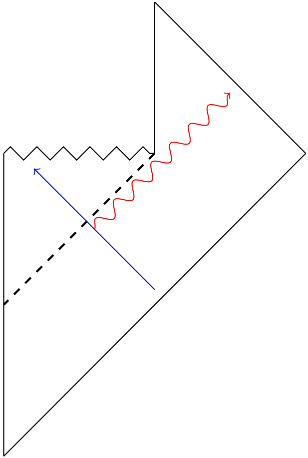
  - evaporating black holes/information loss?



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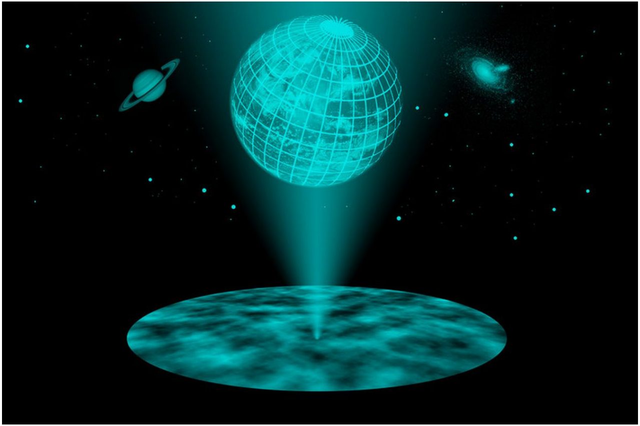

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  - 2: lowest dimension with Riemann curvature and notable topology

$$R_{\mu\nu\lambda\sigma} = \frac{1}{2}\, R \left(g_{\mu\lambda} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\lambda}\right) \qquad R_{\mu\nu} = \frac{1}{2}\, R\, g_{\mu\nu}$$

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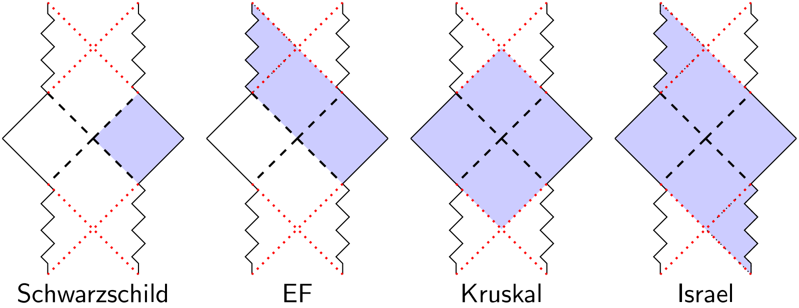

2d sections of Reissner–Nordström black hole Penrose diagram, different coordinate patches

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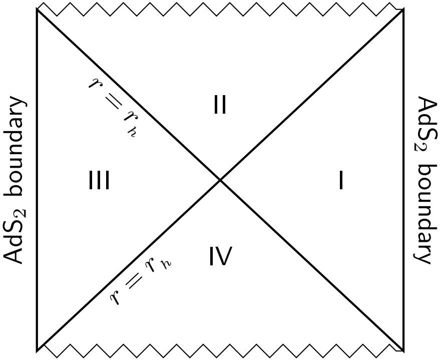

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$$\underbrace{h_{\mu\nu}}_{3} = \underbrace{\nabla_{(\mu}\xi_{\nu)}}_{2} + \underbrace{\frac{1}{2}\,h\,\bar{g}_{\mu\nu}}_{1}$$

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    Einstein equations hold trivially for any 2d metric:

$$\underbrace{R_{\mu\nu} = \frac{1}{2}\, g_{\mu\nu} R}_{\text{true off-shell}}$$

formally: in $d = 2$ we have $d(d-3)/2 = -1$ graviton polarizations

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$$\mathrm{d}s^2 = e^{2\Omega(x^+,\,x^-)}\,\mathrm{d}x^+\,\mathrm{d}x^-$$

Caveat 1: Ω singular at singularities, horizons, asymptotic boundaries
Caveat 2: EF gauge simpler for many purposes
Caveat 3: may need non-proper trafo to achieve conformal gauge

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  - 2d metrics locally conformally flat but beware of premature gauge-fixing

Gravity in 2d provides (often soluble) toy models for quantum gravity, black hole evaporation and holography

Outline

Local geometry

- Often useful: Eddington–Finkelstein (EF) gauge

$$\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = -2\,\mathrm{d}u\,\mathrm{d}r - K(u,\,r)\,\mathrm{d}u^2$$

 used this gauge for classical, semiclassical, and quantum 2d dilaton gravity

 analogous to temporal/axial gauge used by Wolfgang for nonabelian gauge theories

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  $K = 1$: Minkowski spacetime ($u = t - r$)

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> Local properties (curvature, Killing horizons) captured by $K(u,\,r)$

Global geometry

- EF gauge also useful for uncovering global properties

 used this gauge to find all global solutions of $R^2 + T^2$ gravity

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- focus on metrics with Killing vector ∂_u (why? see later!)

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  in example: $r_\pm = M \pm \sqrt{M^2 - Q^2}$

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  in example: metric asymptotically Minkowski for $r \to \infty$

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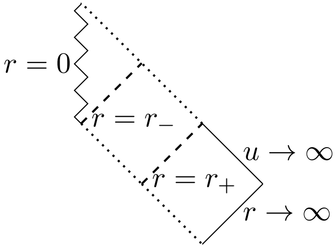

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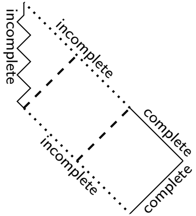

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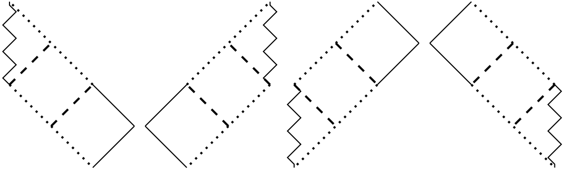

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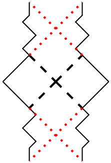

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Global properties (horizons, Penrose diagrams) captured by $K(u, r)$

Asymptotic geometry

- many physical situations: have an actual or asymptotic boundary


while this rarely featured in Wolfgang's work, it was among the last research topics we discussed in 2007


Wolfgang studied boundary conditions with Lau '96; Bergamin, DG, Vassilevich '06 [his last paper]

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implies $R = -\frac{2}{\ell^2} + \dots$ $\Rightarrow$ vanilla asymptotically AdS_2 bc's!

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- metric fluctuations allowed by bc's: $\delta g_{ur} = \delta g_{rr} = 0$, $\delta g_{uu} = \mathcal{O}(1)$

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- need to provide boundary conditions on fields, including metric
- "natural" boundary conditions (field $\to 0$) bad for metric!
- instead: fall-off conditions (adapted to physical situation)
- example in EF gauge (asymptotic boundary at $r \to \infty$):

$$\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = -2\,\mathrm{d}u\,\mathrm{d}r - \Big(\frac{r^2}{\ell^2} + \mathcal{O}(1)\Big)\,\mathrm{d}u^2$$

- metric fluctuations allowed by bc's: $\delta g_{ur} = \delta g_{rr} = 0$, $\delta g_{uu} = \mathcal{O}(1)$
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$$(\mathcal{L}_\xi g)_{\mu\nu} \stackrel{!}{=} \mathcal{O}(\delta g_{\mu\nu})$$

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$$\xi = \xi[\epsilon] = \epsilon(u)\,\partial_u - r\,\epsilon'(u)\,\partial_r + \ell^2\,\epsilon''(u)\,\partial_r + \dots$$

we shall derive this result in the end

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In holographic context AKVs are global symmetries of dual QFT

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Also: asymptotic properties (AKVs, ASA) captured by $K(u,\,r)$

Outline

Motivations (2d or not 2d?)

2d geometry

2d quantum gravity

2d holography

Action for dilaton gravity in 2d

- to quantize, we need more than geometry/kinematics: action!

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- action for Katanaev–Volovich model in 2d dilaton gravity formulation

$$I[X,\, g_{\mu\nu}] = \frac{k}{4\pi} \int \mathrm{d}^2x\sqrt{-g}\left(XR - \alpha(\partial X)^2 - \beta X^2 - \gamma\right)$$

X: dilaton field
$g_{\mu\nu}$: metric
α, β, γ, k: coupling constants

I can solve this globally in EF-gauge!

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- to quantize, we need more than geometry/kinematics: action!
- action for Jackiw–Teitelboim model in 2d dilaton gravity formulation

$$I[X,\, g_{\mu\nu}] = \frac{k}{4\pi}\int \mathrm{d}^2x\sqrt{-g}\, X\big(R-\Lambda\big)$$

X: dilaton field
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- to quantize, we need more than geometry/kinematics: action!
- action for generic power counting renormalizable 2d dilaton gravity

$$I[X,\, g_{\mu\nu}] = \frac{k}{4\pi} \int \mathrm{d}^2x \sqrt{-g}\, \big(XR - U(X)\,(\partial X)^2 - V(X)\big)$$

X: dilaton field
$g_{\mu\nu}$: metric
$U(X)$, $V(X)$: dilaton potentials (may contain coupling constants)

I can solve this globally in EF-gauge: $\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r - K(r)\,\mathrm{d}u^2$

Wolfgang's general solution has a function K of the form

$$K(X) = e^{Q(X)}\big(w(X) - 2M\big) \qquad \mathrm{d}r = e^{Q(X)}\,\mathrm{d}X$$

with $Q(X) = \int^X U(y)\,\mathrm{d}y$ and $w(X) = -\frac{1}{2}\int^X e^{Q(y)}V(y)\,\mathrm{d}y$

$\Rightarrow$ generalized Birkhoff theorem: Killing vector ∂_u for all solutions

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- to quantize, we need more than geometry/kinematics: action!
- action for generalized 2d dilaton gravity DG, Ruzziconi, Zwikel '21

$$I[X,\, g_{\mu\nu}] = \frac{k}{4\pi} \int \mathrm{d}^2x \sqrt{-g} \left(XR - \mathcal{V}(X,\, (\partial X)^2)\right)$$

X: dilaton field
$g_{\mu\nu}$: metric
$\mathcal{V}(X,\, (\partial X)^2)$: free function (may contain coupling constants)

one can proof that this is the most general action possible, without adding matter degrees of freedom or destroying the gravity-nature of the theory; the proof employs consistent deformations using BRST methods

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You can solve this globally in EF-gauge!

- Further generalizations:
  1. boundary terms and holographic renormalization DG, McNees '07
  2. Schwarzian-type boundary actions Maldacena, Stanford '16; González, DG, Salzer '18
  3. Carrollian 2d dilaton gravity Ecker, DG, Hartong, Pérez, Prohazka, Salzer, Troncoso '20 & '23

Various approaches to 2d quantum gravity

- Quantize perturbatively on fixed background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \qquad \langle T^\mu{}_\mu \rangle = \frac{c}{24\pi}\,\bar{R} \qquad \bar{\nabla}_\mu \langle T^\mu{}_\nu \rangle = 0$$

$\bar{g}_{\mu\nu}$: background metric, $\bar{R}$: background Ricci scalar
c: central charge of matter part (trace anomaly)
last equality: covariant conservation equation of EMT $T_{\mu\nu}$


derived Hawking effect in this way, see review with Vassilevich '99

Various approaches to 2d quantum gravity

- Quantize perturbatively on fixed background
- Define 2d gravity as matrix model

see e.g. Di Francesco, Ginsparg, Zinn-Justin '93

Various approaches to 2d quantum gravity

- Quantize perturbatively on fixed background
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- Use holography

see final part of talk

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- Integrate out geometry exactly

Vienna School approach Kummer, Liebl, Vassilevich, DG, Fischer, Bergamin, Hofmann

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Focus first on Vienna School approach and then on holography

Integrating out geometry

For details see DG, Kummer, Vassilevich hep-th/0204253

Distinguishing features of Vienna School approach:

Integrating out geometry

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- Spherically reduce Einstein gravity with matter to 2d dilaton gravity

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- Calculate S-matrix
- Optionally: reconstruct geometry from solving constraints $\Rightarrow$ virtual black holes!

S-matrix for s-wave gravitational scattering

Massless free scalar field scatters on own gravitational energy

Implementing program for Einstein-massless Klein–Gordon model yields lowest order 4pt (unitary and CPT-invariant) S-matrix

$$T(q,q',k,k') = \frac{\delta(k+k'-q-q')}{|kk'qq'|^{3/2}}\,\tilde{T}$$

with $s = k + k' = q + q'$, $t = k - q$, $u = k' - q$ and

$$\tilde{T} = stu\,\ln\frac{t^2u^2}{s^4} + \frac{1}{stu}\sum_{p\in\{k,k',q,q'\}} p^2 \ln\frac{p^2}{s^2}\Big(3kk'qq' - \frac{1}{2}\sum_{r\neq p}\sum_{v\neq r,p} r^2 v^2\Big)$$

S-matrix obtained in Fischer, DG, Kummer, Vassilevich '01

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- only one delta function (no separate momentum conservation)

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- forward scattering poles e.g.,

$$\lim_{t \to 0} T \propto \frac{1}{t} \, \frac{\ln \frac{(s-u)^2}{(s+u)^2}}{|s^2 - u^2|} + \mathcal{O}(t)$$

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- crossing symmetry $T(s,t,u) = T(t,s,u) = T(u,t,s)$
- scale covariance: $T(\lambda q, \lambda q', \lambda k, \lambda k') = \lambda^{-4}\,T(q,q',k,k')$

Physical consequences of S-matrix

- we integrated out geometry exactly and maintain only matter dof

Wolgfgang had this vision already in '98, the start of my PhD; it took until '01 to realize it

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- can reconstruct geometry and get virtual black holes

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r - \Big[1 - \delta(u-u_0)\theta(r_0-r)\Big(\frac{2m(r_0)}{r} + a(r_0)\,r + d(r_0)\Big)\Big]\,\mathrm{d}u^2$$

with distributional contributions

$$m(r_0) = c_0 r_0^3 + c_1 r_0^2 \qquad a(r_0) = 3c_0 r_0 - 2c_1 \qquad d(r_0) = c_0 r_0^2$$

that account for δ-insertions of kinetic matter sources at u_0, r_0

Generalizations

- Vertices for arbitrary 2d dilaton gravity models

done with Vassilevich, DG '02

Generalizations

- Vertices for arbitrary 2d dilaton gravity models
- 1-loop calculations

  indications that non-local loops vanish

  see my contribution to the 14th International Hutsulian Workshop on Mathematcial Theories and their Physical and Technical Applications '02

Generalizations

- Vertices for arbitrary 2d dilaton gravity models
- 1-loop calculations
- 1-loop corrections to specific heat

consequence for CGHS model: specific heat no longer infinite

calculated with Vassilevich, DG '03

Generalizations

- Vertices for arbitrary 2d dilaton gravity models
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- 1-loop corrections to specific heat
- Fermions instead of scalars

same procedure works

done with my first student in Leipzig, Rene Meyer, DG '06

Generalizations

- Vertices for arbitrary 2d dilaton gravity models
- 1-loop calculations
- 1-loop corrections to specific heat
- Fermions instead of scalars
- Higher dimensions

done in a recent master thesis by my student Florian Ecker '21

Generalizations

- Vertices for arbitrary 2d dilaton gravity models
- 1-loop calculations
- 1-loop corrections to specific heat
- Fermions instead of scalars
- Higher dimensions
- missing: other matter interactions, more S-matrix calculations, Mellin amplitudes, asymptotic symmetries, flux-balance laws, soft theorems, ...

There is a lot of opportunity for students
Consider joining the Vienna School!

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Heuristics of holography

- Entropy S of typical substances is extensive:

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Daring idea by 't Hooft and Susskind in '90s:

Holographic Principle

Theory with gravity in $d+1$ dimensions equivalent to theory without gravity in d dimensions

Heuristics of AdS/CFT Maldacena '97 — more than 20thousand citations

AdS/CFT = most concrete and best-studied implementation of holographic principle

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- idea: geometrize RG-flow and use E as additional coordinate

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AdS/CFT = most concrete and best-studied implementation of holographic principle

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$$\mathrm{d}s^2 = f_1(E)\,\mathrm{d}E^2 + f_2(E)\,\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu \qquad \mu,\nu = 0..(D-1)$$

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$$x^\mu \to \lambda x^\mu \qquad\qquad E \to E\lambda^{-1}$$

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- **This metric is AdS$_{D+1}$! UV of QFT, $E\to\infty$, is IR of gravity!**

Holographic dictionary and applications

Holographic Dictionary Gubser, Klebanov, Polyakov '98; Witten '98

$$\left\langle \exp\Big(\int j(x)\,\mathcal{O}(x)\Big)\right\rangle_{\text{CFT}} = Z_{\text{gravity}}\Big[\phi(x,z)|_{z\to 0} = j(x)\Big]$$

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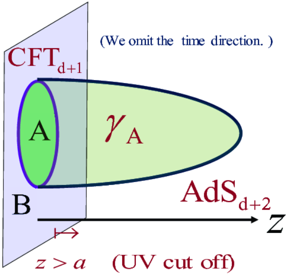

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- example type I: map strongly coupled quantum (field) theory [complicated] to weakly coupled classical gravity theory [simple]
  heavy ion collisions at LHC, neutron stars, cold atoms, viscous hydrodynamics, holographic superconductors, strange metals, ...

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- example type II: map quantum gravity [complicated] to weakly coupled quantum field theory [simple]

  microscopic understanding of black holes, information paradox, black hole evaporation, quantum information aspects of black holes, ...

Constraining dual field theory from gravity side: asymptotic symmetries

Asymptotic symmetries are key concept in gravity
They allow to constrain the dual field theory

- Symmetries in geometries characterized by Killing vectors ξ that solve Killing equation

$$(\mathcal{L}_\xi g)_{\mu\nu} = \xi^\alpha \partial_\alpha g_{\mu\nu} + g_{\mu\alpha} \partial_\nu \xi^\alpha + g_{\nu\alpha} \partial_\mu \xi^\alpha = 0$$

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$\mathcal{O}(\delta g_{\mu\nu})$: fluctuations allowed by asymptotic fall-off conditions

e.g. asymptotically AdS, asymptotically flat, asymptotically dS, ...

AdS_2: menagerie of possibilities DG, McNees, Salzer, Valcárcel, Vassilevich '17

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Key insight for holography

Asymptotic symmetry algebra generates global symmetries of dual QFT

Derivation of asymptotic symmetry algebra for AdS_2 example

- boundary and gauge fixing conditions for metric (set $\ell = 1$):

$$g_{rr} = 0 \qquad g_{ru} = -1 \qquad g_{uu} = -r^2 + \underbrace{\mathcal{T}(u)}_{\text{state-dependent}}$$

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terms of order r^2 cancel

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terms of order r cancel if $\eta = \epsilon''$

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  this is what we wanted to prove, i.e.,

$$\xi = \epsilon(u)\,\partial_u - r\,\epsilon'(u)\,\partial_r + \epsilon''(u)\,\partial_r$$

  are the AKVs preserving the form of the metric above

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terms of order 1 yield infinitesimal Schwarzian derivative

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First glimpse of AdS_2/CFT_1!

Open key questions in holography

Continuing the Vienna School in the 21st century

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