Soft hair

on black holes and cosmological horizons in any dimension

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Punchline

$$S = 2\pi\, P_0$$

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Universal and simple entropy law for (higher spin) black holes and cosmologies

$$S = 2\pi\, P_0$$

P_0: zero mode generator in near horizon symmetry algebra

$$[X_n,\, P_m] = i\,\delta_{n,\,m} \quad m \neq 0 \qquad [P_0, \bullet] = 0$$

or equivalently a number of $u(1)$ current algebras

Outline

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Entropy of (higher spin) black holes and Cardyology

Soft Heisenberg hair in spin-2 case

Soft Heisenberg hair for higher spins

Generalizations to arbitrary dimensions

Semi-classical microstates and Hardyology

Outlook

BTZ black hole entropy and Cardy formula

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- CFT_2: Cardy formula reproduces S_{BH}

$$S_{\mathrm{BH}} = \frac{A}{4G} = 2\pi \sum_{\pm} \sqrt{\frac{c^\pm L_0^\pm}{6}} = S_{\mathrm{Cardy}}$$

where $L_0^\pm$ are expectations values (for state whose entropy is calculated) of zero mode Virasoro generators

$$[L_n^\pm,\, L_m^\pm] = (n-m)\, L_{n+m}^\pm + \frac{c^\pm}{12}\, n^3\, \delta_{n+m,\,0}$$

and $c^\pm$ the left- and right central charges

see work by Strominger, Carlip, ...

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> Asymptotic Virasoro symmetries crucial for holographic Cardy formula

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- Example 2: Virasoro $\oplus\, u(1)$ current algebra
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$$t \to t\lambda^z \qquad x \to x\lambda$$

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$$S = 2\pi\,(1+z)\,\Delta^{1/(1+z)}\,\exp\big[z/(1+z)\,\ln\big(\Delta_0[1/z]/z\big)\big]$$

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Cardy formula not universal

Generalizations to higher spins in AdS_3

- 3d higher spin theories described by Chern–Simons action

  Blencowe '89; Bergshoeff, Blencowe, Stelle '90

  Note: higher spin holography (Sezgin, Sundell '02; Klebanov, Polyakov '02; Gaberdiel, Gopakumar '10) will not appear in this talk

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Henneaux, Rey; Campoleoni, Fredenhagen, Pfenninger, Theisen '10

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- Cardy-like (?) formula for their entropy

$$S = 2\pi\sqrt{2\pi k}\left(\sqrt{\mathcal{L}_+}\cos\left[\frac{1}{3}\arcsin\left(\frac{3}{8}\sqrt{\frac{3k}{2\pi\mathcal{L}_+^3}}\mathcal{W}_+\right)\right] + \sqrt{\mathcal{L}_-}\cos\left[\frac{1}{3}\arcsin\left(\frac{3}{8}\sqrt{\frac{3k}{2\pi\mathcal{L}_-^3}}\mathcal{W}_-\right)\right]\right)$$

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- not evident from asymptotic symmetries

$$[L_n^\pm,\, L_m^\pm] = (n-m)\, L_{n+m}^\pm + \frac{c^\pm}{12}\, n^3\, \delta_{n+m,\,0} \qquad [L_n^\pm,\, W_m^\pm] = (2n-m)\, W_{n+m}^\pm$$

$$[W_n^\pm,\, W_m^\pm] = \frac{96}{c}\,(n-m)\left(L^\pm\right)^2_{n+m} + (n-m)(2n^2+2m^2-nm-8)\, L_{n+m} + \frac{c^\pm}{12}\, n^5\, \delta_{n+m,\,0}$$

Flat space higher spins

- flat space HST: İnönü–Wigner contraction from HST in AdS_3

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- has flat space cosmological solutions with horizons and entropy (Gary, Grumiller, Riegler, Rosseel '14)

$$S = 2\pi\, L_0 \sqrt{\frac{c_M}{2M_0}} \cdot \frac{2R - 3 - 12P\sqrt{R}}{(R-3)\sqrt{4 - 3/R}}$$

P, R: spin-3 zero-mode charges

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How to obtain them from higher spin symmetries?

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Guideline

Perhaps near horizon physics more universal than asymptotic physics

Outline

Near horizon boundary conditions

- Near horizon line-element with Rindler acceleration κ:

$$\mathrm{d}s^2 = -\kappa^2\rho^2\,\mathrm{d}t^2 + \mathrm{d}\rho^2 + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

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- e.g. implement these bc's in CS formulation of AdS_3 Einstein gravity

$$I_{\text{CS}} = \pm \sum_{\pm} \frac{k}{4\pi} \int \langle A^\pm \wedge \mathrm{d}A^\pm + \tfrac{2}{3} A^\pm \wedge A^\pm \wedge A^\pm \rangle$$

with $sl(2)$ connections $A^\pm$ and $k = \ell/(4G)$ with AdS radius $\ell = 1$

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$$A^\pm = b_\pm^{-1}(\mathrm{d}+a^\pm)b_\pm \qquad\qquad a^\pm = \big(\kappa\,\mathrm{d}t \pm \mathcal{J}^\pm(\varphi)\,\mathrm{d}\varphi\big)\,L_0$$

technical note: diagonal gauge convenient, but not necessary

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- two towers of charges (like in Brown–Henneaux case)

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- equivalently ($X_n = J_n^+ - J_n^-$, $P_n = \frac{i}{kn}\,(J_{-n}^+ + J_{-n}^-)$, $P_0 = J_0^+ + J_0^-$):

$$\text{Heisenberg algebras:} \quad [X_n,\, P_m] = i\, \delta_{n,\,m} \quad m \neq 0 \qquad [P_0, \bullet] = 0$$

Soft Heisenberg hair

Notion of "soft hair" introduced by Hawking, Perry, Strominger '16

- Vacuum descendants $|\psi\rangle$

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$$\boxed{\text{Soft hair} = \text{zero energy excitations on horizon}}$$

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- works for warped black holes in massive gravity
- also may work for higher spins $\rightarrow$ check this now!

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see paper with Perez, Prohazka, Tempo and Troncoso

- use again CS ($sl(2) \to sl(3)$ with principal embedding of $sl(2)$)

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- twice the number of commuting elements: $P_0, P_0^{(3)}, X_0, X_0^{(3)}$

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- use again CS ($sl(2) \to sl(3)$ with principal embedding of $sl(2)$)
- technical key step: diagonal gauge for connection, $A \sim L_0, W_0$
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$$[X_n,\, P_m] = [X_n^{(3)},\, P_m^{(3)}] = i\,\delta_{n,m} \text{ for } m \neq 0\,.$$

equivalently: four $u(1)$ current algebras generated by $J_n^\pm$ and $J_n^{(3)\,\pm}$

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- same result for entropy!!!

$$S = 2\pi\, P_0 = 2\pi\,\big(J_0^+ + J_0^-\big)$$

fineprint: result above holds for branch continuously connected to BTZ black holes; other branches have additionally linear dependence on zero-mode charges $J_0^{(3)\,\pm}$

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  zero modes: quadratic and cubic relations (solve for J_0 and $J_0^{(3)}$)

$$L_0 \sim J_0^2 + \left(J_0^{(3)}\right)^2 \qquad W_0 \sim \left(J_0^{(3)}\right)^3 + J_0^2 J_0^{(3)}$$

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- complicated looking Cardy-type higher spin result recovered through twisted Sugawara construction induced by near horizon bc's insertion into entropy formula above recovers spin-3 entropy law

$$S = 2\pi\sqrt{2\pi k} \sum_{\pm} \sqrt{\mathcal{L}_\pm} \cos\left[\tfrac{1}{3} \arcsin\left(\tfrac{3}{8}\sqrt{\frac{3k}{2\pi\mathcal{L}_\pm^3}}\mathcal{W}_\pm\right)\right]$$

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see paper with Ammon, Prohazka, Riegler, Wutte

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- complication of Cardy-type formula again fully captured by twisted Sugawara-like results for higher spin currents

$$\begin{aligned}
\mathcal{L} &= \mathcal{J}\mathcal{P} + \mathcal{J}^{(3)}\mathcal{P}^{(3)} + \mathcal{P}' \\
\mathcal{M} &= \mathcal{J}^2 + \mathcal{J}^{(3)\,2} + \mathcal{J}' \\
\mathcal{U} &= \mathcal{J}^2\mathcal{P}^{(3)} + \mathcal{J}^{(3)\,2}\mathcal{P}^{(3)} + \mathcal{J}\mathcal{J}^{(3)}\mathcal{P} + \mathcal{J}'\mathcal{P}^{(3)} + \mathcal{J}^{(3)}\mathcal{P}' \\
&\quad + \mathcal{J}\mathcal{P}^{(3)\,\prime} + \mathcal{J}^{(3)\,\prime}\mathcal{P} + \mathcal{P}^{(3)\,\prime\prime} \\
\mathcal{V} &= \mathcal{J}^2\mathcal{J}^{(3)} + \mathcal{J}^{(3)\,3} + \mathcal{J}'\mathcal{J}^{(3)} + \mathcal{J}\mathcal{J}^{(3)\,\prime} + \mathcal{J}^{(3)\,\prime\prime}
\end{aligned}$$

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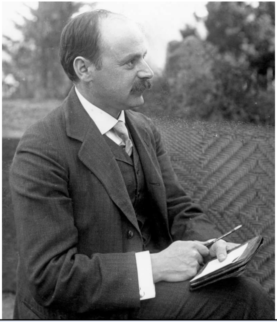
- higher dimensions? see remainder of talk!

Consider arbitrary $D > 3$ but restrict to spin-2 Einstein gravity



Outline

Near horizon boundary conditions in any dimensions

near horizon line-element:

$$\mathrm{d}s^2 = -\kappa^2\,\rho^2\,\mathrm{d}t^2 + \mathrm{d}\rho^2 + \Omega_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b + \dots$$

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near horizon Killing vectors:

$$\xi^t = \rho\,\epsilon^t + \mathcal{O}(\rho^3) \qquad \xi^\rho = \mathcal{O}(\rho^2) \qquad \xi^a = \epsilon^a + \mathcal{O}(\rho^2)$$

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near horizon charges:

$$\delta Q[\epsilon^t,\,\epsilon^a] = \int \mathrm{d}^{D-2}x\,\big[\epsilon^t\,\delta\mathcal{P} + \epsilon^a\,\delta\mathcal{J}_a\big]$$

with supertranslations

$$\mathcal{P} := \frac{\sqrt{\Omega}}{8\pi G}$$

and superrotations

$$\mathcal{J}_a := \Omega_{ab}\,\frac{\pi^{\rho b}_{(0)}}{8\pi G}$$

$\pi^{\rho b}_{(0)}$ are canonical momenta of metric

Possibilities for near horizon charges

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3. Change from 1-form densities $\mathcal{J}_a$ to 1-forms $\mathcal{J}_a^{\text{H}}$

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get following near horizon symmetries

$$\delta\mathcal{P} = \frac{1}{8\pi G}\,\partial_a\epsilon^a_{\mathrm{H}}$$

$$\delta\mathcal{J}^{\mathrm{H}}_a = \frac{1}{8\pi G}\left[\partial_a\epsilon^t_{\mathrm{H}} - \frac{\epsilon^b_{\mathrm{H}}}{\mathcal{P}}\left(\partial_a\mathcal{J}^{\mathrm{H}}_b - \partial_b\mathcal{J}^{\mathrm{H}}_a\right)\right]$$

and associated charges

$$Q_{\mathrm{H}}[\epsilon^t_{\mathrm{H}},\,\epsilon^a_{\mathrm{H}}] = \int \mathrm{d}^{D-2}x\,\left[\epsilon^t_{\mathrm{H}}\,\mathcal{P} + \epsilon^a_{\mathrm{H}}\,\mathcal{J}^{\mathrm{H}}_a\right]$$

Soft hair and entropy
Focus on third case

Reminder:

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$$H := Q_{\mathrm{H}}[\epsilon^t_{\mathrm{H}} = \kappa,\, \epsilon^a_{\mathrm{H}} = 0] = \kappa\int \mathrm{d}^{D-2}x\,\mathcal{P} \equiv \kappa\,\mathcal{P}_0$$

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Recover universal entropy result in any spacetime dimension greater than two

Heisenberg algebra?

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near horizon symmetry algebra above simplifies to Heisenberg:

$$\{\mathcal{Q}(x),\,\mathcal{P}(y)\} = \frac{1}{8\pi G}\,\delta^{(D-2)}(x-y)$$

note: factor $1/(4G)$ playing role of Planck's constant h

Outline

Microstate counting from near horizon symmetries

Works at least in three spacetime dimensions!

- start with Lifshitz scaling formula ($t \to t\lambda^z$, $\varphi \to \varphi\lambda$)

$$S = 2\pi\,(1+z)\,\sum_{\pm}\Delta_{\pm}^{1/(1+z)}\,\exp\big[z/(1+z)\,\ln\big(\Delta_0^{\pm}[1/z]/z\big)\big]$$

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- can exploit Cardy-method also to get log-corrections to entropy

$$S = S_{\text{BH}} - \frac{1}{2}\,\ln S_{\text{BH}} + \dots$$

(see paper with Perez, Tempo, Troncoso '17)

Note: factor different from the $-\frac{3}{2}$ found for Brown–Henneaux bc's

Soft hair and semi-classical microstates?

- Generic descendant of vacuum:

$$|\Psi(\{n_i^\pm\})\rangle = \prod_{\{n_i^\pm>0\}} \big(\mathcal{J}^+_{-n_i^+}\mathcal{J}^-_{-n_i^-}\big)|0\rangle$$

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- Exploited this property to provide controlled cut-off on soft hair spectrum! (Bohr-type quantization conditions)

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Input: large c; quantization of c in integers; quantization of conical defects in integers over c; black hole/particle correspondence

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- Full set of semi-classical BTZ black hole microstates:

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- Subleading log corrections also correct! (reproduce factor $-\frac{1}{2}$)

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Numerous further research avenues from soft Heisenberg hair

Thanks for your attention!

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- spin-2 case: Hamid Afshar, Stephane Detournay, Hernán González, Philip Hacker, Wout Merbis, Alfredo Perez, David Tempo, Ricardo Troncosos
- higher spins: Martin Ammon, Alfredo Perez, Stefan Prohazka, Max Riegler, David Tempo, Ricardo Troncoso, Raphaela Wutte
- semi-classical microstates: Hamid Afshar, Shahin Sheikh-Jabbari, Hossein Yavartanoo

Papers (can be clicked in PDF):
spin-2 in three dimensions: 1603.04824, 1611.09783, 1705.10605, 1711.07975
higher spins: 1607.05360, 1703.02594
semi-classical microstates: 1607.00009, 1608.01293, 1705.06257, 1708.06378, 1805.11099
spin-2 in higher dimensions: 1709.09667, 180x.xxxxx

Example: Kerr black hole

Near horizon metric for Kerr:

$$\mathrm{d}s^2 = -\kappa^2\rho^2\,\mathrm{d}t^2 + \mathrm{d}\rho^2 + 2\rho\,\frac{\frac{r_-}{r_+}\,\sin\theta\cos\theta}{1+\frac{r_-}{r_+}\cos^\theta}\,\mathrm{d}\rho\,\mathrm{d}\theta + r_+^2\left[\left(1+\frac{r_-}{r_+}\,\cos^2\theta\right)\,\mathrm{d}\theta^2 + \frac{(1+\frac{r_-}{r_+})^2\sin^2\theta}{1+\frac{r_-}{r_+}\cos^2\theta}\,\mathrm{d}\varphi^2\right] + \dots$$

Near horizon charges for Kerr black holes:

$$\mathcal{P} = \frac{r_+(r_+ + r_-)}{8\pi G}\,\sin\theta$$

$$\mathcal{J}_a^{\mathrm{H}} = \delta_a^\varphi\,r_-\,\frac{r_-(r_- - r_+)\cos^2\theta - r_+(3r_+ + r_-)}{8\pi G\,\sqrt{r_+r_-}\,(r_+ + r_-\cos^2\theta)^2}\,\sin^2\theta$$

superrotation field strength is not identically zero iff $r_- \neq 0$:

$$\partial_\theta\mathcal{J}_\varphi^{\mathrm{H}} = \frac{\sqrt{\frac{r_-}{r_+}}(1+\frac{r_-}{r_+})^2(\frac{r_-}{r_+}\cos^2\theta - 3)\sin(2\theta)}{\left(1+\frac{r_-}{r_+}\cos^2\theta\right)^3}$$