Horizon strings as 3d black hole microstates

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Based on work with Bagchi & Sheikh-Jabbari, 2210.10794

$$S_{\mathrm{BTZ}} = \frac{A}{4} - \frac{3}{2}\ln A + \dots$$

- Only semiclassical data in BH entropy to LO/NLO
- Semiclassical construction of BH microstates?

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Worldsheet of tensionless null strings collectively represents black hole horizon

Near horizon symmetries

Adami, Sheikh-Jabbari, Taghiloo, Yavartanoo, Zwikel '20

- Non-extremal null hypersurfaces

$$\mathrm{d}s^2 = -r\,V(v,\,\phi)\;\mathrm{d}v^2 + 2\eta(v,\,\phi)\;\mathrm{d}v\,\mathrm{d}r + R^2(v,\,\phi)\,\big(\,\mathrm{d}\phi + U(v,\,\phi)\;\mathrm{d}v\big)^2 + \dots$$

preserved by near horizon Killing vectors

$$\xi = T(v,\,\phi)\,\partial_v + L(v,\,\phi)\,\partial_\phi + W(v,\,\phi)\,r\,\partial_r + \dots$$

generating $\mathsf{diff}_2 \oplus \mathsf{Weyl}$

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- partially gauge-fixed non-extremal null hypersurfaces

$$\mathrm{d}s^2 = -2a\,r\;\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}r + R^2(\phi)\;\mathrm{d}\phi^2 + 4\omega(\phi)\,r\,\mathrm{d}v\,\mathrm{d}\phi + \dots$$

  generate two $\hat{u}(1)$ current algebras as near horizon symmetries

$$[J_n^\pm,\,J_m^\pm] = n\,\delta_{n+m,\,0} \qquad J_n^\pm \sim \oint \mathrm{d}\phi\,e^{in\phi}\big(R(\phi) \pm \omega(\phi)\big)$$

  suggesting as near horizon vacuum $J_n^\pm|0\rangle = 0$, $n \in \mathbb{Z}^+$

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- precisely matches one of the three possible tensionless string vacua

Classical tensionless null strings I

Worldsheet action and gauge fixing; see Bagchi et al., '13-'23

- Worldsheet action

$$S = \frac{\kappa}{2} \int \mathrm{d}\tau \, \mathrm{d}\sigma \, V^a \, V^b \, \partial_a X^\mu \, \partial_b X^\nu \, G_{\mu\nu}(X)$$

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- residual worldsheet diffeos ($\delta_\xi V^a = 0$)

$$\xi = \big(h(\sigma) + \tau f'(\sigma)\big)\, \partial_\tau + f(\sigma)\, \partial_\sigma$$

generate conformal Carroll$_2$ $\simeq$ BMS$_3$

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Equations of motion, constraints, and closed string mode expansion

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- L_n, M_n obey BMS$_3$ algebra provided

$$\{A_n^\mu,\, B_m^\nu\} = -2in\, \delta_{n,\, m}\, G^{\mu\nu}$$

- As usual, quantum constraints not enforced as operator statements but as statements on physical states

$$\langle \text{phys}' | \mathcal{O}_n | \text{phys} \rangle = 0 \quad \text{with} \quad \mathcal{O}_n = L_n, M_n$$

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- reminiscent of natural near horizon vacuum!

Horizon strings

Quantum tensionless null strings on near horizon BTZ with oscillator vacuum

- Use target space metric $(\phi \sim \phi + 2\pi)$

$$\mathrm{d}s^2 = -2\,\mathrm{d}x^+\,\mathrm{d}x^- + R_h^2\,\mathrm{d}\phi^2 = -2\,\mathrm{d}x^+\,\mathrm{d}x^- + \mathrm{d}\varphi^2$$

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$$X^\varphi = R_h\,\omega\,\sigma + \frac{n}{\kappa R_h}\,\tau + \sum_{m\neq 0}\frac{i}{m}\,\big(A_m^\varphi - im\tau B_m^\varphi\big)e^{-im\sigma}$$

with winding number $\omega \in \mathbb{Z}$ and momentum number $n \in \mathbb{Z}$

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- Call tensionless null strings with properties above "horizon strings"

Hilbert space of horizon strings, level matching, and mass formula

- generic near horizon string state labelled by momentum p^μ (including momentum number n), oscillator levels $\{r_i^\pm\}$, and winding ω

$$|\Psi\rangle = |p^\mu,\, \{r_i^\pm\},\, \omega\rangle$$

low-level states:

$$\begin{aligned}
\text{Vacuum:} \quad & |0\rangle \\
\text{Level 1:} \quad & J_{-1}^{+}|0\rangle,\, J_{-1}^{-}|0\rangle \\
\text{Level 2:} \quad & J_{-2}^{+}|0\rangle,\, J_{-2}^{-}|0\rangle,\, J_{-1}^{+}J_{-1}^{-}|0\rangle,\, (J_{-1}^{+})^2|0\rangle,\, (J_{-1}^{-})^2|0\rangle \\
\vdots \quad & \vdots \\
\text{Level } N: \quad & J_{-N}^{+}|0\rangle,\, J_{-N}^{-}|0\rangle,\, \dots,\, (J_{-1}^{+})^N|0\rangle,\, (J_{-1}^{-})^N|0\rangle
\end{aligned}$$

Level $N = r^+ + r^-$ with $r^\pm = \sum_i i r_i^\pm$

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where we defined $m = \sqrt{2p^+p^-}$ we also assume $p^+ = p^-$

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- physical states of given mass m labeled by integers $r_i^\pm, \omega, n$ subject to level matching and mass formula

BTZ BH microstates I

Definition and various sectors

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  Donnay, Giribet, Gonzalez, Pino '15; Afshar et al. '16

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- insertion into mass formula at level N yields

$$\kappa R_h^2 = N + \frac{n^2}{\kappa R_h^2} \approx N + \frac{n^2}{N} + \mathcal{O}(n^4/N^3)$$

BTZ BH microstates I

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Proposal for BTZ microstates: $|m\rangle_{\text{BTZ}} = |n,\,\{r_i^\pm\},\,\omega\rangle$

- remaining task: fix m in terms of macroscopic geometric input R_h
- exploit near horizon first law $E = T\,S$ with $S \propto R_h$ and $E \propto J_0^+ + J_0^-$
- due to near horizon first law mass scales linearly in horizon radius

$$m = \kappa\,R_h$$

- insertion into mass formula at level N yields

$$\kappa R_h^2 = N + \frac{n^2}{\kappa R_h^2} \approx N + \frac{n^2}{N} + \mathcal{O}(n^4/N^3)$$

- soft sector: $n = 0$

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- generic sector: none of the above; $N \gg n$

BTZ BH microstates II

Dominant contribution from Hardy–Ramanujan-type of combinatorics

- partition function in generic sector for fixed level N

$$Z^{\text{fix}}(N) = \sum_{l=1}^{\frac{N}{2}} \Pi\Big(\frac{N}{2} - l\Big)\,\Pi\Big(\frac{N}{2} + l\Big)\,\tau(2l)$$

$\Pi(k)$: number of integer partitions of k
$\tau(k)$: number of divisors of k

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- implies typical ranges of n and ω between $\mathcal{O}(1)$ and $\mathcal{O}(N^{3/4})$
- consequently, should allow fluctuations of mass $m \to m + \mathcal{O}(1)$ implying Gaussian fluctuations of level, $\Delta N = \mathcal{O}(\sqrt{N})$

Final result for partition function and entropy

- taking into account mass fluctuations yields

$$Z(N) = \sum_{N_0=N}^{N+\mathcal{O}(\sqrt{N})} Z^{\text{fix}}(N_0) \approx \frac{1}{N^{3/4}}\,\exp\left(2\pi\sqrt{\frac{N}{3}}\right)$$

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- comparison with Bekenstein–Hawking works provided we fix $\kappa = \frac{3}{16}$

Outlook

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- generalization to higher dimension using tensionless null branes?

Outlook by ChatGPT

In the study of black holes, tensionless null strings have been proposed as potential microstates of BTZ black holes. These tensionless null strings, when considered on black hole horizons, offer valuable insights into the derivation of the Bekenstein-Hawking entropy and its logarithmic corrections. Furthermore, they provide a foundation for exploring potential generalizations in higher dimensions using tensionless null branes.

By examining tensionless null strings on black hole horizons, researchers have made significant progress in understanding the microscopic origins of black hole entropy. The Bekenstein-Hawking entropy is a fundamental concept in black hole thermodynamics, relating the entropy of a black hole to its horizon area. The successful derivation of this entropy is a crucial step in reconciling general relativity with quantum mechanics.

When tensionless null strings are considered as microstates of BTZ black holes, their properties can account for the Bekenstein-Hawking entropy. These strings carry no tension, which allows them to exist without breaking or distorting under the influence of external forces. Their presence on the black hole horizon provides a mechanism to encode the entropy associated with the black hole.

Moreover, the study of tensionless null strings reveals logarithmic corrections to the Bekenstein-Hawking entropy. These corrections, often referred to as quantum corrections, arise due to the quantum nature of gravity and provide a more refined understanding of black hole thermodynamics.

Looking ahead, tensionless null branes offer a potential avenue for generalizing these concepts to higher dimensions. By extending the analysis from strings to branes, which are higher-dimensional analogues, researchers can explore the behavior of tensionless null objects in diverse spacetime dimensions. This opens up new possibilities for investigating black hole microstates and understanding the entropy associated with higher-dimensional black holes.

In summary, considering tensionless null strings on black hole horizons as microstates of BTZ black holes has contributed to the successful derivation of the Bekenstein-Hawking entropy and its logarithmic corrections. This research provides valuable insights into the microscopic origins of black hole entropy and paves the way for exploring generalizations in higher dimensions using tensionless null branes.

Poem by ChatGPT

In black holes' depths, a tale unfolds,
Tensionless strings, their secrets hold.
On horizons, they dance and sway,
Microstates of BTZ, they convey.

Bekenstein-Hawking's entropy revealed,
A union of quantum, gravity sealed.
Logarithmic corrections grace the stage,
Whispers of quantum's ancient sage.

Tensionless branes, in higher realms,
Extend the tale, where mystery overwhelms.
Microstates' symphony, yet to be explored,
In dimensions vast, their essence soared.

In verses woven, the cosmic ballet,
Where knowledge blossoms, like stars at play.
The riddles of holes, our souls engage,
In poetry's embrace, the universe's page.