Soft Heisenberg Hair

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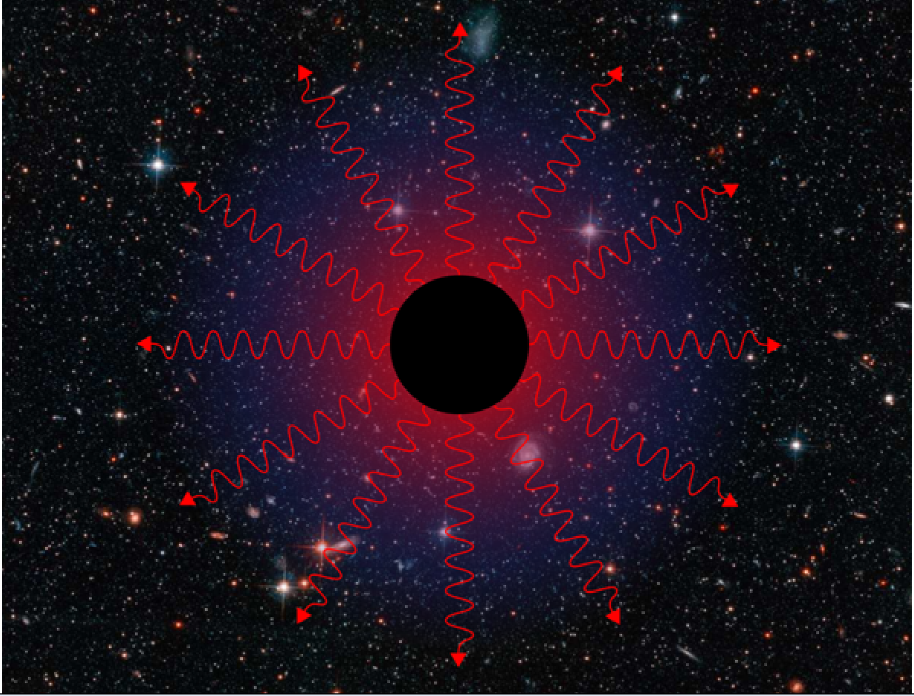

Two simple punchlines

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$$[X_n,\, P_m] = i\,\delta_{n,\,m}$$

fundamental not only in quantum mechanics
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based on work with

- Hamid Afshar, Shahin Sheikh-Jabbari [IPM Teheran]
- Martin Ammon [U. Jena]
- Stephane Detournay, Wout Merbis, Stefan Prohazka, Max Riegler [ULB]
- Hernán González, Philip Hacker, Raphaela Wutte [TU Wien]
- Alfredo Perez, David Tempo, Ricardo Troncoso [CECS Valdivia]
- Hossein Yavartanoo [ITP Beijing]

Outline

Motivation

Problems (and possible resolutions)

Near horizon boundary conditions and soft hair

Proposal for semi-classical BTZ microstates

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Preamble: snapshot of quantum gravity

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Conservative approach to quantum gravity based on following premises:

- **General Relativity correct** as classical approximation to QuGr

  Numerous experimental evidence that General Relativity correct classical theory of gravity:

  - Tests of equivalence principle
  - Classical tests of Schwarzschild metric
  - Solar system precision tests
  - Gravitational lensing
  - Frame dragging/Lense–Thirring
  - Binary pulsars
  - Existence of black holes
  - Gravitational waves
  - Cosmological evidence for FLRW

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Conservative approach to quantum gravity based on following premises:

- **General Relativity correct** as classical approximation to QuGr
- **Quantum mechanics correct** as non-relativistic limit of QuGr

  - Sometimes suggested: perhaps issues with QuGr absent if gravity not quantized
  - New problematic issue then arises

    $$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R \sim T_{\mu\nu}$$

    l.h.s.: classical; r.h.s.: quantum mechanical
  - Logically possible, but modifies rules of quantum mechanics
  - For more than a century no deviations of quantum mechanics found

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Conservative approach to quantum gravity based on following premises:

- **General Relativity correct** as classical approximation to QuGr
- **Quantum mechanics correct** as non-relativistic limit of QuGr
- **Special Relativity correct**

  - Sometimes suggested Lorentz violation at Planck scale
  - Modified dispersion relations

    $$\omega^2 \sim k^2\big(1 + \omega/\alpha + \dots\big)$$

    feature new parameters $\alpha, \dots$ with dimension of energy
  - Fermi collaboration: $\alpha > \mathcal{O}(10\ m_{\text{Planck}})$
  - Logically possible, but again more than century of attempts found no deviations from Special Relativity

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    - tested experimentally to high precision (e.g. $g-2$)

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Semi-classical predictions such as Hawking effect and Bekenstein–Hawking black hole entropy are trustworthy

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$$S = S_{\text{BH}} - q\,\ln S_{\text{BH}} + \mathcal{O}(1) \qquad q = \text{number depending on matter}$$

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currently "template for experimental results" in quantum gravity

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  [at least any theory of quantum gravity claiming to reproduce (semi-)classical Einstein gravity in limit of small Newton constant]

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Perhaps no need for full knowledge of quantum gravity to account microscopically for black hole entropy (of sufficiently large black holes)

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$$S_{\text{Cardy}} = 2\pi \left(\sqrt{c\Delta^+/6} + \sqrt{c\Delta^-/6}\right) = \frac{A}{4G} = S_{\text{BH}}$$

c: left/right central charges of CFT_2
$\Delta^\pm$: left/right energies of state whose entropy is counted

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- Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...)

  warped CFT: Detournay, Hartman, Hofman '12
  Galilean CFT: Bagchi, Detournay, Fareghbal, Simon '13; Barnich '13

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Kerr/CFT: Guica, Hartman, Song, Strominger ’09; Compere ’12

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Hope: near horizon symmetries allow for Cardyology

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additional motivation: information loss

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Perhaps no need for full knowledge of quantum gravity to construct microstates (of sufficiently large non-extremal black holes)
[at least for some observer, not necessarily an asymptotic one]

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  (locally) diffeomorphic geometries may be physically inequivalent

  Famous example: BTZ black hole is locally AdS_3, but canonical boundary charges (e.g. mass, angular momentum) differ
  Bañados, Henneaux, Teitelboim, Zanelli '93

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  Donnay, Giribet, Gonzalez, Pino '16
  Afshar, Detournay, Grumiller, Merbis, Perez, Tempo, Troncoso '16

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Hope: soft hair could address black hole entropy puzzles and microstates in a semi-classical framework

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Note: this problem may be obvious even to laypersons

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Problem: how?

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Same conceptual problems as in higher dimension, but technically more manageable

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Properties of Einstein gravity in 2+1 dimensions with negative cc (AdS_3)

- Second order bulk action:

$$I_{\text{EH}} = \frac{1}{16\pi G} \int \mathrm{d}^3x \sqrt{-g} \left(R + \frac{2}{\ell^2}\right)$$

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- Spectrum of physical states includes BTZ black holes

$$\mathrm{d}s^2 = -\frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2\ell^2}\,\mathrm{d}t^2 + \frac{r^2\ell^2\,\mathrm{d}r^2}{(r^2-r_+^2)(r^2-r_-^2)} + r^2\Big(\mathrm{d}\varphi - \frac{r_+r_-}{\ell r^2}\,\mathrm{d}t\Big)^2$$

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- BTZ BH entropy given by Bekenstein–Hawking

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- Spectrum of physical states includes BTZ black holes

$$\mathrm{d}s^2 = -\frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2\ell^2}\,\mathrm{d}t^2 + \frac{r^2\ell^2\,\mathrm{d}r^2}{(r^2-r_+^2)(r^2-r_-^2)} + r^2\,\Big(\mathrm{d}\varphi - \frac{r_+r_-}{\ell r^2}\,\mathrm{d}t\Big)^2$$

- BTZ BH entropy given by Bekenstein–Hawking and Cardy formula

$$S_{\text{BH}} = \frac{A}{4G} = \frac{2\pi r_+}{4G} = 2\pi\,\Big(\sqrt{c\Delta^+/6} + \sqrt{c\Delta^-/6}\Big)$$

  $\Delta^\pm = (r_+ \pm r_-)^2/(16\ell G) \propto \ell M \pm J$ (M: mass, J: angular momentum)

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See Afshar, Detournay, DG, Merbis, Perez, Tempo, Troncoso '16 for details

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- This is somewhat unusual, but convenient for our purposes!

Explicit form of our boundary conditions in metric formulation

Note: everything much simpler in Chern–Simons formulation!

Boundary conditions as near horizon expansion of metric

$$
\begin{aligned}
g_{tt} &= -a^2 r^2 + \mathcal{O}\left(r^3\right) \\
g_{\varphi\varphi} &= \gamma^2 + \left(\gamma^2 - \ell^2\omega^2\right)\frac{r^2}{\ell^2} + \mathcal{O}\left(r^3\right) \\
g_{t\varphi} &= a\omega r^2 + \mathcal{O}\left(r^3\right) \\
g_{rr} &= 1 + \mathcal{O}\left(r^2\right) \quad g_{rt} = \mathcal{O}\left(r^2\right) \quad g_{r\varphi} = \mathcal{O}\left(r^2\right)
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$$g_{tt} = -\tfrac{1}{4}\,a^2 r^2 + \tfrac{1}{2}\ell^2 a^2 + \mathcal{O}\left(\tfrac{1}{r}\right)$$

$$g_{\varphi\varphi} = \left(\gamma^2 - \ell^2\omega^2\right)\frac{r^2}{4\ell^2} + \tfrac{1}{2}\left(\gamma^2 + \ell^2\omega^2\right) + \mathcal{O}\left(\tfrac{1}{r}\right)$$

$$g_{t\varphi} = \tfrac{1}{4}\,a\omega r^2 - \tfrac{1}{2}\,a\omega\ell^2 + \mathcal{O}\left(\tfrac{1}{r}\right)$$

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Boundary conditions in Chern–Simons formulation

$$A^\pm = b_\pm^{-1}\big(\,\mathrm{d}+\mathfrak{a}^\pm\big)b_\pm$$

with fixed $\mathfrak{sl}_2$ group element

$$b_\pm = \exp\Big(\pm\frac{r}{2\ell}\left(L_1 - L_{-1}\right)\Big)$$

and 1-form ($\mathcal{J}^\pm = \gamma/\ell \pm \omega$)

$$\mathfrak{a}^\pm = L_0\left(\pm\mathcal{J}^\pm\,\mathrm{d}\varphi - a\,\mathrm{d}t\right) \qquad \delta\mathcal{J}^\pm \neq 0 \quad \delta a = 0$$

Consequences of our near horizon boundary conditions

To reduce clutter consider henceforth constant Rindler acceleration, $a = \text{const.}$

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Map

$$P_0 = J_0^+ + J_0^- \qquad P_n = \frac{i}{kn}\left(J_{-n}^+ + J_{-n}^-\right) \text{ if } n \neq 0 \qquad X_n = J_n^+ - J_n^-$$

yields Heisenberg algebra (with Casimirs $X_0,\, P_0$)

$$[X_n,\, X_m] = [P_n,\, P_m] = [X_0,\, P_n] = [P_0,\, X_n] = 0$$
$$[X_n,\, P_m] = i\delta_{n,m} \quad \text{if } n \neq 0$$

Map explains word “Heisenberg” in title and provides first punchline

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- For real J_0 all states in theory regular and have horizon

Whole spectrum (subject to reality) compatible with regularity!

Could be used as defining property of our bc's

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  Near horizon Hamiltonian defined as diffeo charge generated by unit translations ∂_v in (advanced) time direction

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- Consequence: soft hair!

$$H|\psi\rangle = E|\psi\rangle \quad \Rightarrow \quad H|\tilde{\psi}\rangle = E|\tilde{\psi}\rangle$$

where state $\tilde{\psi}$ is state ψ dressed arbitrarily with soft hair

$$|\tilde{\psi}\rangle = \prod_{n_i^\pm \in \mathbb{Z}^+} J^+_{n_i^+} J^-_{n_i^-} |\psi\rangle$$

Explains word "soft hair" in title

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- Entropy formula remarkably simple

$$S = 2\pi \left(J_0^{+} + J_0^{-}\right) = T^{-1}\, H$$

also remarkably universal:
generalizes to flat space, higher spins, higher derivatives!

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- Relations to asymptotic Virasoro charges $L^{\pm}$ and sources $\mu^{\pm}$

$$L \sim J^2 + J' \qquad \mu' - \mu J \sim a$$

Twisted Sugawra construction emerges! (yields Brown–Henneaux c)

Outline

Motivation

Problems (and possible resolutions)

Near horizon boundary conditions and soft hair

Proposal for semi-classical BTZ microstates

Outlook

Assumptions

For technical details see Afshar, DG, Sheikh-Jabbari, Yavartanoo '17

1. Central charges quantized in integers

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   Needed due to relations like

$$\mathcal{J}_{cn} \sim \mathcal{W}_n^0$$

   Note non-local relation

$$\mathcal{W} \sim e^{-2\int \mathcal{J}}$$

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Note twisted periodicity conditions
$$\mathcal{W}^\nu(\varphi + 2\pi) = e^{-2\pi\nu i}\,\mathcal{W}^\nu(\varphi)$$

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   Justifiable through explicit stringy construction in D1-D5 system

   Maldacena, Maoz '00; Lunin, Maldacena, Maoz '02

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   Identify states in Hilbert space $\mathcal{H}_{\text{BTZ}}$ as (composite) states in $\mathcal{H}_{\text{CG}}$
   $$\sum_p \mathcal{J}_{nc-p}\mathcal{J}_p \sim \sum_p J_{n-p}J_p + incJ_n$$

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   Justification 2: gives nice result

List of all semi-classical BTZ black hole microstates

- Given a BTZ black hole with mass M and angular momentum J (as measured by asymptotic observer) define parameters

$$\Delta_{\pm} = \frac{1}{2}\left(\ell M \pm J\right) = \frac{c}{6}\left(J_0^{\pm}\right)^2$$

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- Full set of semi-classical BTZ black hole microstates given by

$$|\mathcal{B}(\{n_i^\pm\}); J_0^\pm\rangle = \prod_{\{n_i^\pm\}}\left(\mathcal{J}^+_{-n_i^+}\mathcal{J}^-_{-n_i^-}\right)|0\rangle$$

BTZ black hole entropy from counting all semi-classical microstates

We proposed (after some Bohr-type semi-classical quantization conditions) explicit set of BTZ black hole microstates

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$$S = \ln N = \ln p(c\Delta^{+}) + \ln p(c\Delta^{-})$$

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Now let us count these microstates (not "Cardyology" but "Hardyology")

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- Entropy given by Boltzmann's formula

$$S = \ln N = \ln p(c\Delta^{+}) + \ln p(c\Delta^{-})$$

- Solved long ago by Hardy, Ramanujan; asymptotic formula (large N):

$$\ln p(N) = 2\pi\sqrt{N/6} - \ln N + \mathcal{O}(1)$$

BTZ black hole entropy from counting all semi-classical microstates

We proposed (after some Bohr-type semi-classical quantization conditions) explicit set of BTZ black hole microstates

Now let us count these microstates (not "Cardyology" but "Hardyology")

- Straightforward combinatorial problem: partition of integers $p(c\Delta^\pm)$
- Entropy given by Boltzmann's formula

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- Solved long ago by Hardy, Ramanujan; asymptotic formula (large N):

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- Our final result for semi-classical BTZ black hole entropy is

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Outline

Motivation

Problems (and possible resolutions)

Near horizon boundary conditions and soft hair

Proposal for semi-classical BTZ microstates

Outlook

Summary, loose ends and generalizations

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- Soft resolution of information loss problem?

  Neglecting soft gravitons generates information loss Carney, Chaurette, Neuenfeld, Semenoff '17

  Conjectured resolution of information loss problem: include soft gravitons

  Strominger '17

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Thanks for your attention!