Art and Geometry in 2D

From Wallpapers to Black Holes

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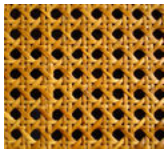
Outline

Outline

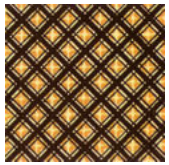
Wallpaper patterns are culturally universal



Viennise cane, Austria

Thomb, Thebes, Egypt

Bathroom linoleum, U.S.



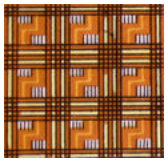
Cloth, Otaheite, Tahiti

Floor tiling, Prague, Czech

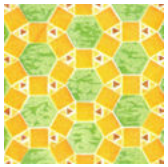
Metalwork, India

Ornament, Persia

Renaissance earthenware

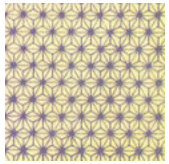


Cloth, Sandwich Islands



Byzantine marble pavement

Street, Zakopane, Poland



Painted porcelain, China

Soffitt of Arch, Alhambra, Spain

Bronze vessel, Nimroud, Assyria

Wood fence, contemporary, Europe/U.S.

Why are these patterns so universal?

Possible explanations:

- Exchange between cultures?
- Common esthetic principles among all humans?
- Simple deeper structure responsible for universality?

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Reason for universality

- Wall paper patterns are mathematical patterns
- Mathematics is universal

Note: *Description* of Mathematics is non-universal

$$1 + 3 = 4 \leftrightarrow I + III = IV \leftrightarrow 1 + 11 = 100 \leftrightarrow \bullet + \bullet\bullet\bullet = \bullet\bullet\bullet\bullet$$

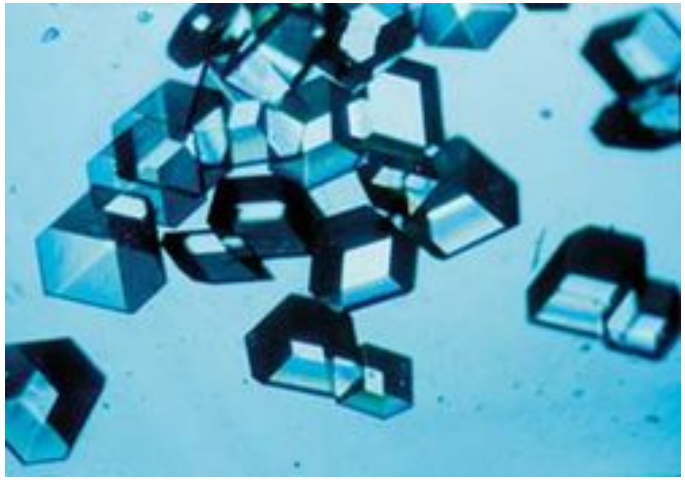
Important observation

Art and Nature exhibit the same kind of universality

Mathematics describes the underlying structure

Wallpaper groups

Giant's Causeway in Ireland



Insulin crystals

- Wallpaper patterns emerge from translations, rotations and reflections
- Same patterns arise e.g. in crystals
- Categorized by group theory: 17 different wallpaper patterns

Wallpaper groups

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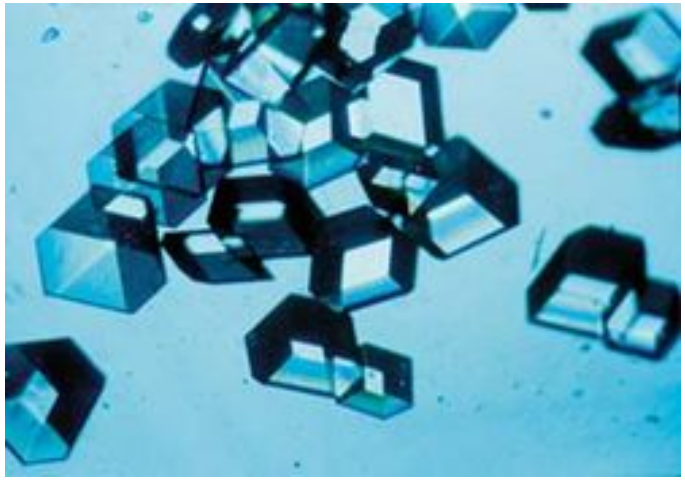

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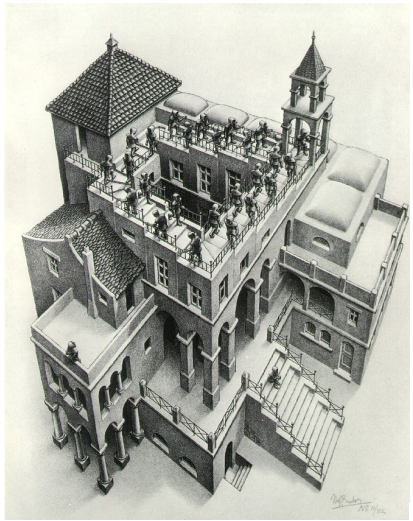
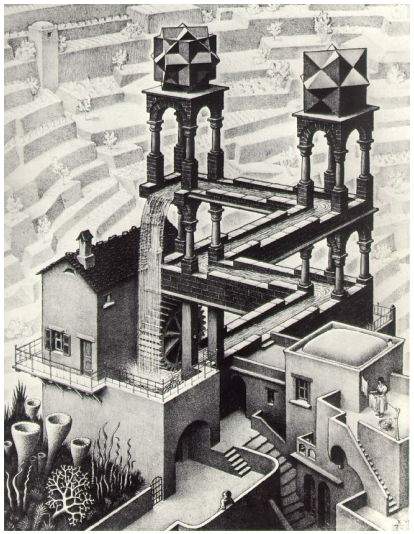
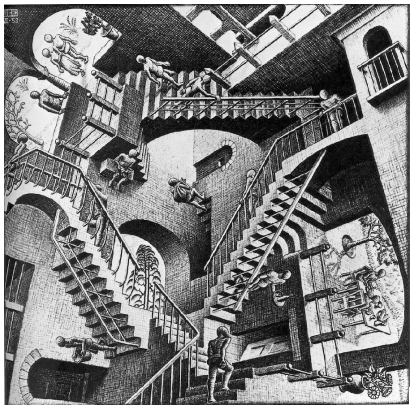
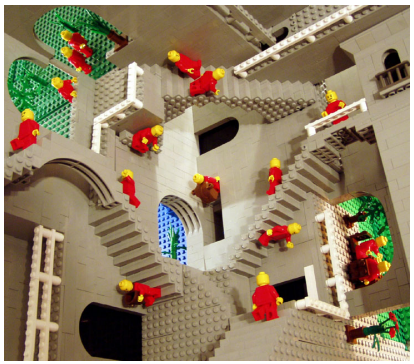
Well-known examples





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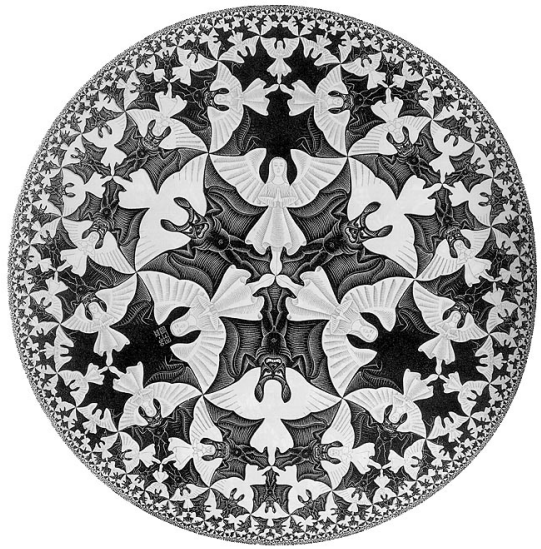
Hyperbolic wallpaper



Infinitely many repetitions of the basic patterns in finite area

Possible in non-Euclidean (hyperbolic) geometry!

Note: Standard intuition may fail (straight lines, parallels) in non-Euclidean geometry!

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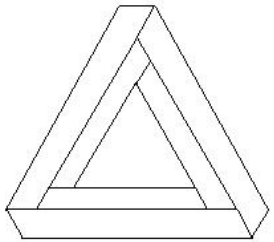
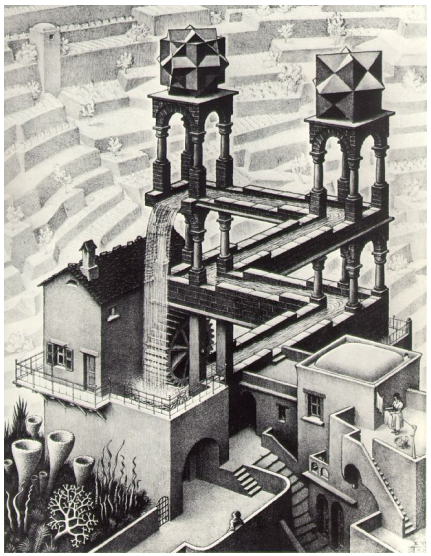
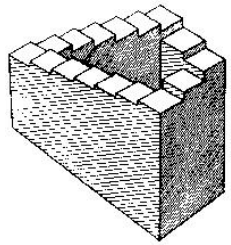
Outline

Two examples

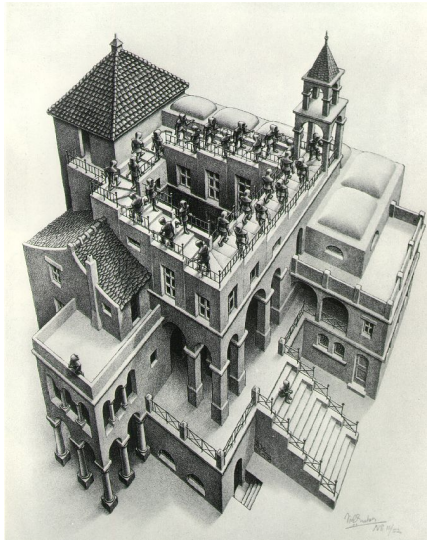


Penrose triangle





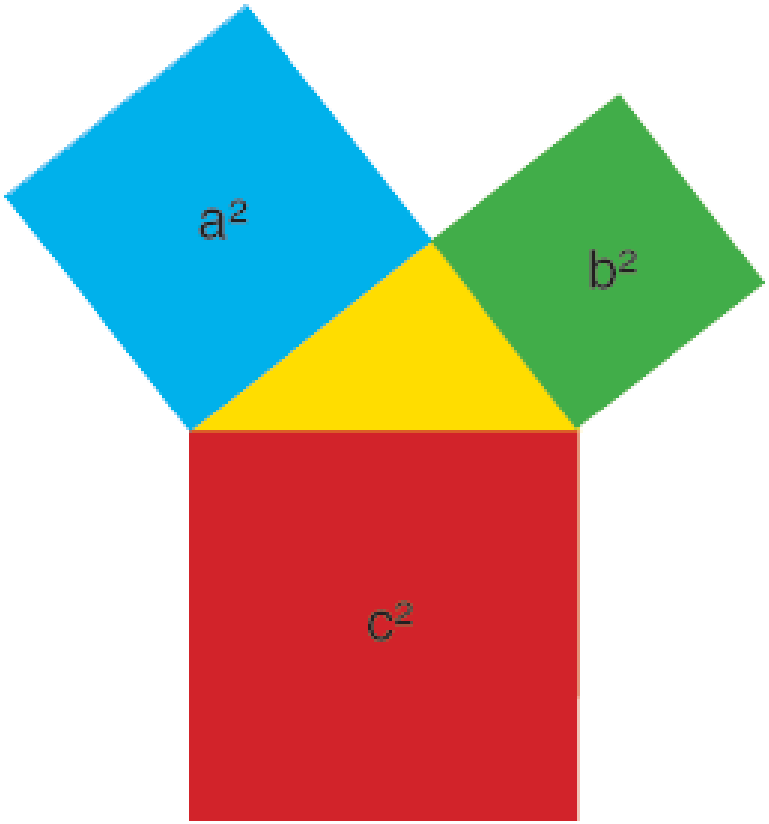
Penrose stair

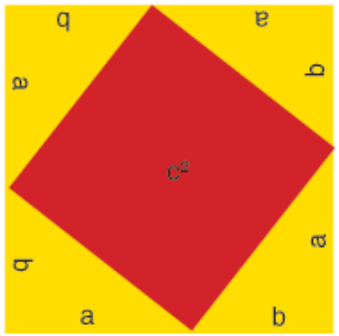


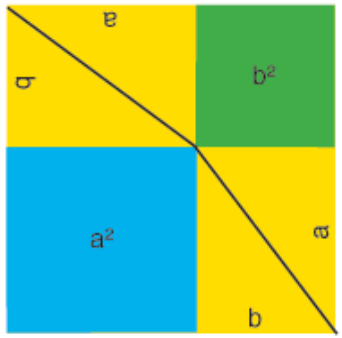
Euclidean geometry

Back to Pythagoras

$$a^2 + b^2 = c^2 \tag{1}$$







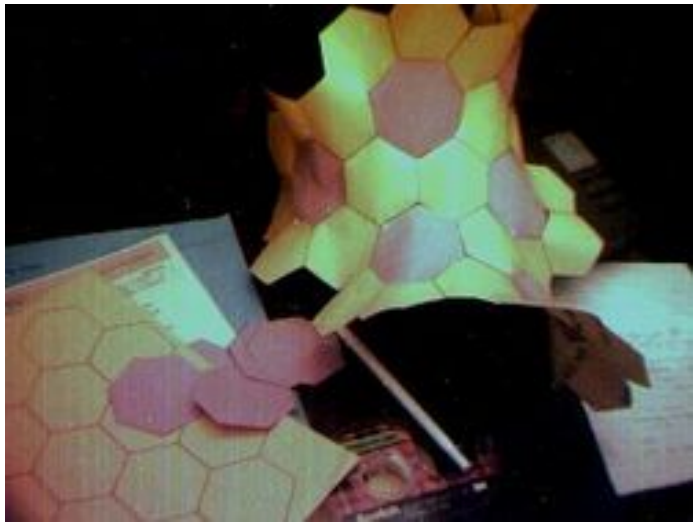
Graphical proof of (1)
Note: Very popular theorem!
More than 300 different proofs are known!

Hyperbolic geometry

Lines of zero length (a.k.a. light-rays)

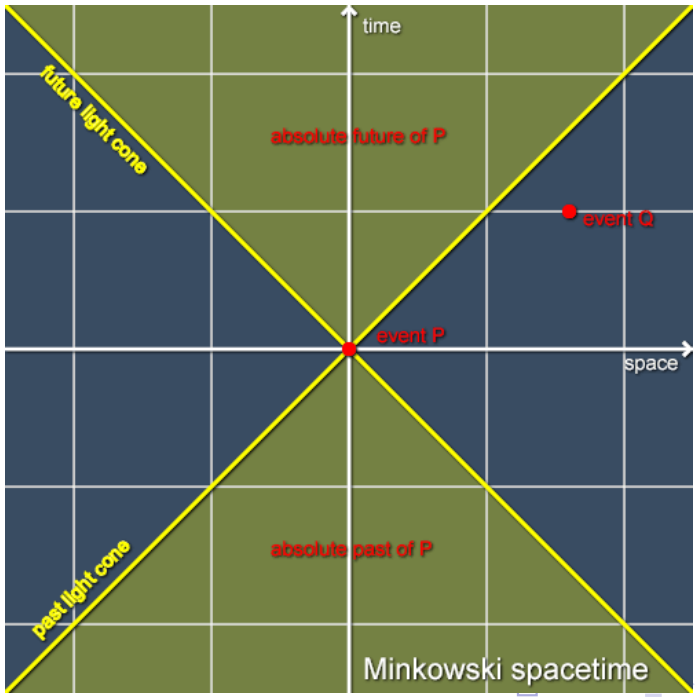
$$a^2 - b^2 = c^2 \tag{2}$$

Consequence: Lines of zero length exist which are *not* just points – very counter-intuitive!



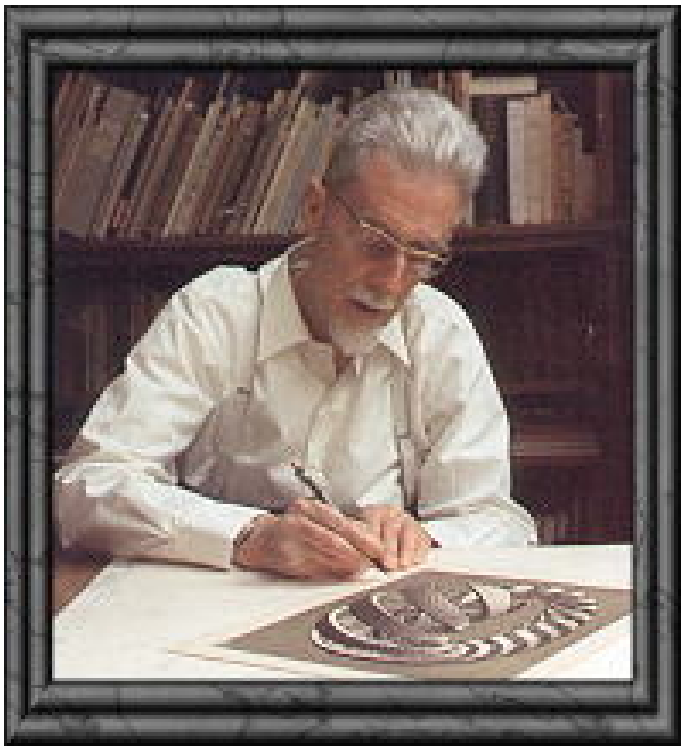
Hyperbolic Soccerball – count the hexagons

and compare with a real Soccerball!

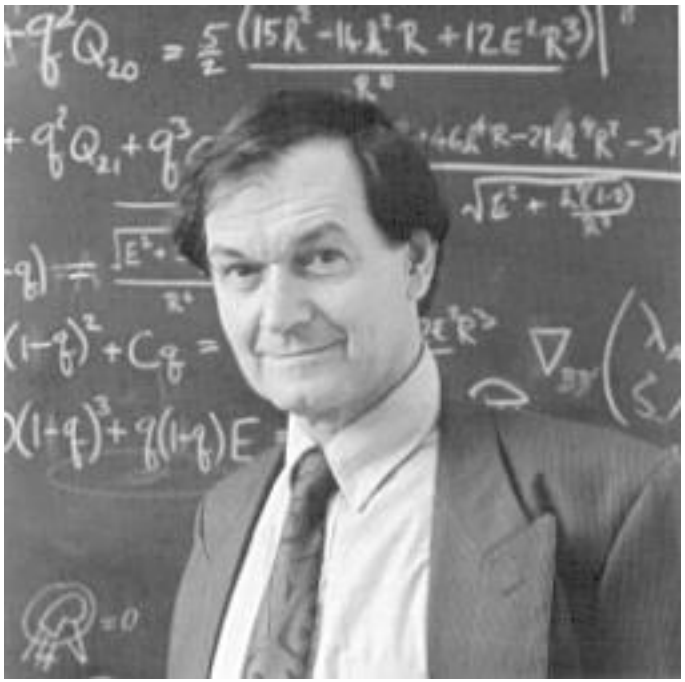


Escher and Penrose

Visualizers of hyperbolic geometry with Art and Mathematics



M.C. Escher, 1898–1972



R. Penrose, 1931–????

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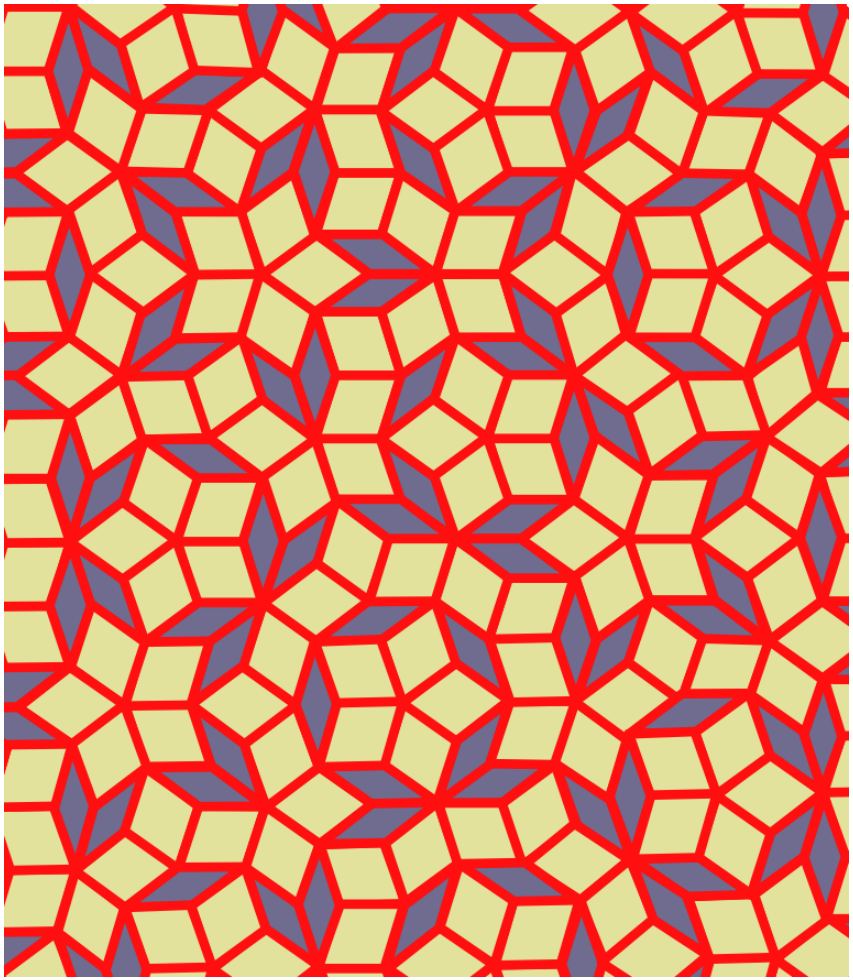
Addendum: Penrose tilings

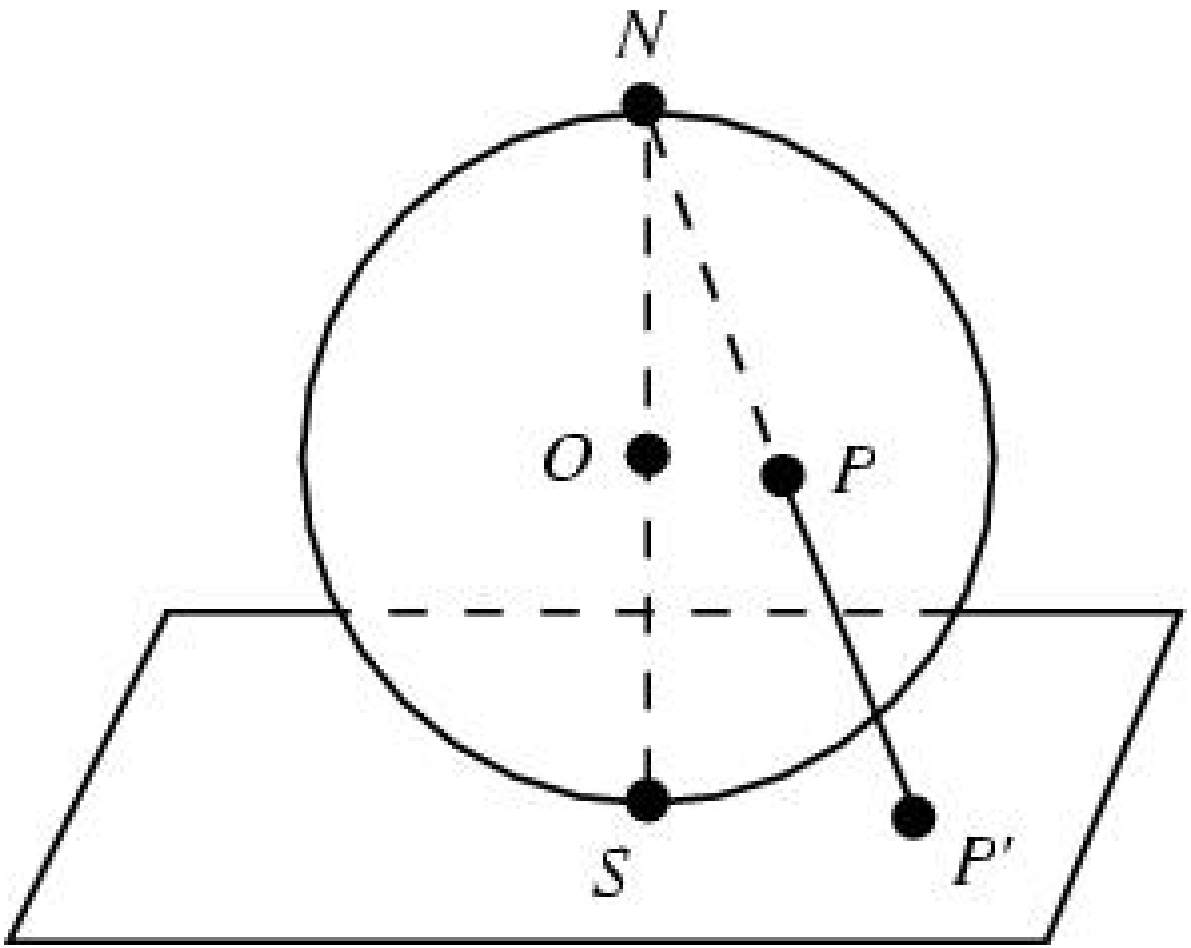


- Discovered 1973
- Penrose tilings are *not* standard wallpaper patterns
- No translational invariance!
- Nature: Quasi-crystals
- Art: Escher died before discovery

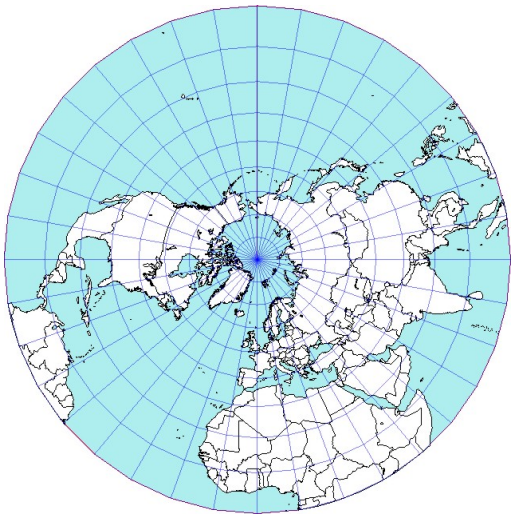
Outline

Stereographic projection

Moving infinity to the North Pole



Stereographic projection preserves angles, but not distances!



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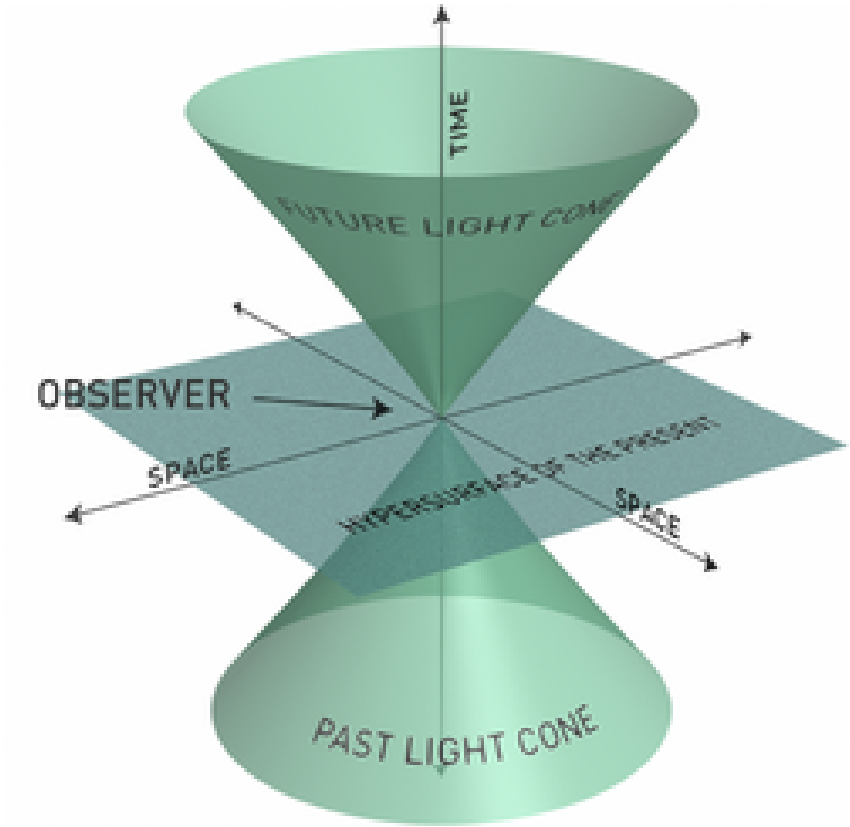
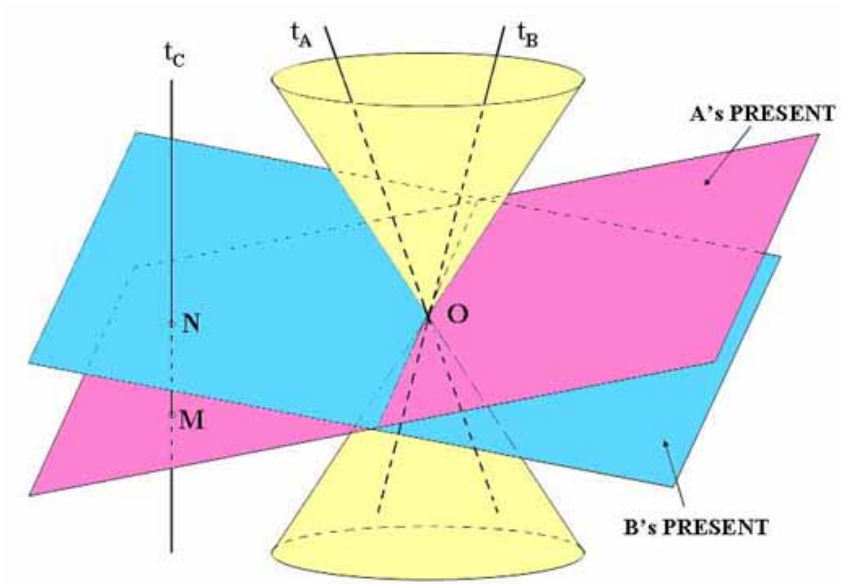
Minkowskian geometry: Hyperbolic Spacetime in 4D

Nothing moves faster than light!





A: event M simultaneous with O

B: event N simultaneous with O

Relativity!

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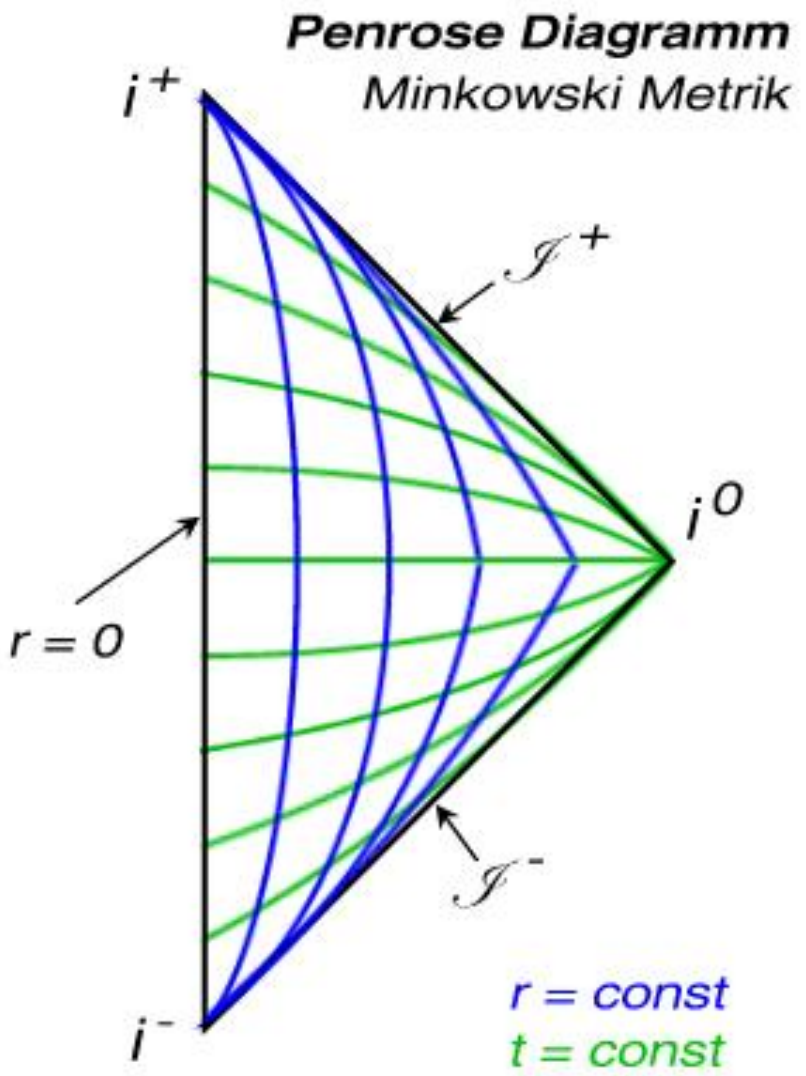
Conformal compactification

Causal structure of space-time



- Diagram: Minkowski (flat)
- Hyperbolic analogue of stereographic projection!
- Infinity not just North Pole (=point) but “celestial sphere” (=lightcone at ∞)
- Angels preserved means causal structure is preserved
- Distances are not preserved

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Outline

Fishy analogy

Above: Black Hole
(Artist's impression)
Left: Waterfall

Analogy:
Infinity $\leftrightarrow$ Lake

Horizon $\leftrightarrow$ Point of no return

Singularity $\leftrightarrow$ Waterfall

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Collapsing Stars

Black Holes exist in Nature

- Stars (like our Sun) will eventually burn out
- Collapse into dense object
- Depending on initial mass: White Dwarf, Neutron Star or...
- Black Hole!
- Observable through interactions with matter
- Black Holes come in various sizes

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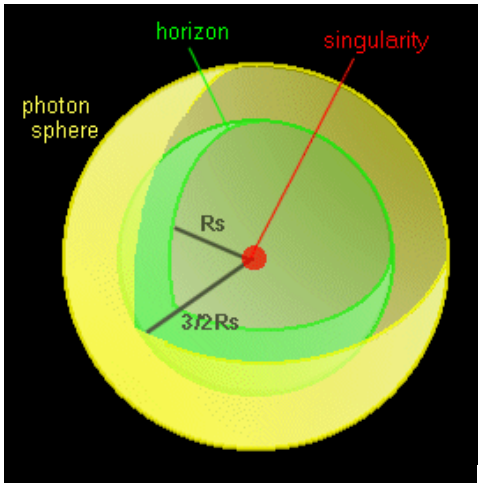
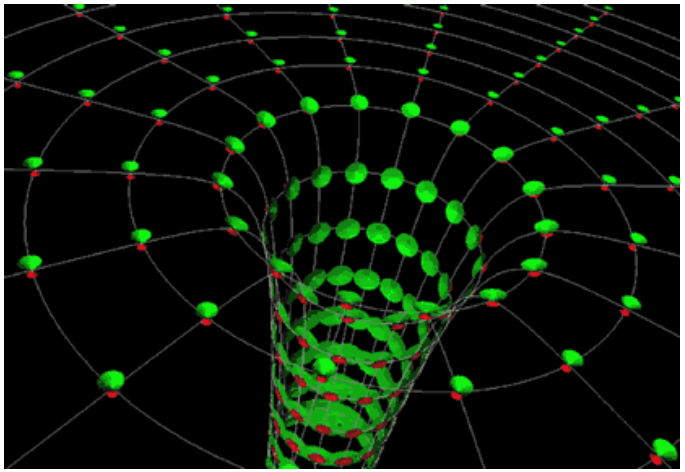
Outline

Why just two dimensions?

Schwarzschild Black Hole





- Spherical symmetry reduces 4D to 2D!
- 2D: Time and surface radius
- Exact solution of Einstein equations: Schwarzschild
  (General: Einstein equations determine geometry from matter
  Schwarzschild: no matter – (unique) vacuum solution!)

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Schwarzschild Black Hole

Visualisation of Black Holes needed!

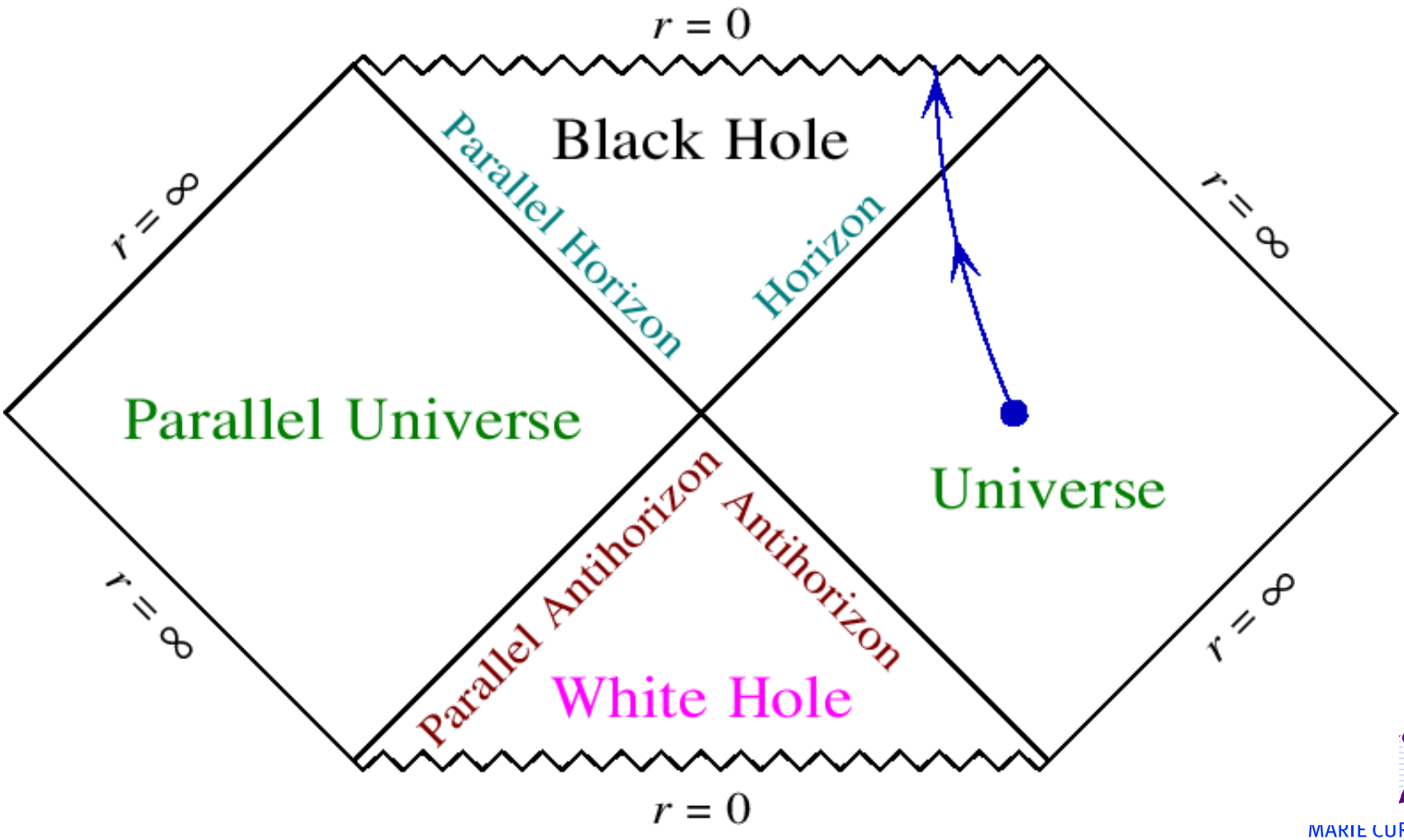
Schwarzschild metric:

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\Omega^2_{S^2}$$

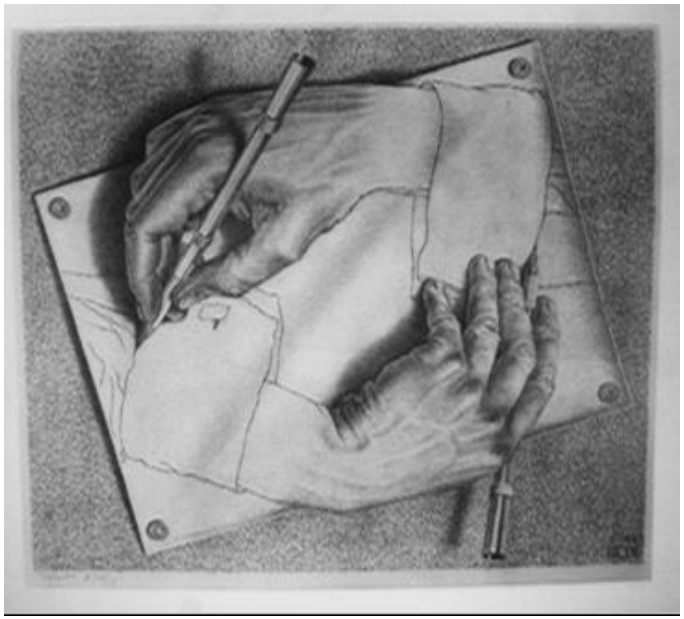
- Constant M: total mass
- Coordinate r: surface radius
- Coordinate t: time (note staticity)
- Last term: 2-sphere
- Relevant term $1 - \frac{2M}{r}$
- Asymptotic region: $r \to \infty$ (“far from waterfall”)
- Event horizon: $r = 2M$ (“point of no return”)
- Singularity: $r = 0$ (“waterfall”)

Visualisation of Black Holes

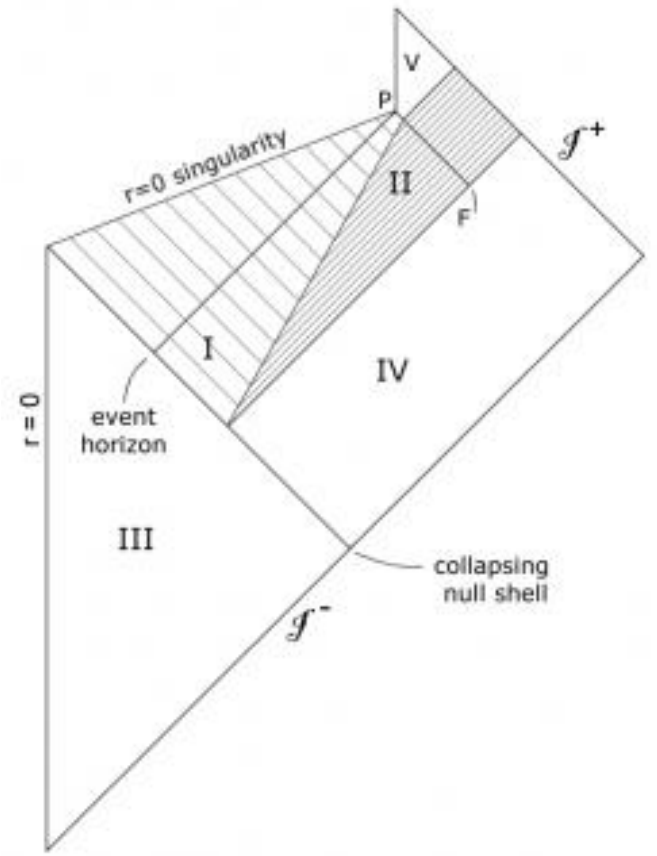
Quantum gravity?



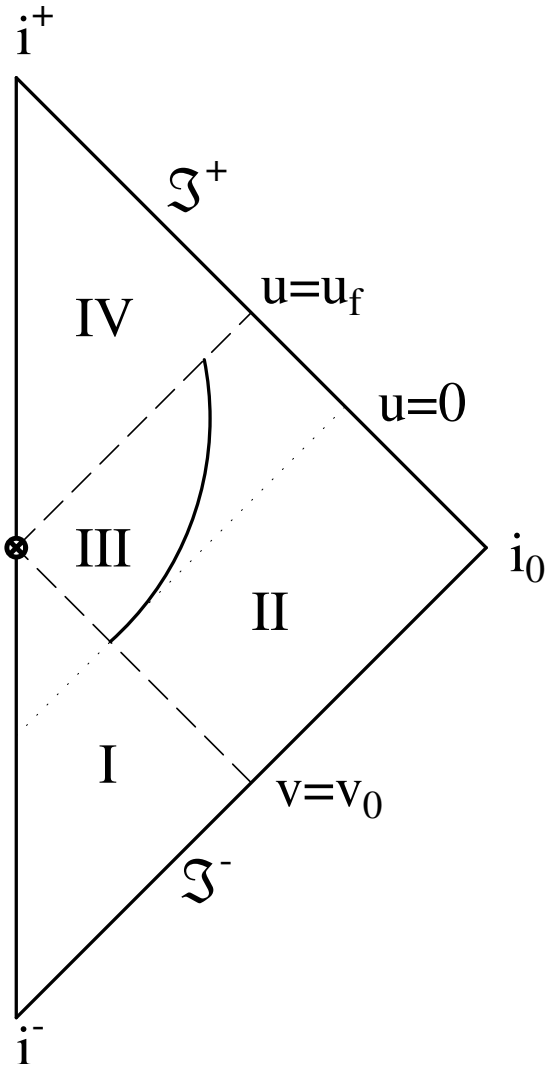
Quantum gravity
Information loss problem



- Quantum Theory: no information loss!
- Gravity: information loss!
- Incompatible?



Visulization of a possible solution



- Main question: What is the global structure of an evaporating Black Hole?
- Hakwing (1970ies – 2004): Previous picture (Information lost)
- Hakwing (since 2004): No information loss (but no picture)
- Suggestion: solution looks as depicted

Artistic challenge

Find better method of visualization!

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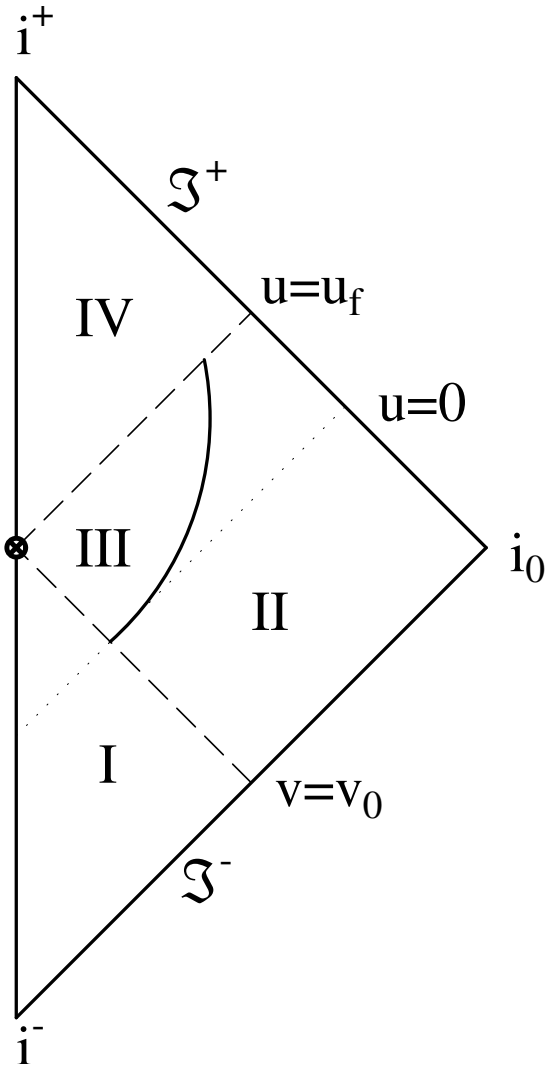

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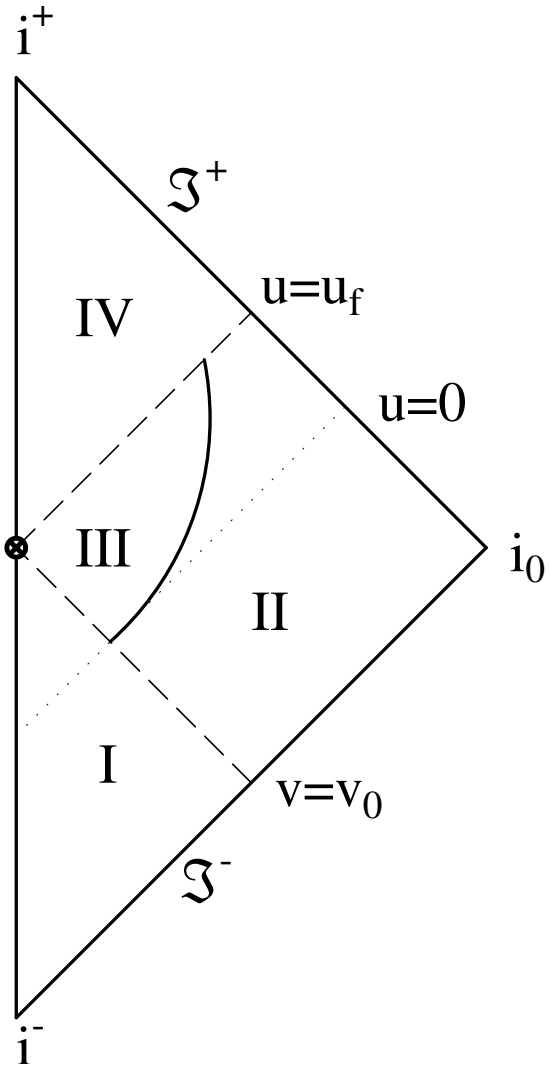

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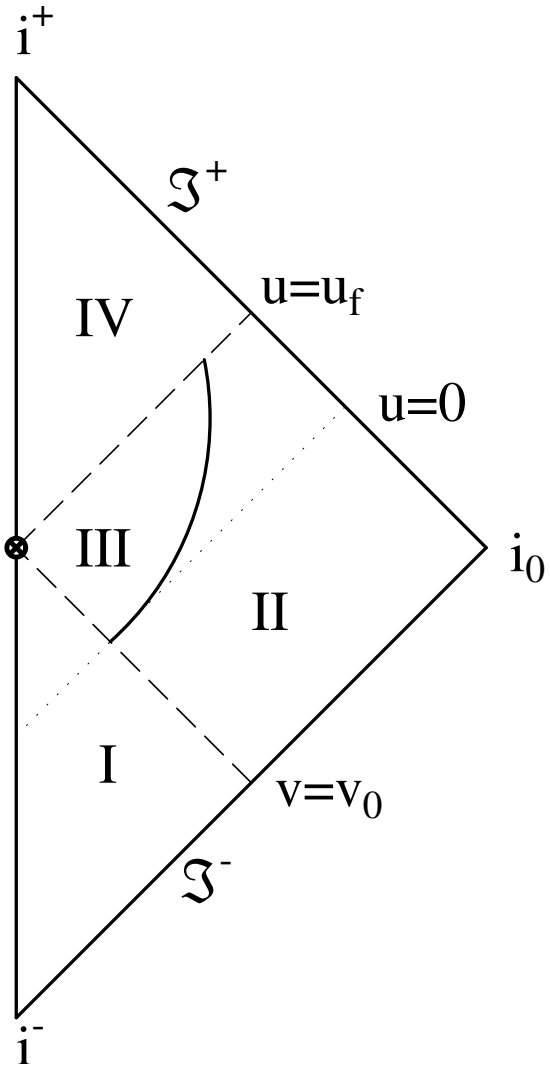

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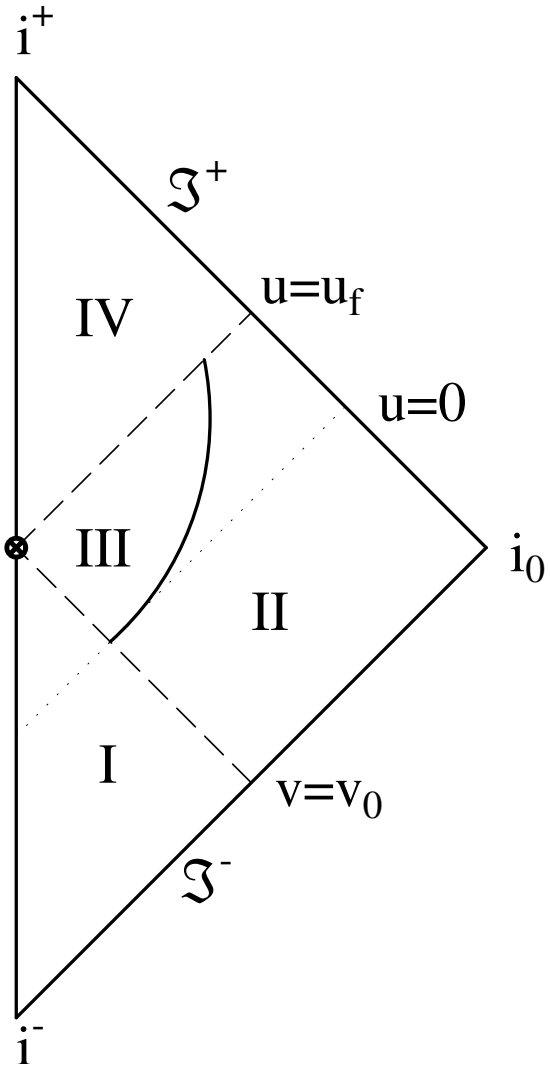

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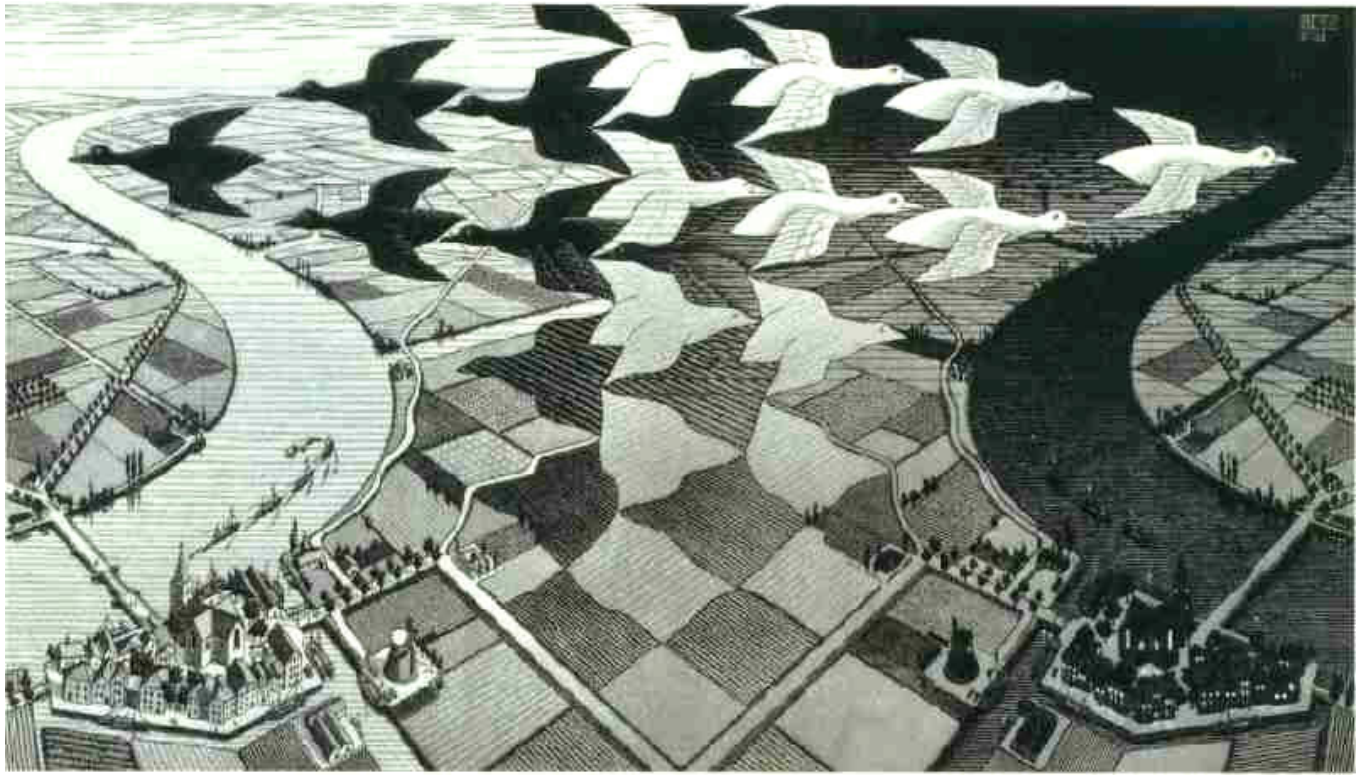
Main message

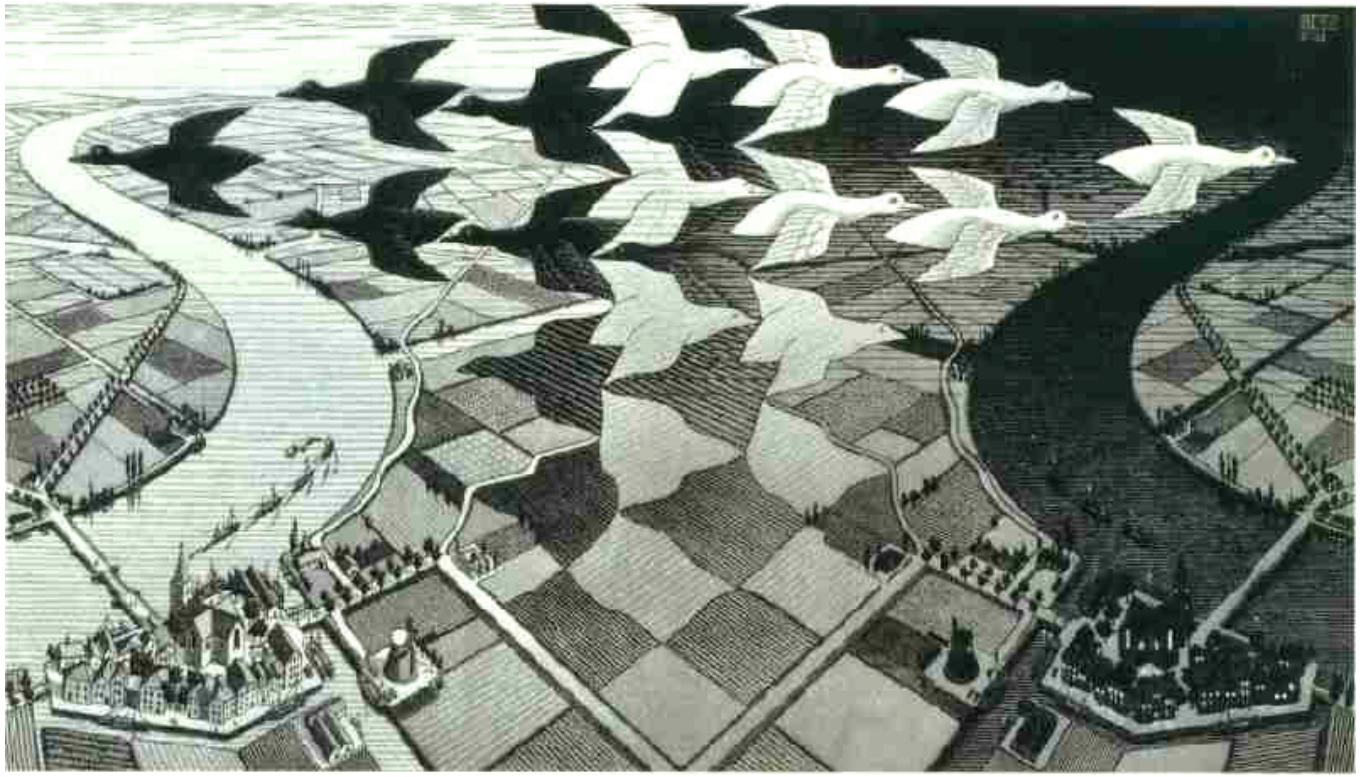


Art and Technique: No contradiction!

Etymology: Greek *tekhnikos* actually means

Art, skill, craft, method, system

Main message



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