Black Holes in Nuclear and Particle Physics

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Outline

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Black Holes in Gravitational Physics

Black Holes in Non-Gravitational Physics

Case Study: Holographic Renormalization

Black Holes: A Brief History of Quotes

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- S. Hughes (2008): "Most physicists and astrophysicists accept the hypothesis that the most massive, compact objects seen in many astrophysical systems are described by the black hole solutions of general relativity."

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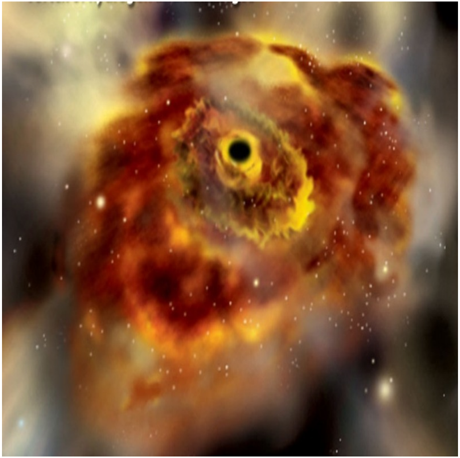
Do they exist?

Let me answer this without getting philosophical, by appealing to data

Some Black Hole Observational Data

OJ287, about 18 billion solar masses



Artistic impression (NASA Outreach), presented at the Annual meeting of the American Astronomical Society, 2008

- Microscopic BHs: none
- Primordial BHs: none (upper bound)
- Stellar mass BHs in binary systems: many (**17 good candidates (including Cygnus X-1)**, 37 other candidates)
- Isolated stellar mass BHs: some (**1 good candidate**, 3 other candidates)
- Intermediate mass BHs: some (11 candidates)
- Galactic core BHs: many (**Milky Way**, 66 other candidates)

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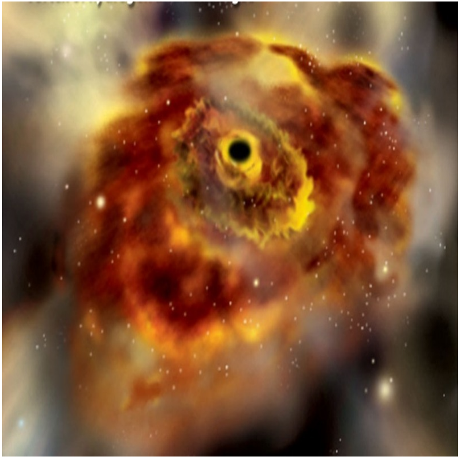

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Black holes are the simplest explanation of data! Thus, by Occam's razor they exist.

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Black Holes in Science Fiction

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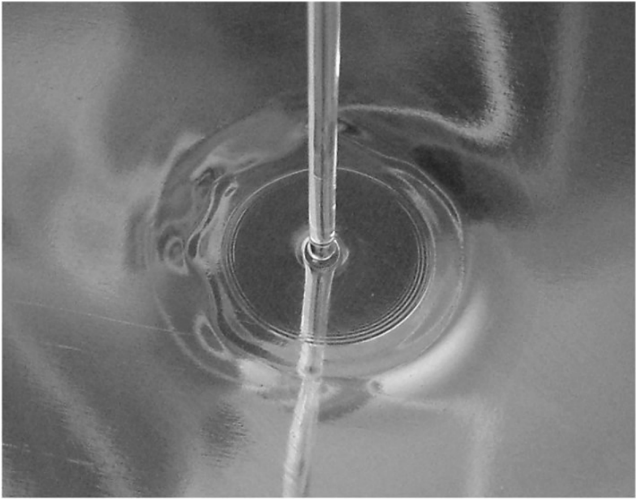
Hydraulic jump as a white hole analog



Picture by Piotr Pieranski, taken from a paper by G. Volovik

Some literature:

- C. Barcelo, S. Liberati, M. Visser, `gr-qc/0505065`
- G. Volovik, `gr-qc/0612134`
- T. Philbin et al, `arXiv:0711.4796`
- M. Visser, S. Weinfurtner, `arXiv:0712.0427`

Condensed Matter Analogs

Deaf and Dumb holes (W. Unruh 1981), Part II: Formulas

Idea: Linearize perturbations in continuity equation

$$\partial_t \rho + \nabla \cdot (\rho v) = 0$$

and Euler equation

$$\rho\big(\partial_t v + (v \cdot \nabla) v\big) = -\nabla p$$

and assume no vorticity, $v = -\nabla\phi$, and barotropic equation of state

$$\nabla h = \frac{1}{\rho} \nabla p$$

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Then the velocity-potential ϕ obeys the relativistic (!) wave-equation

$$\Box\phi = \frac{1}{\sqrt{-g}}\partial_\mu\big(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\big) = 0$$

with the acoustic metric

$$g_{\mu\nu}(t,x) = \frac{\rho}{c}\begin{pmatrix} -(c^2 - v^2) & -v^T \\ -v & \mathbb{I} \end{pmatrix}$$

where the speed of sound is given by $c^{-2} = \partial\rho/\partial p$.

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Deaf and Dumb holes (W. Unruh 1981), Part III: Reality Check

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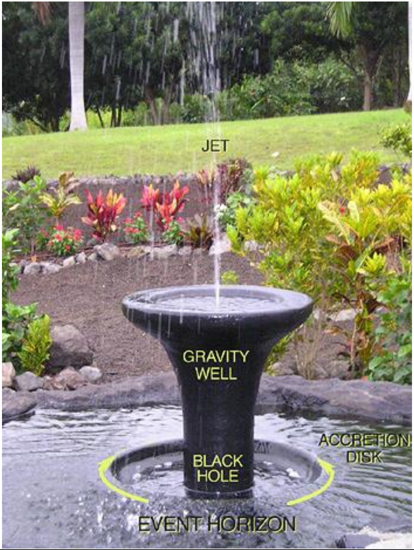

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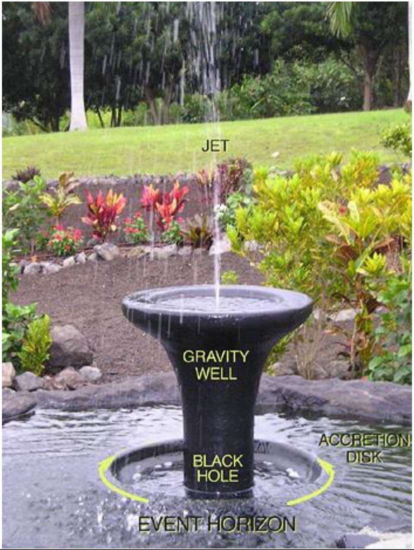

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- Black hole analogs are very useful for pedagogic demonstrations

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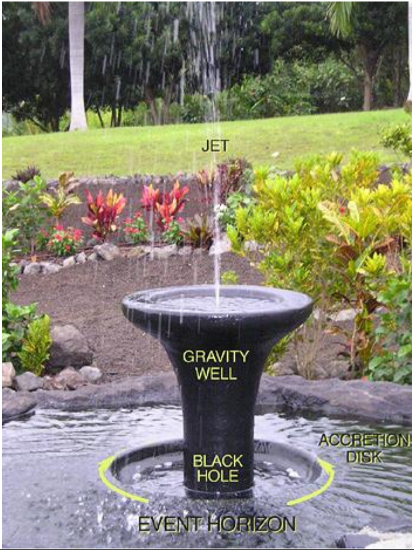

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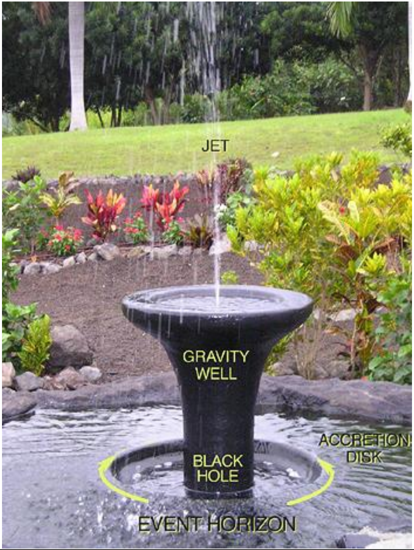

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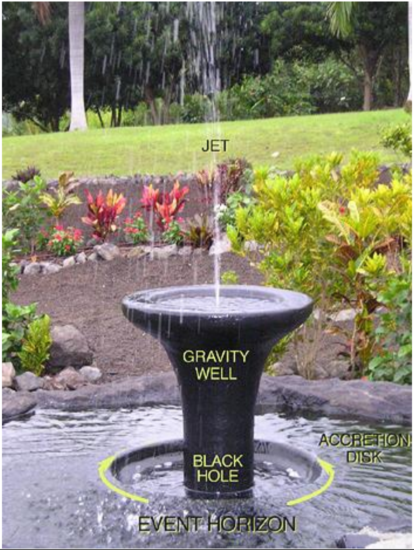

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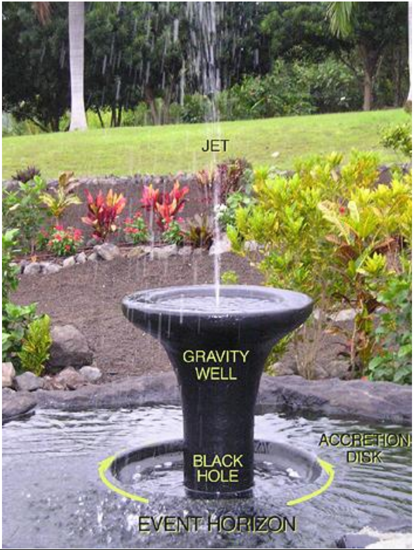

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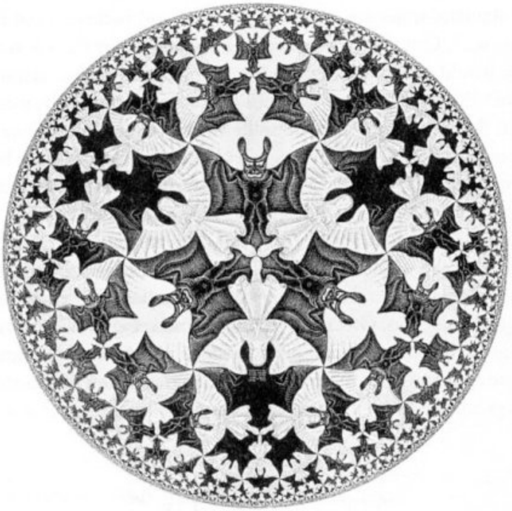
Q: Are there other unexpected applications of black holes?

AdS/CFT Correspondence!

Relates strings on AdS to specific gauge theories at the boundary of AdS



Open Universe Looking from inside, boundary at infinity
Limit Circle IV, by M. C. Escher

Applications

- RHIC physics
- Black hole physics
- Scattering at strong coupling
- Cold atoms
- Superconductors
- Quantum gravity
- ... to be discovered!

Note: J. Maldacena's paper `hep-th/9711200` second most cited paper ever (SPIRES). First is Steven Weinberg's "A Model of Leptons".

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AdS_2

... the simplest gravity model where the need for holographic renormalization arises!

Bulk action:

$$I_B = -\frac{1}{2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{g} \left[X \left(R + \frac{2}{\ell^2} \right) \right]$$

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There is an important catch, however: Boundary terms tricky!

Boundary terms, Part I

Gibbons–Hawking–York boundary terms: quantum mechanical toy model

Let us start with an bulk Hamiltonian action

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As expected $I_E = \int_{t_i}^{t_f} [p\dot{q} - H(q,p)]$ is standard Hamiltonian action

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Gibbons–Hawking–York boundary terms in gravity — something still missing!

That was easy! In gravity the result is

$$I_{GHY} = -\int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{\gamma}\, X\, K$$

where γ (K) is determinant (trace) of first (second) fundamental form. Euclidean action with correct boundary value problem is

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$\delta I_E \neq 0$ for some variations that preserve boundary conditions!!!

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Holographic renormalization: quantum mechanical toy model

Key observation: Dirichlet boundary problem not changed under

$$I_E \to \Gamma = I_E - I_{CT} = I_{EH} + I_{GHY} - I_{CT}$$

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Works if $S(q,t)$ is Hamilton's principal function!

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Holographic renormalization in AdS_2 gravity

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In case of AdS_2 gravity this Ansatz yields

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Boundary terms, Part IV

Holographic renormalization in AdS_2 gravity

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$\delta\Gamma = 0$ for all variations that preserve the boundary conditions!

Thermodynamics of Black Holes as a Simple Application

Consider small perturbation around classical solution

$$I_E[g_{cl} + \delta g, X_{cl} + \delta X] = I_E[g_{cl}, X_{cl}] + \delta I_E + \dots$$

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Everything goes wrong with I_E!

In particular, do not get correct free energy $F = T I_E = -\infty$ or entropy

$$S = \infty$$

Thermodynamics of Black Holes as a Simple Application

Consider small perturbation around classical solution

$$\Gamma[g_{cl} + \delta g, X_{cl} + \delta X] = \Gamma[g_{cl}, X_{cl}] + \delta\Gamma + \dots$$

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Accessibility of the semi-classical approximation requires

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2. $\delta\Gamma[g_{cl}, X_{cl}; \delta g, \delta X] = 0 \quad \rightarrow \quad$ ok in AdS gravity!

Everything works with Γ!

In particular, do get correct free energy $F = TI_E = M - TS$ and entropy

$$S = 2\pi X\big|_{\text{horizon}} = \text{Area}/4$$

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Thank you for your attention!

... and thanks to my collaborators!

Some literature on AdS_2 holography

- D. Grumiller and R. McNees, "Thermodynamics of black holes in two (and higher) dimensions," JHEP **0704**, 074 [arXiv:hep-th/0703230].
- T. Hartman and A. Strominger, "Central charge for AdS_2 quantum gravity," [arXiv:0803.3621 [hep-th]].
- M. Alishahiha and F. Ardalan, "Central charge for 2D gravity on AdS_2 and AdS_2/CFT_1 correspondence," [arXiv:0805.1861 [hep-th]].
- R. Gupta and A. Sen, "AdS(3)/CFT(2) to AdS(2)/CFT(1)," [arXiv:0806.0053 [hep-th]].
- A. Castro, D. Grumiller, R. McNees and F. Larsen, "Holographic description of AdS_2 black holes," [arXiv:0810.xxxx [hep-th]].

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