Gravity in three dimensions and holography

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*supported by STAR

Outline

Holography: An Introduction

3D gravity

Towards $AdS_3/LCFT_2$

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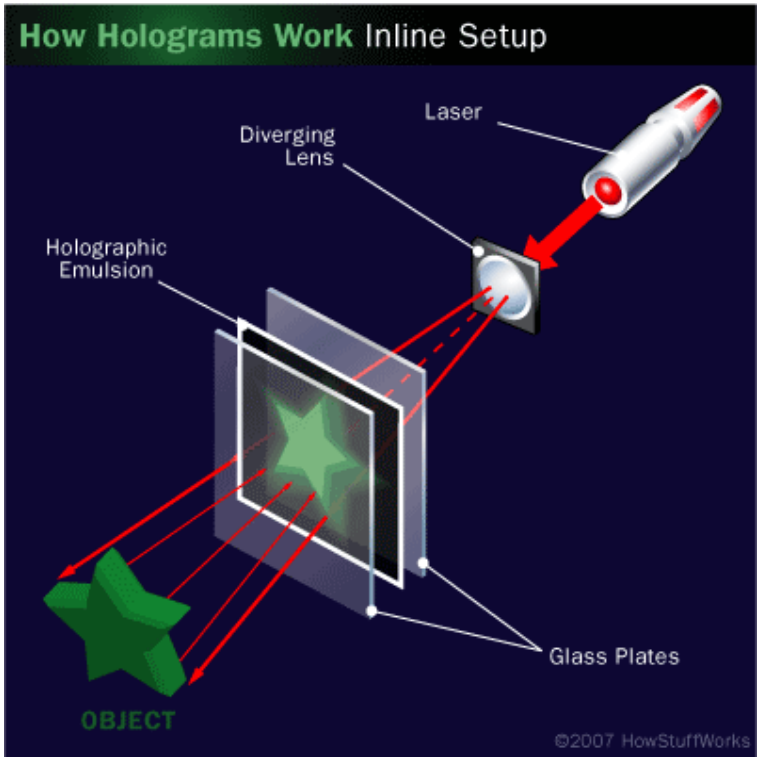
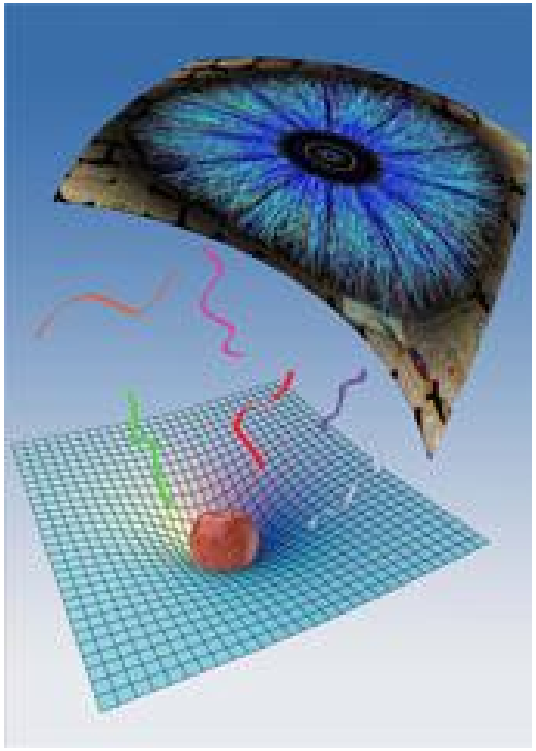
Holography – Main Idea

aka gauge/gravity duality, aka AdS/CFT correspondence





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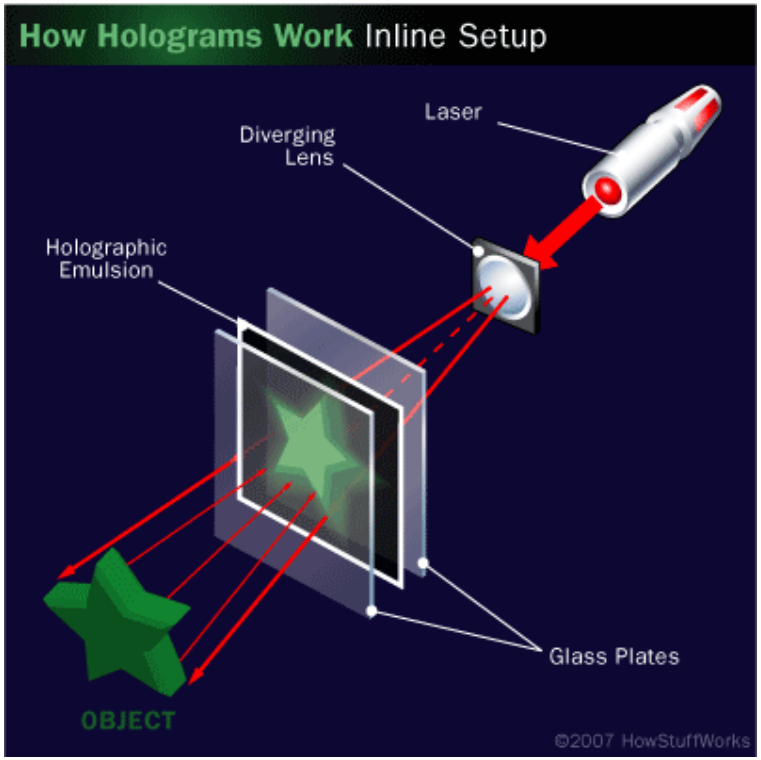

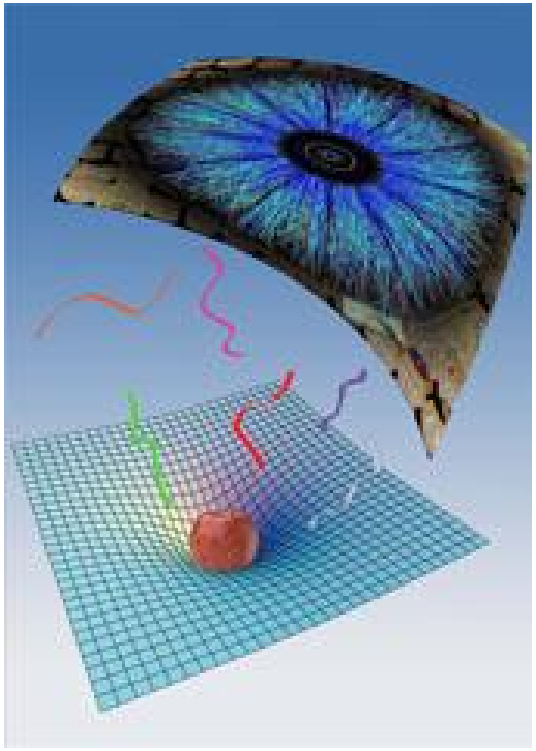

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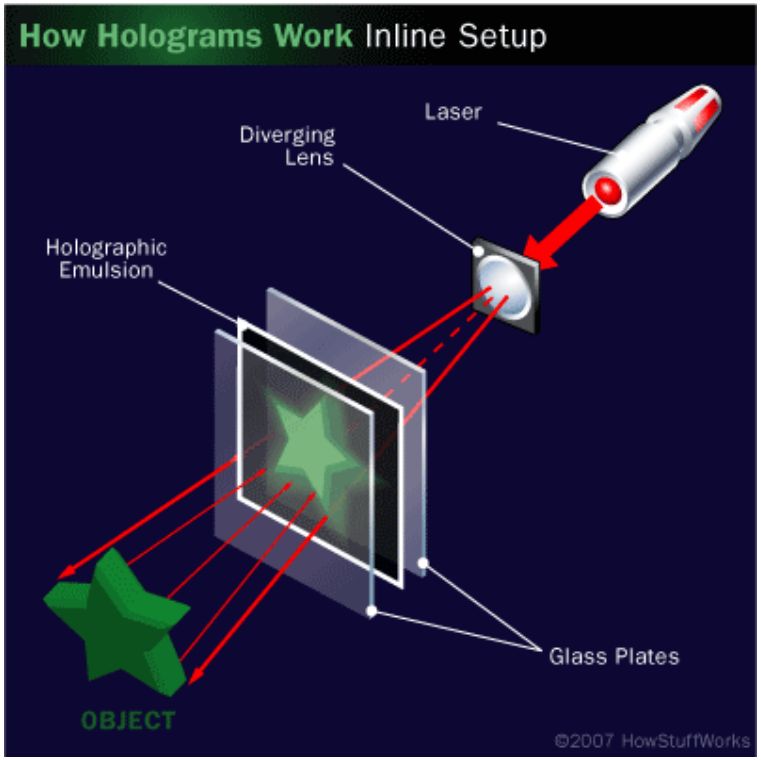

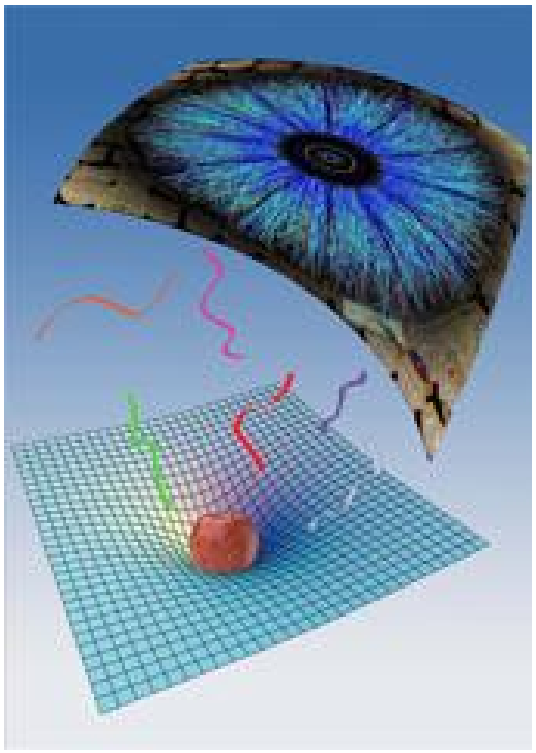

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- The number of dimensions is a matter of perspective
- We can choose to describe the same physical situation using two different formulations in two different dimensions
- The formulation in higher dimensions is a theory with gravity
- The formulation in lower dimensions is a theory without gravity

Why Gravity?

The holographic principle in black hole physics

Boltzmann/Planck: entropy of photon gas in d spatial dimensions

$$S_{\text{gauge}} \propto \text{volume} \propto L^d$$

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$$\text{e.g.} \quad \langle T_{\mu\nu} \rangle_{\text{gauge}} = T^{BY}_{\mu\nu} \qquad \delta\text{action} = \int \mathrm{d}^d x \sqrt{|h|}\, T^{BY}_{\mu\nu}\, \delta h^{\mu\nu}$$

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We can expect many new applications in the next decade!

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Why gravity in three dimensions?

"As simple as possible, but not simpler"

Gravity simpler in lower dimensions

$11D$: 1144 Weyl, 66 Ricci, $5D$: 35 Weyl, 15 Ricci, $4D$: 10 Weyl, 10 Ricci
$3D$: no Weyl, 6 Ricci, $2D$: no Weyl, 1 Ricci

2D gravity: black holes!

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Applications:

- Solve conceptual problems of (quantum) gravity
- Approximate geometry of cosmic strings/particles confined in plane
- Holographic tool for 2D condensed matter systems

pioneering work by Deser, Jackiw and Templeton in 1980ies
2007 Witten rekindled interest in 3D gravity

Cosmological topologically massive gravity (CTMG)

Action!

$$I_{\text{CTMG}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x \sqrt{-g} \left[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\, \varepsilon^{\lambda\mu\nu}\, \Gamma^{\rho}{}_{\lambda\sigma} \left(\partial_\mu \Gamma^{\sigma}{}_{\nu\rho} + \frac{2}{3}\, \Gamma^{\sigma}{}_{\mu\tau} \Gamma^{\tau}{}_{\nu\rho} \right) \right]$$

Equations of motion:

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Properties of CTMG

- Gravitons (topologically massive spin 2 excitations)
- Black holes (BTZ)
- Asymptotically anti-deSitter solutions (AdS/CFT!?)
- Higher derivative terms (third derivatives in EOM)
- Parity violating Chern–Simons term
- Related theory 2009: new massive gravity

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AdS/CFT – but which CFT?

Chiral versus logarithmic

Pre-cursor of AdS/CFT: Brown–Henneaux 1986

3D quantum gravity on AdS dual to 2D CFT with $c_L = c_R = 3/2G_N$

Constant time slice of EAdS_3

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- Characterized by central charges
- $c_L = c_R$ in Einstein gravity
- $c_L \neq c_R$ in CTMG
- $c_L = (1 - 1/\mu\ell)\, 3/2G_N$
- Chiral point: $\mu\ell = 1$
- At chiral point $c_L = 0$

A tempting conjecture

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CFT dual to CTMG exists and is chiral

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Another tempting conjecture

Observation (E: energy, J: angular momentum):

$$(E+J)\begin{pmatrix} \log \\ \text{left} \end{pmatrix} = \begin{pmatrix} 2 & \frac{1}{2} \\ 0 & 2 \end{pmatrix}\begin{pmatrix} \log \\ \text{left} \end{pmatrix},$$

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- Until recently unknown which of these alternatives is realized!

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Results:

$$\langle \mathrm{right}(z,\bar{z})\,\mathrm{right}(0)\rangle = \frac{c_R}{2\bar{z}^4} \tag{1}$$

$$\langle \mathrm{left}(z,\bar{z})\,\log(0)\rangle = -\frac{b}{2z^4} \tag{2}$$

$$\langle \log(z,\bar{z})\,\log(0)\rangle = \frac{2b\,\ln\left(m^2|z|^2\right)}{z^4} \tag{3}$$

These are precisely the 2-point correlators of a logarithmic CFT!

3-point correlators also consistent with logarithmic CFT conjecture

Conclusion

- Cosmological topologically massive gravity at the chiral point is an intersting gravitational theory in three dimensions
- Its dual CFT was conjectured to be logarithmic in work with Niklas Johansson 2008
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  Exciting possibility: gravity duals to strongly coupled logarithmic CFTs in condensed matter physics
  Examples: turbulence, critical polymers, percolation, disordered systems, sandpile model, quantum Hall effect, ...

It seems we have uncovered yet-another interesting chapter in the epic AdS/CFT saga...

Literature

- W. Li, W. Song and A. Strominger, [arXiv:0801.4566].
- D. Grumiller and N. Johansson, [arXiv:0805.2610].
- A. Maloney, W. Song and A. Strominger, [arXiv:0903.4573].
- K. Skenderis, M. Taylor and B. van Rees, [arXiv:0906.4926].
- D. Grumiller and I. Sachs, "$AdS_3/LCFT_2$ Correlators in Cosmological Topologically Massive Gravity [arXiv:09xx.xxxx].

Thank you for your attention!

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