Daniel Grumiller November 13$^{\text{th}}$ 2018

Gravity and holography in lower dimensions I

(7.1) **Equivalence of first and second order formulations**
Show that the second order action of dilaton gravity, Eq. (17) in chapter 5 of the lecture notes, leads to a theory that in the bulk is classically equivalent to the one descending from the first order action, Eq. (18).

(7.2) **Schwarzschild–Tangherlini and generalized Birkhoff**
Spherically reducing General Relativity from D dimensions to a two-dimensional dilaton gravity action (17) yields the potentials given in (20) [all cross-refs. refer to chapter 5 in the lecture notes]. Derive for any spacetime dimension $D > 3$ all classical solutions in a basic Eddington–Finkelstein patch. Prove that all solutions have a Killing vector that asymptotically is time-like.

(7.3) **Asymptotically AdS$_2$**
Consider two-dimensional metrics of the asymptotic form

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 - \big(e^{2\rho/\ell} + \gamma^{(2)}(t) + \mathcal{O}(e^{-2\rho/\ell})\big)\,\mathrm{d}t^2$$

and determine all asymptotic Killing vectors (if not exact expressions then at least their leading order contributions in a large ρ expansion), assuming $\delta\gamma^{(2)} \neq 0$.

Derive which of them preserve the asymptotic form of the dilaton for

(a) constant dilaton boundary conditions (bc's), $X = X_0 = \text{const.}$
(b) linear dilaton bc's, $X = e^{\rho/\ell} + \mathcal{O}(e^{-\rho/\ell})$
(c) generalized linear dilaton bc's, $X = \mathcal{O}(e^{\rho/\ell}) + \mathcal{O}(e^{-\rho/\ell})$.

These exercises are due on November 20$^{\text{th}}$ 2018.

Hints:

- If $U(X) = 0$ the exercise is rather trivial, so assume $U(X) \neq 0$. It is best to start with the first order action (18) and useful to split the dualized connection 1-form into a Levi–Civitá part and a torsion dependent part, $\omega = \omega_{\text{LC}} + \omega_T$ with $\omega_{\text{LC}} = e_a * \mathrm{d}e^a$. Then vary the first order action with respect ω_T (which in path integral slang means you have integrated out a variable appearing linearly in the Lagrangian, namely ω_T, which yields a functional δ-function that can be used to integrate out the auxiliary field X^a) and use this equation of motion to find an expression for X^a in terms of $*(e^a \wedge \mathrm{d}X)$. Then use the fact that you have already integrated out ω_T and that ω_{LC} by definition obeys $\mathrm{d}e^a + \epsilon^a{}_b\, \omega_{\text{LC}} \wedge e^b = 0$, so that the remaining terms in the action consist solely of $X\, \mathrm{d}\omega_{\text{LC}}$ (which in second order form is proportional to $XR\sqrt{-g}\,\mathrm{d}^2x$) and of $-\epsilon\,(V(X) + \frac{1}{2}\,X^aX_aU(X))$ (where X^aX_a through the relation you established is proportional to $(\nabla X)^2$). This provides already the final answer.

  Note: The procedure outlined in section 2.2 of `hep-th/0204253` also works, but in my opinion is unnecessarily contrived.

- You can use all the results of section 5.5.1 — if you just insert into (49) the exercise is rather short. To bring your line-element into a standard form you may need to exploit the freedom to choose the integration constant in the definition of the function $Q = \int U$, which allows to rescale e^Q by an arbitrary factor.

- Find all asymptotic Killing vectors ξ for the given class of metrics, along the lines of chapter 2 of the lecture notes. Note that you could determine exact expressions, but it is sufficient to determine the components ξ^t and ξ^ρ to leading order in a large ρ expansion (in the latter case this is already the exact result). Then verify for each of the three cases which of the Killing vectors preserve the asymptotic form of the dilaton field, $\mathcal{L}_\xi X = \mathcal{O}(\delta X)$, where δX denotes the allowed fluctuations of the dilaton. Note that in case (a) no such fluctuations are allowed, in case (b) fluctuations are allowed to order $\mathcal{O}(e^{-\rho/\ell})$ and in case (c) to order $\mathcal{O}(e^{\rho/\ell})$.

  Historical note: generalized linear dilaton bc's to the best of my knowledge were first introduced, though not fully understood, in `1311.7413`. The next step was `1509.08486`, where these bc's were discussed in more depth for the charged JT model but still not fully understood. Triggered by the excitement about the SYK model, see `1604.07818`, `1606.01857` and references therein, the generalized linear dilaton bc's were finally understood in the second order formulation in `1708.08471` and in the first order formulation in `1802.01562`.