Daniel Grumiller October 30^{th} 2018

Gravity and holography in lower dimensions I

(5.1) **AdS**

Does the following set of $\mathrm{sl}(2,\mathbb{R})$ connections (with generators L_n obeying $[L_n, L_m] = (n-m)L_{n+m}$) lead to locally AdS_3 solutions of Einstein gravity? How does the metric look like?

$$A^{\pm} = b^{\mp 1}\big[\,\mathrm{d} - \big(\kappa\,\mathrm{d}t \mp J^{\pm}(\varphi)\;\mathrm{d}\varphi\big)\,L_0\big]\,b^{\pm 1} \qquad b = e^{\frac{r}{2\ell}\,(L_{+1}-L_{-1})}$$

Note: κ is some (state-independent) constant, while $J^{\pm}$ are (state-dependent) functions of $\varphi \sim \varphi + 2\pi$.

(5.2) **BTZ**

Do rotating BTZ black holes have an ergo region? Are you sure?

(5.3) **TMG**

Which solutions of Einstein gravity are solutions of TMG? Which solutions of TMG are solutions of Einstein gravity? Provide one example of a solution to TMG that is not a solution of Einstein gravity.

These exercises are due on November 7^{th} 2018.

Hints:

- If the gauge flatness conditions hold you know the solution must be locally AdS_3. If you prefer to use an explicit basis, here is a standard choice:

$$L_0 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad L_+ = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad L_- = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}$$

  If you are efficient and refer to appropriate parts of the lectures then it should take a single line to answer the first question. For the second question you just need to insert into the definition of the metric in terms of the Chern–Simons connections $A^\pm$ and use Baker–Campbell–Hausdorff (or just expand the exponentials) to get the r-dependence.

- There are two ways you could define an ergo region, hence the follow-up question of how sure you are (I am referring here to Schwarzschild-like coordinates when mentioning t and φ): 1. with respect to the asymptotic time variable t, checking for zero of the norm of the Killing vector ∂_t; 2. with respect to an arbitrary Killing vector $\partial_t + A\,\partial_\varphi$ (where A is some constant chosen such that this Killing vector remains timelike everyhwere outside the event horizon). The first case is straightforward and leads to a short answer. The second case is more interesting and leads to an even shorter answer (by one letter).

- Both answers are relatively straightforward, if you look at the relevant field equations. Regarding the example, there are numerous options. If you are completely lost, here is a suggestion: consider conformal gravity (which is a special case of TMG) and recall the defining property of its solutions. Then write down one such solution that has non-constant Ricci scalar, which means it cannot possibly be a solution of three-dimensional Einstein gravity. If for some reason you would like to know how the Ricci scalar behaves under Weyl rescalings have a look at equation (D.9) in Wald's book "General Relativity".