Daniel Grumiller October 23rd 2018

Gravity and holography in lower dimensions I

(4.1) **Curvature a la Cartan**
Take the metric

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r - \frac{r^2 - r_+^2}{\ell^2}\,\mathrm{d}u^2 + r^2\,\mathrm{d}\varphi^2\,,$$

construct a dreibein, solve for the spin-connection by imposing vanishing torsion and calculate the curvature 2-form. Which geometry is this locally? (And which geometry is it globally?)

(4.2) **Transformation of non-abelian curavture 2-form**
Take the non-abelian gauge curvature 2-form

$$F = \mathrm{d}A + A \wedge A$$

where A is some non-abelian gauge connection 1-form. Derive how F transforms under non-abelian gauge transformations

$$A \to \hat{A} = g^{-1}\,\mathrm{d}g + g^{-1}Ag$$

generated by some group element g. If $F = 0$ what does this imply for the transformed $\hat{F}$?

(4.3) **Einstein–Hilbert–Palatini from Chern–Simons**
Prove that the Chern–Simons action (11) with the gauge connection (14)-(18) in the lecture notes leads to the Einstein–Hilbert–Palatini action (5). What is the Chern–Simons level k in terms of Newton's constant G?

These exercises are due on October 30th 2018.

Hints:

- You could answer both questions almost without doing any calculations. However, for calculating the curvature 2-form you need to do some calculations, so it is best to start with dreibein, then solve torsion equals zero for the spin-connection and then insert into the definition of the curvature 2-form. If you know what you must get in advance you have a nice cross-check on possible mistakes. Part of the calculation is analogous to the example in the lecture notes, so you could use similar techniques, but in one dimension higher. Specifically, I suggest you use again the ansatz (34) in the lecture notes together with a suitable ansatz for the third dreibein, $e^3 = r\,\mathrm{d}\varphi$, with the Minkowski metric $\eta_{\pm\mp} = 1 = \eta_{33}$ (all other components of η_{ab} vanish). As a final hint: in case you did all the calculations but still have difficulties in answering the questions recall that a result like $R^{ab} \propto e^a \wedge e^b$ implies that you have a constant curvature spacetime. Alternatively, check how the metric looks in Schwarzschild-type of gauge and you should be able to deduce its local and global properties.

- The calculation is straightforward and should fit into less than a handful of lines. Do not forget to answer the question.

- Ditto (well, maybe two handful...)