Daniel Grumiller March 10^{th} 2021

Gravity and holography in lower dimensions II

(2.1) **Jordan block structure in $\mathbf{AdS_3/log\ CFT_2}$**
Prove that the Hamiltonian $H = L_0 + \bar{L}_0 = i\partial_t$ has a Jordan block structure in the CFT dual to critical topologically massive gravity when acting on the pair of logarithmic mode ($\psi^{\log}$) and left-moving quasi-primary (ψ^L) with SL(2) weights $(2,0)$. (notation alert: $L_0, \bar{L}_0$ are the Virasoro zero modes)

(2.2) **$\mathbf{QNEC_2}$ constraint on entanglement with kink**
Assume that entanglement entropy (expressed as a function of the size of the entangling interval) in a (relativistic, unitary) QFT_2 has a kink, meaning that it is continuous, but has at least one point where its first derivative jumps. Prove that the QNEC_2 inequality implies an inequality for the slopes on the left and right sides of the kink.

(2.3) **Domain walls and holographic RG flow**
Consider an AdS_3 domain wall metric

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + e^{2A(\rho)}\big(-\mathrm{d}t^2 + \mathrm{d}x^2\big)$$

that models an RG flow from the UV ($\rho \to \infty$) to the IR ($\rho \to -\infty$), in the sense that this geometry can be dual to the ground state of a QFT_2. Show that this metric has the expected Killing vectors and deduce conditions on the function $A(\rho)$ such that the domain wall really can describe an RG flow between CFT_2 fixed points in the UV and IR.

These exercises are due on April $\mathbf{13^{\text{th}}}$ 2021.

Hints/comments:

- If you exploit the result announced in the lectures, $\psi^{\log} = -2(it + \ln\cosh\rho)\,\psi^L$ and recall what it means for ψ^L to have SL(2) weights $(2, 0)$ then this exercise is not much longer than a 2×2 matrix. You should find (21) in `1302.0280`.

- Recall the QNEC$_2$ inequality

$$2\pi\,\langle T_{++}\rangle \geq \frac{\mathrm{d}^2 S(\lambda, L+\lambda)}{\mathrm{d}\lambda^2}\bigg|_{\lambda=0} + \frac{6}{c}\left(\frac{\mathrm{d}S(\lambda, L+\lambda)}{\mathrm{d}\lambda}\bigg|_{\lambda=0}\right)^2$$

  and make a suitable ansatz for EE to acommodate the kink. Before calculating, think what a kink, i.e., a term of the form $x\theta(x)$, will generate on the right hand side of QNEC, and you will be half there. Try to answer this question first without cheating, but if you must, feel free to have a look at `2007.10367`.

- For the first part ask yourself: which Killing vectors should you expect if this geometry is supposed to be dual to the ground state of a QFT$_2$? For the second part ask yourself: what do you know about monotonicity properties inherent to an RG flow and in particular about the relation between UV and IR values of the central charge? Then recall that at a fixed point the central charge is given by the Brown–Henneaux value, $c = 3\ell/(2G)$, where ℓ is the AdS radius and G Newton's constant and note that you can relate the AdS-radius to the function $A(\rho)$. There is not much you need to calculate.