Daniel Grumiller October 19^{th} 2020

Gravity and holography in lower dimensions I

(2.1) **Landau pole and dimensional transmutation.**
Take the QED β-function for the finestructure constant α

$$\beta(\alpha) = \frac{\alpha^2}{3\pi}$$

and, using the experimental input $\alpha(80GeV) \approx 1/128$, derive the UV energy scale where you have a Landau pole, i.e., a divergence of the coupling strength. Then take the QCD β-function for the strong 'finestructure constant' α_s

$$\beta(\alpha_s) = -\frac{7\alpha_s^2}{2\pi}$$

and, using the experimental input $\alpha_s(80GeV) \approx 0.11$, derive the IR energy scale where the QCD coupling constant blows up. Note: This feature is responsible for 'dimensional transmutation', because you start with a theory that is scale invariant in the UV, introduce an energy cutoff, and use the RG-flow captured by the β-function to derive a mass scale in the IR.

(2.2) **Marginal and relevant interaction terms in Thirring model.**
Take a QFT in two dimensions consisting of one fermion with kinetic term $i\bar{\psi}\partial\!\!\!/\psi$. Add all local interaction terms (without derivatives) that are either marginal or relevant. The resulting Lagrangean describes the Thirring model. What happens if you instead take a scalar field in two dimensions with kinetic term $(\partial\phi)^2$?

(2.3) **Conformal quantum mechanics**
Show that the action

$$I[Q] = \int \mathrm{d}t \left(\frac{1}{2}\,\dot{Q}^2 - \frac{g}{2Q^2}\right)$$

is invariant under

$$t \to t' = \frac{at+b}{ct+d} \qquad\qquad Q \to Q' = \frac{Q}{ct+d}$$

where $ad-bc=1$ and $a,b,c,d \in \mathbb{R}$. Identify for which values of a,b,c,d this transformation corresponds to a time translation, $t \to t+t_0$, and for which values it corresponds to a rescaling of time, $t \to \lambda t$.

These exercises are due on October 27$^{\text{th}}$ 2020.

Hints:

- Recall the definition of β as derivative of the coupling constant with respect to the logarithm of energy and solve the differential equation with the initial conditions given in the exercise to determine the approximate mass scales. Rounding errors can have a huge impact here, so do not worry if your result for the QCD scale is off by a factor of order unity from results in the literature. The energy scale of the Landau pole should be ridiculously high.

- Use the kinetic term and the fact that the action is dimensionless (since $\hbar = 1$) to deduce the length dimension of ψ, and then write down all the terms that are powers of $\bar{\psi}\psi$ and have either coupling constants that are dimensionless (these are the marginal terms) or have negative length dimension (a.k.a. positive mass dimension — these are the relevant terms). For the scalar field part of the exercise you should not find many marginal terms but a lot of relevant ones.

- Just insert the trafo into the action and recall that you are allowed to partially integrate and drop total derivative terms. It seems easier if you start with the action in terms of primed quantities and then insert to express it in terms of unprimed ones. Note that the Jacobian factor $\partial t'/\partial t$ simplifies using the $SL(2,\mathbb{R})$ condition $ad - bc = 1$. The last parts of the exercise are straightforward.