Daniel Grumiller October 22nd 2019

Black Holes I — Exercise sheet 2

(2.1) **Heavy nucleons and Black Earth**
In the lectures you have learned how to estimate the Chandrasekhar limit, and that the Earth is too light to collapse to a black hole. However, if nucleons were considerably more heavy, it would be possible for the Earth to collapse to a black hole. Calculate how heavy a nucleon has to be for this to happen. Discuss what would happen if nucleons were as heavy as the Planck mass.

(2.2) **Hydraulic jump as white hole analog**
A "hydraulic jump" is basically water falling down onto some surface. Consider a kitchen sink (for instance, *your* kitchen sink). Turn on the water as strong as possible, so that it falls onto a flat surface[1]. You should observe two clearly distinct regions with different depths of water. Observe what happens if you slowly reduce the water flow. Summarize your most relevant observations and explain in what sense the shallow region in the center is a white hole. If you want, consider the fishy Gedankenexperiment you heard in the lectures and transpose it into the outside region of the white hole analog.

(2.3) **Kepler problem revisited**
Consider Newtonian gravity for this example. Take some test-particle (say, a planet in the Sun's gravitational field or a satellite around the Earth) and consider motion around the central object. Write energy conservation in the form

$$\frac{\dot r^2}{2} + V^{\text{eff}} = E = \text{const.}$$

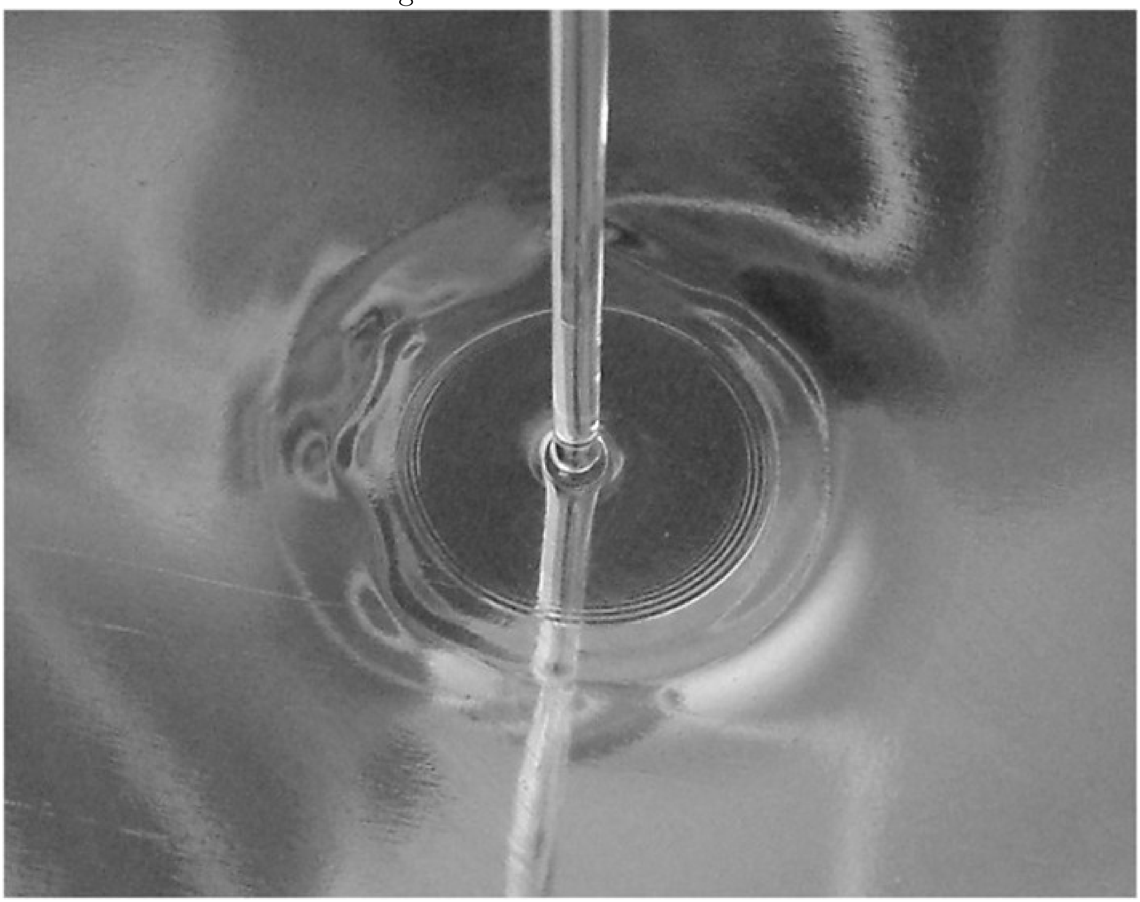
Here r is the radial distance of the test-particle from the central object, dot means time-derivative and E is a constant of motion (the energy per mass), which you may set to zero. What is V^{eff} in Newtonian gravity? What is the condition on V^{eff} for circular motion? Which constant of motion is hidden in V^{eff}? Are all values of r possible for circular orbits?

These exercises are due on November 5th 2019.

[1] If this is not possible in your sink try to find one where this is possible; the one in this seminar room works, for example.

Hints:

- If you use Planck units then the mass of the Earth is about $3 \cdot 10^{32}$ and the Planck mass is (by definition) 1. If you prefer pounds or kilogram please google up the corresponding values of masses.
- You should observe something like this:

  As to why the central region is analog to a white hole: try to excite shallow water waves in the outside region and send signals into the white hole region, without destroying the latter.
- You should obtain two terms in V^{eff}, one coming from gravitational attraction (so it comes with a minus sign) and one from the centrifugal barrier (which has a positive sign).