Daniel Grumiller October 1st 2019

Black Holes I — Exercise sheet 1

(1.1) **Newtonian black holes**
Describe mathematically what happens when you send a light-ray from the surface of a Newtonian black hole outward. By "Newtonian black hole" I mean a star whose escape velocity exceeds the speed of light, but that is nevertheless described by Newtonian physics [note: as far as we know such stars do not exist in Nature — so this exercise is a Gedankenexperiment]. Discuss in what sense this star really is "black", and how mass and radius have to be related.

(1.2) **The unbearable weakness of gravity**
Compare the Coulomb force with the gravitational force, by considering positronium (a meta-stable bound state of an electron and its anti-particle, the positron). More concretely, calculate the electrostatic force and the gravitational force between an electron and a positron, separated at a distance of an Ångstrom. How heavy would an electron have to be so that in the positronium system the Coulomb force equals to the Newtonian gravitational force? If you want, answer also the following questions: (why) is gravity negligible in particle physics? (why) are all other forces besides gravity negligible in the solar system?

(1.3) **Planck units**
Planck units (sometimes called "natural units") are physical units where for convenience various conversion factors introduced by humans are set to unity: $c = \hbar = G_N = k_B = 1$, where c is Einstein's constant (vacuum speed of light), $\hbar$ is Planck's constant, G_N is Newton's constant and k_B is Boltzmann's constant. Calculate the following quantities in Planck units: age of the Universe, radius of visible Universe, cosmological constant (see 2011 Nobel prize), mass of the Sun, mass of the proton, mass of a human cell, the current room temperature and, if you want, miscellaneous further quantities like your age, your mass, your height etc.
Compare the results with the same results expressed in some "unnatural units", e.g. the ones used on a ship, where time is measured in seconds, speed in knots, vertical lengths in meters, horizontal lengths in sea miles, and so on.

These exercises are due on October 22nd 2019.

Hints:

- For the first example you need Newton's gravity law and classical mechanics. It is a bit simpler (but not necessary) to work with energies.
- For the second example you need Newton's gravity and Coulomb's law.
- For the third example you need the values of various conversion factors. You can google them up or find them in most textbooks.
  Note: a key aspect of Planck units is that they simplify otherwise unnecessarily complicated formulas and allow to highlight their actual physical content. For example, the Hawking temperature of a Schwarzschild black hole is given by $T = 1/(8\pi M)$ in Planck units and by $T = \hbar c^3/(8\pi G_N M k_B)$ in SI units. In this case the physical key observation is that temperature is inverse to the black hole mass, which is a bit harder to spot with all the superfluous conversion factors attached in SI units. See the correct 33% (by M. Duff) in `physics/0110060`.