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Gravity and holography in lower dimensions I

(1.1) **Reminder of $d+1$ split of metric**

Take the (Euclidean) asymptotically AdS_3 metric

$$
\begin{aligned}
g_{rr} &= \frac{\ell^2}{r^2} + \mathcal{O}(1/r^4) & g_{rt} &= \mathcal{O}(1/r^3) \\
g_{tt} &= \frac{r^2}{\ell^2} + \mathcal{O}(1) & g_{r\varphi} &= \mathcal{O}(1/r^3) \\
g_{\varphi\varphi} &= r^2 + \mathcal{O}(1) & g_{t\varphi} &= \mathcal{O}(1) + O(1/r)
\end{aligned}
$$

in the limit of large radius $r \gg 1$ and calculate for a $r = \text{const.}$ hypersurface the normal vector n_μ, the induced volume form $\sqrt{\gamma}$ and the trace of extrinsic curvature K.

(1.2) **Reminder of variational principle**

Show that the (Euclidean) action

$$\Gamma = -\frac{1}{16\pi G}\int \mathrm{d}^3x\sqrt{g}\left(R + \frac{2}{\ell^2}\right) - \frac{1}{8\pi G}\int \mathrm{d}^2x\sqrt{\gamma}\left(\alpha K + \frac{\beta}{\ell}\right)$$

with the boundary conditions of exercise (1.1) has a well-defined variational principle only if $2\alpha = 1 - \beta$.

(1.3) **Dimension as parameter**

Discuss the possibilities for theoretical calculations that could emerge when considering General Relativity in D spacetime dimension, keeping D arbitrary (potentially allowing for analytic continuation to non-integer D). Focus then specifically on the Schwarzschild–Tangherlini solution (where $r_h = \text{const.}$ is the horizon radius)

$$\mathrm{d}s^2 = -\left(1 - \frac{r_h^{D-3}}{r^{D-3}}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1 - \frac{r_h^{D-3}}{r^{D-3}}} + r^2\,\mathrm{d}\Omega^2_{S^{D-2}}$$

and discuss some of its main features, in particular for $D \to \infty$, $D \to 3$ and $D \to 2$. (As usual, $\mathrm{d}\Omega^2_{S^{D-2}}$ denotes the line-element of the round $(D-2)$-sphere).

These exercises are due on October 9th 2018.

Hints/comments:

- You can look up various definitions in the lecture notes of Black Holes II, section 10.1. The results that you should get are

$$
\begin{aligned}
n_\mu &= \delta^r_\mu \, \frac{\ell}{r} + \mathcal{O}(1/r^3) \\
\sqrt{\gamma} &= \frac{r^2}{\ell} + \mathcal{O}(1) \\
K &= \frac{2}{\ell} + \mathcal{O}(1/r^2) \, .
\end{aligned}
$$

- Recall that "well-defined variational principle" is synonymous for "the first variation of the full action vanishes on-shell (including all boundary contributions) for all variations that preserve the specified boundary conditions". Solving this exercise is lengthy, but useful — not just to build your character, but to get more familiar with the required tools. If you get stuck you find guidance in Eqs. (1)-(14) of `1402.3687`.

- Regarding the first part: when D is arbitrarily large, $1/D$ is arbitrarily small. Concerning the second part, check what happens with typical gradients as $D \to \infty$, think what happens to the Killing norm as $D \to 3$ (is the limit unique?) and give some physical interpretation of the force law you find for $D \to 2$ (in particular, do you have the expected Coulomb-like behavior and if so, does this correspond to a fall-off behavior or a confining behavior).