Daniel Grumiller May 18$^{\text{th}}$ 2020

Black Holes II — Exercise sheet 7

(17.1) **Schwarzschild black hole and thermal reservoir**
Couple the Schwarzschild black hole to a finite thermal reservoir of radiation within the volume V at the Hawking temperature $T_H = 1/(8\pi M)$. Show that for sufficiently small volumina $V < V_c$ this system is thermodynamically stable, whereas for sufficiently large volumina $V > V_c$ it is unstable. Calculate V_c.

(17.2) **Information loss problem in condensed matter physics**
Consider a piece of coal at zero temperature and a laser beam (a pure quantum state with some finite energy and entropy) in vacuum as initial state. Provided the laser beam is directed toward the piece of coal it will eventually be absorbed and scattered by the coal. In this (complicated) process the coal will heat up a little bit. Suppose that the coal is a nearly perfect black body. Then the final state will be the scattered pure radiation and the outgoing thermal black-body radiation emitted by the piece of coal. Thus, we appear to have an evolution of a pure initial state into a final state that is not pure. Information is lost, similar to what happens in the case of an evaporating black hole. How is this information loss problem resolved in condensed matter physics?

(17.3) **Third Law**
The third law of black hole mechanics states that physical processes that lead to

$$\kappa \to 0$$

are not possible in finite time. Discuss for a Schwarzschild black hole how you could attempt to violate the third law and why such attempts do not work. Generalize this discussion to Reissner–Nordström black holes.

These exercises are due on May 26$^{\text{th}}$ 2020.

Hints:

- For the finite reservoir of radiation you need the Stefan–Boltzmann law $E_{\text{res}} = \sigma V T^4$, where E_{res} is the energy of the radiation and $\sigma = \pi^2/15$. The relation between energy E_{res} and entropy S_{res} for a radiation gas is given by $E_{\text{res}} = \frac{3}{4} S_{\text{res}} T$. Use the Bekenstein–Hawking result for the entropy, $S_{BH} = A/4$, and show that the total entropy $S = S_{\text{res}} + S_{BH}$ is extremized for a total energy of $E = E_{\text{res}} + M$ if $T = T_H$. A simple way to extremize entropy under the given conditions is to add to the total entropy the energy constraint multiplied with a Lagrange multiplier β. Then vary that entropy with respect to the Lagrange multiplier and with respect to the black hole mass, keeping fixed the total energy E:

$$\delta S = \delta\big(S_{\text{res}} + S_{BH} + \beta(E_{\text{res}} + M - E)\big) = 0$$

  Prove now that the extremum is a maximum if and only if $V < V_c$, where

$$V_c = \frac{15}{32\pi^3\, T^5}$$

  Consider what this result implies for thermodynamic (in-)stability.

- Compare with exercise (8.3) of Black Holes I.

- Remember how surface gravity is related to mass and consider what you would have to do with the mass of a Schwarzschild black hole in order to make surface gravity vanish. For the Reissner–Nordström case start with a sub-extremal black hole $|Q| < M$ and try to make it extremal by dropping charged particles into it. Note that the particle only falls into the black hole if gravitational attraction overcomes the electrostatic repulsion.