Daniel Grumiller April 28th 2020

Black Holes II — Exercise sheet 4

(14.1) **Linearizing around global (A)dS**
Assume as background (A)dS, i.e., a maximally symmetric Riemann-tensor with non-zero Ricci scalar,

$$\bar{R}_{\alpha\beta\gamma\delta} = -\lambda \left(\bar{g}_{\alpha\delta}\bar{g}_{\beta\gamma} - \bar{g}_{\alpha\gamma}\bar{g}_{\beta\delta}\right) \qquad \lambda \neq 0$$

and write down the linearized equations of motion (for Einstein gravity with a cosmological constant) for some small fluctuation $h_{\mu\nu}$. Split the fluctuation into TT-part, gauge-part and trace part and write down separately the linearized equations of motion for each of these parts.

(14.2) **Accessibility of de-Donder gauge**
Prove that de-Donder gauge is accessible.

(14.3) **Quadrupole formula**
Derive the quadrupole formula

$$\tilde{h}^{ij} = \frac{2G}{r}\,\frac{\mathrm{d}^2 Q^{ij}(t)}{\mathrm{d}t^2}\Big|_{t\to t-|\vec{x}-\vec{x}'|}$$

where $\tilde{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\,\eta_{\mu\nu}\,h^\alpha_\alpha$ and

$$Q^{ij}(t) = \int \mathrm{d}^3x'\, x'^i x'^j T^{00}(t,\, \vec{x}')\,.$$

These exercises are due on May 5th 2020.

Hints:

- One immediate result you should check is that the Ricci scalar obeys $\bar{R} = \lambda\, D(D-1)$. Recall the Einstein equations with cosmological constant, $R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R + g_{\mu\nu}\, \Lambda = 0$ and take their trace to establish a relation between the parameter λ and the cosmological constant Λ. Then linearize the Einstein equations in the presence of such a cosmological constant, using the formulas for the linearized Ricci tensor and scalar derived in the lectures. Note that $\bar{\nabla}_\mu$ does not commute with $\bar{\nabla}_\nu$ when acting on a vector or 2-tensor.

- "Accessible" means that this gauge choice is always possible, i.e., any perturbation $h_{\mu\nu}$ that is not in de-Donder gauge can be braught into de-Donder gauge by a regular gauge transformation.

- Start with the first term in the multipole expansion

$$\tilde{h}_{ij} = \frac{4G}{r} \int \mathrm{d}^3x'\, T_{ij}(t - |\vec{x} - \vec{x}'|,\, \vec{x}') + \dots$$

  use the conservation equation $\partial_\mu T^{\mu\nu} = 0$ and the identity (which you should derive first)

$$\int \mathrm{d}^3x'\, T^{\mu i}(t,\, \vec{x}') = -\int \mathrm{d}^3x'\, x'^i\, \partial_j T^{\mu j}(t,\, \vec{x}')$$

  where $\mu = 0, 1, 2, \dots$ and $i, j = 1, 2, \dots$