3D quantum gravity, logarithmic CFT and its chiral truncation

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Outline

Holography: An Introduction

3D gravity

Which 3D theory?

Logarithmic CFT dual

Open issues

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Holography: An Introduction

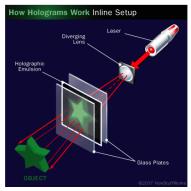
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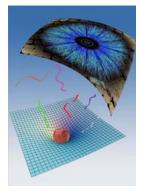
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Holography – Main Idea aka gauge/gravity duality, aka AdS/CFT correspondence





One of the most fruitful ideas in contemporary theoretical physics:

- The number of dimensions is a matter of perspective
- We can choose to describe the same physical situation using two different formulations in two different dimensions
- > The formulation in higher dimensions is a theory with gravity
- > The formulation in lower dimensions is a theory without gravity

Why Gravity? The holographic principle in black hole physics

Boltzmann/Planck: entropy of photon gas in d spatial dimensions $S_{\rm gauge} \propto {\rm volume} \propto L^d$

Bekenstein/Hawking: entropy of black hole in d spatial dimensions

 $S_{\rm gravity} \propto {\rm area} \propto L^{d-1}$

Daring idea by 't Hooft/Susskind in 1990ies:

Any consistent quantum theory of gravity could/should have a holographic formulation in terms of a field theory in one dimension lower

Discovery by Maldacena 1997:

Holographic principle is realized in string theory in specific way

e.g.
$$\langle T_{\mu\nu} \rangle_{\text{gauge}} = T^{BY}_{\mu\nu} \qquad \delta \text{action} = \int d^d x \sqrt{|h|} T^{BY}_{\mu\nu} \,\delta h^{\mu\nu}$$

Why should I care?

...and why were there > 6300 papers on holography in the past 12 years?

- Many applications!
- Tool for calculations
- Strongly coupled gauge theories (difficult) mapped to semi-cassical gravity (simple)
- Quantum gravity (difficult) mapped to weakly coupled gauge theories (simple)
- \blacktriangleright Sometimes both limits accessible: integrability of N=4 SYM
- Examples of first type: heavy ion collisions at RHIC and LHC, superfluidity, type II superconductors (?), cold atoms (?), ...
- Examples of the second type: microscopic understanding of black holes, information paradox, Kerr/CFT (?), 3D quantum gravity (?), ...

We can expect many new applications in the next decade!

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Why gravity in three dimensions? "As simple as possible, but not simpler"

Gravity simpler in lower dimensions

11D: 1144 Weyl, 66 Ricci, 5D: 35 Weyl, 15 Ricci, 4D: 10 Weyl, 10 Ricci 3D: no Weyl, 6 Ricci, 2D: no Weyl, 1 Ricci

2D gravity: black holes!

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Applications:

- Solve conceptual problems of (quantum) gravity
- Approximate geometry of cosmic strings/particles confined in plane
- Holographic tool for 2D condensed matter systems

pioneering work by Deser, Jackiw and Templeton in 1980ies 2007 Witten rekindled interest in 3D gravity

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Attempt 1: Einstein-Hilbert As simple as possible... but not simpler!

Let us start with the simplest attempt. Einstein-Hilbert action:

$$I_{\rm EH} = \frac{1}{16\pi G} \int \mathrm{d}^3 x \sqrt{-g} \, R$$

Equations of motion:

 $R_{\mu\nu} = 0$

Ricci-flat and therefore Riemann-flat - locally trivial!

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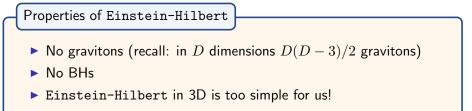
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Attempt 2: Topologically massive gravity Deser, Jackiw and Templeton found a way to introduce gravitons!

Let us now add a gravitational Chern-Simons term. TMG action:

$$I_{\rm TMG} = I_{\rm EH} + \frac{1}{16\pi G} \int d^3x \sqrt{-g} \, \frac{1}{2\mu} \, \varepsilon^{\lambda\mu\nu} \, \Gamma^{\rho}{}_{\lambda\sigma} \left(\partial_{\mu} \Gamma^{\sigma}{}_{\nu\rho} + \frac{2}{3} \, \Gamma^{\sigma}{}_{\mu\tau} \Gamma^{\tau}{}_{\nu\rho} \right)$$

Equations of motion:

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$$C_{\mu\nu} = \frac{1}{2} \,\varepsilon_{\mu}{}^{\alpha\beta} \nabla_{\alpha} R_{\beta\nu} + (\mu \leftrightarrow \nu)$$

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Properties of TMG

- Gravitons! Reason: third derivatives in Cotton tensor!
- No BHs
- TMG is slightly too simple for us!

Attempt 3: Einstein-Hilbert-AdS

Bañados, Teitelboim and Zanelli (and Henneaux) taught us how to get 3D BHs

Add negative cosmological constant to Einstein-Hilbert action:

$$I_{\Lambda \rm EH} = \frac{1}{16\pi G} \int \mathrm{d}^3 x \sqrt{-g} \left(R + \frac{2}{\ell^2} \right)$$

Equations of motion:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{1}{\ell^2} g_{\mu\nu} = 0$$

Particular solutions: BTZ BH with line-element

$$\mathrm{d}s_{\mathrm{BTZ}}^{2} = -\frac{(r^{2} - r_{+}^{2})(r^{2} - r_{-}^{2})}{\ell^{2}r^{2}} \,\mathrm{d}t^{2} + \frac{\ell^{2}r^{2}}{(r^{2} - r_{+}^{2})(r^{2} - r_{-}^{2})} \,\mathrm{d}r^{2} + r^{2}\left(\mathrm{d}\phi - \frac{r_{+}r_{-}}{\ell r^{2}} \,\mathrm{d}t\right)^{2}$$

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- No gravitons
- Rotating BH solutions that asymptote to AdS₃!
- Adding a negative cosmological constant produces BH solutions!

Cosmological topologically massive gravity CTMG is a 3D theory with BHs and gravitons!

We want a 3D theory with gravitons and BHs and therefore take CTMG action

$$I_{\rm CTMG} = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left[R + \frac{2}{\ell^2} + \frac{1}{2\mu} \varepsilon^{\lambda\mu\nu} \Gamma^{\rho}{}_{\lambda\sigma} \left(\partial_{\mu} \Gamma^{\sigma}{}_{\nu\rho} + \frac{2}{3} \Gamma^{\sigma}{}_{\mu\tau} \Gamma^{\tau}{}_{\nu\rho} \right) \right]$$

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 ${\sf Properties} \ {\sf of} \ {\sf CTMG}$

Gravitons!

BHs!

CTMG is just perfect for us. Study this theory!

Einstein sector of the classical theory

Solutions of Einstein's equations

$$G_{\mu\nu} = 0 \qquad \leftrightarrow \qquad R = -\frac{6}{\ell^2}$$

also have vanishing Cotton tensor

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Line-element of pure AdS:

$$ds_{AdS}^{2} = \bar{g}_{\mu\nu} dx^{\mu} dx^{\nu} = \ell^{2} \left(-\cosh^{2}\rho d\tau^{2} + \sinh^{2}\rho d\phi^{2} + d\rho^{2} \right)$$

Isometry group: $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$ Useful to introduce light-cone coordinates $u = \tau + \phi$, $v = \tau - \phi$ Cotton sector of the classical theory

Solutions of CTMG with

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Few exact solutions of this type are known.

Cotton sector of the classical theory

Solutions of CTMG with

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Few exact solutions of this type are known. Perhaps most interesting solution:

► Warped AdS (stretched/squashed), see Bengtsson & Sandin Line-element of space-like warped AdS:

$$ds_{\text{warped AdS}}^{2} = \frac{\ell^{2}}{\nu^{2} + 3} \left(-\cosh^{2}\rho \,d\tau^{2} + \frac{4\nu^{2}}{\nu^{2} + 3} \,(\mathrm{d}u + \sinh\rho \,\mathrm{d}\tau)^{2} + \mathrm{d}\rho^{2} \right)$$

Sidenote: null-warped AdS in holographic duals of cold atoms:

$$ds_{\text{null warped AdS}}^{2} = \ell^{2} \left(\frac{dy^{2} + 2 dx^{+} dx^{-}}{y^{2}} \pm \frac{(dx^{-})^{2}}{y^{4}} \right)$$

Definition: CTMG at the chiral point is CTMG with the tuning

$\mu\,\ell=1$

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Calculating the central charges of the dual boundary CFT yields

$$c_L = \frac{3}{2G} \left(1 - \frac{1}{\mu \ell} \right), \qquad c_R = \frac{3}{2G} \left(1 + \frac{1}{\mu \ell} \right)$$

Thus, at the chiral point we get

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Notes:

- Abbreviate "CTMG at the chiral point" as CCTMG
- CCTMG sometimes called "chiral gravity" (misnomer!)

Linearization around AdS background

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$$G_{\mu\nu}^{(1)} + \frac{1}{\mu} C_{\mu\nu}^{(1)} = (\mathcal{D}^R \mathcal{D}^L \mathcal{D}^M h)_{\mu\nu} = 0$$

with three mutually commuting first order operators

$$(\mathcal{D}^{L/R})_{\mu}{}^{\nu} = \delta^{\nu}_{\mu} \pm \ell \,\varepsilon_{\mu}{}^{\alpha\nu} \bar{\nabla}_{\alpha} \,, \qquad (\mathcal{D}^{M})_{\mu}{}^{\nu} = \delta^{\nu}_{\mu} + \frac{1}{\mu} \varepsilon_{\mu}{}^{\alpha\nu} \bar{\nabla}_{\alpha}$$

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Three linearly independent solutions to linearized EOM:

$$\left(\mathcal{D}^L h^L\right)_{\mu\nu} = 0, \qquad \left(\mathcal{D}^R h^R\right)_{\mu\nu} = 0, \qquad \left(\mathcal{D}^M h^M\right)_{\mu\nu} = 0$$

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At chiral point left (L) and massive (M) branches coincide!

Li, Song & Strominger found all regular nonrmalizable solutions of linearized EOM for $\mu\ell \neq 1.$

• Primaries: L_0, \bar{L}_0 eigenstates $\psi^{L/R/M}$ with

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$$h_{\mu\nu} = \operatorname{Re} \psi_{\mu\nu}$$

Degeneracy at the chiral point

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 At chiral point: L and M branches degenerate. Get new regular normalizable solution (Grumiller & Johansson)

$$\psi_{\mu\nu}^{\log} = \lim_{\mu\ell \to 1} \frac{\psi_{\mu\nu}^M(\mu\ell) - \psi_{\mu\nu}^L}{\mu\ell - 1}$$

with property

$$\left(\mathcal{D}^L \psi^{\log}\right)_{\mu\nu} = \left(\mathcal{D}^M \psi^{\log}\right)_{\mu\nu} \propto \psi^L, \qquad \left((\mathcal{D}^L)^2 \psi^{\log}\right)_{\mu\nu} = 0$$

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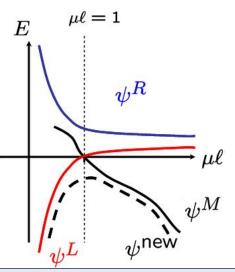
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- With signs defined as in this talk: BHs positive energy, gravitons negative energy
- With signs as defined in Deser-Jackiw-Templeton paper: BHs negative energy, gravitons positive energy
- Either way need a mechanism to eliminate unwanted negative energy objects – either the gravitons or the BHs
- Even at chiral point the problem persists because of the logarithmic mode. See Figure. (Figure: thanks to N. Johansson)

Energy for all branches:



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The LCFT conjecture

Observation:

$$L_0 \begin{pmatrix} \log \\ \text{left} \end{pmatrix} = \begin{pmatrix} 2 & \frac{1}{2} \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \log \\ \text{left} \end{pmatrix},$$
$$\bar{L}_0 \begin{pmatrix} \log \\ \text{left} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \log \\ \text{left} \end{pmatrix}.$$

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Such a Jordan form of L_0 , \overline{L}_0 is defining property of a logarithmic CFT! Logarithmic gravity conjecture (Grumiller & Johansson 2008):

CFT dual to CTMG exists and is logarithmic

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Grumiller, Jackiw, Johansson, Henneaux, Maloney, Martinez, Song, Strominger, Troncoso, ... 2008/2009: Several non-trivial consistency checks that LCFT conjecture could be correct.

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 Construct non-normalizable modes related to left-, right- and log-branches

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► Take the *n*-th variation of the full on-shell action

$$\delta S^{(2)}(\psi_1,\psi_2) =$$
boundary $\delta S^{(n)}(\psi_1,\psi_2,\ldots,\psi_n) =$ bulk

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Compare with LCFT correlators, e.g.

$$\langle \mathcal{O}^L(z,\bar{z}) \mathcal{O}^{\log}(0) \rangle = -\frac{b}{2 z^4} + \dots$$

Skenderis, Taylor & van Rees 2009: n=2 Grumiller & Sachs 2009: n=2,3

D. Grumiller — 3D gravity

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$$\psi_{\mu\nu} = e^{-ihu - i\bar{h}v} F_{\mu\nu}(\rho) \qquad \mathcal{D}^{L/R/M}\psi = 0$$

Constructed with Ivo Sachs in global coordinates:

• Separation Ansatz with $SL(2,\mathbb{R})$ weights h, \bar{h} :

$$\psi_{\mu\nu} = e^{-ihu - i\bar{h}v} F_{\mu\nu}(\rho) \qquad \mathcal{D}^{L/R/M}\psi = 0$$

• Obtain two integration constants: one fixed by regularity at origin $\rho = 0$, the other fixed by overall normalization

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- \blacktriangleright Obtain two integration constants: one fixed by regularity at origin $\rho=0,$ the other fixed by overall normalization
- ► No free parameters! For given weights (h, h̄) modes are either normalizable or non-normalizable (up to degenerate cases)

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• Separation Ansatz with $SL(2,\mathbb{R})$ weights h, \bar{h} :

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- ▶ Normalizable: (2 + n, m) and (n, 2 + m)
- ▶ Non-normalizable: (-1 + n, -1 m) and (-1 n, -1 + m)
- Exploit $SL(2,\mathbb{R})$ algebra to related modes of different weights:

$$\left[\mathcal{D}^{L/R}, L_{\pm}\right]\psi^{L} = \left[\mathcal{D}^{L/R}, \bar{L}_{\pm}\right]\psi^{L} = 0$$

$$\left[\left(\mathcal{D}^{L}\right)^{2}, L_{\pm}\right]\psi^{\log} = \left[\left(\mathcal{D}^{L}\right)^{2}, \bar{L}_{\pm}\right]\psi^{\log} = 0$$

In words: L_{\pm} , $ar{L}_{\pm}$ act as ladder operators

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- Show that all boundary terms are contact terms!
- Exploit trick above and partial integrations to simplify 3-point correlators

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Keep track of all numerical factors:

$$\langle L \log \rangle = -\frac{b}{2 \, z^4} = \langle \mathcal{O}^L(z, \bar{z}) \, \mathcal{O}^{\log}(0) \rangle$$

Without log insertions reduce to Einstein gravity correlators:

$$\langle R R R \rangle \sim \delta^{(3)} S(R, R, R) \sim 2\delta^{(3)} S_{\rm EH}(R, R, R)$$
$$\langle L L L \rangle \sim \delta^{(3)} S(L, L, L) \sim 0\delta^{(3)} S_{\rm EH}(L, L, L) = 0$$

With single log insertions after some manipulations reduce to Einstein gravity correlators:

$$\langle L L \log \rangle \sim \delta^{(3)} S(L, L, \log) \propto \delta^{(3)} S_{\rm EH}(L, L, L)$$

 $\langle L R \log \rangle \sim \delta^{(3)} S(L, R, \log) \sim 0 + \text{contact terms}$

With multiple log insertions calculations still very lengthy:

$$\langle \log \log \log \rangle \sim \delta^{(3)} S(\log, \log, \log) =$$
lengthy!

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All correlators above reproduced on gravity side! Skenderis, Taylor and van Rees (2009)

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3-point correlators:

Calculated 7 of 10 correlators so far — all of them match precisely. Plan to calculate one more. Will not calculate $\langle L\log\log\rangle$ and $\langle \log\log\log\rangle$ (lengthy!)

Outline

Holography: An Introduction

3D gravity

Which 3D theory?

Logarithmic CFT dual

Open issues

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CTMG might be a good gravity dual to strongly coupled LCFTs!

Literature

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Thank you for your attention!

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