Quantum Null Energy Condition

In two dimensions

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FWF

Based on work in progress with
Ecker, Sheikh-Jabbari, Stanzer and van der Schee

Outline

Inequalities in mathematics

- Inequalities are a core part of mathematics

  Boring inequalities (type 1): true, but could be sharpened

  $$p^2 \geq -p^2 \qquad \forall p \in \mathbb{R}$$

  Boring inequalities (type 2): true, but actually an equality

  $$p^2 \geq p^2 \qquad \forall p \in \mathbb{R}$$

  Fine inequalities: true, cannot be sharpened, not always an equality

  $$p^2 \geq 0 \qquad \forall p \in \mathbb{R}$$

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add on both sides $4ab$

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take square root and then divide by 2

$$\frac{a+b}{2} \geq \sqrt{ab}$$

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here u, v are some vector, $||$ is their length and $\cdot$ the inner product

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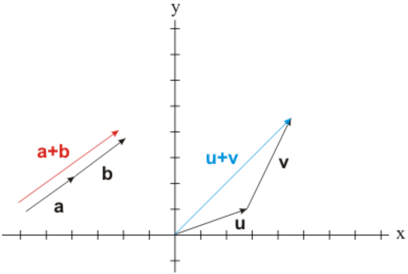

numerous consequences
e.g. triangle inequality

$$|u| + |v| \geq |u + v|$$

graphic proof evident

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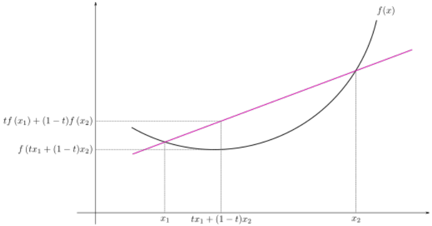

special case of Jensen's inequality: secant always above convex curve between intersection points x_1, x_2

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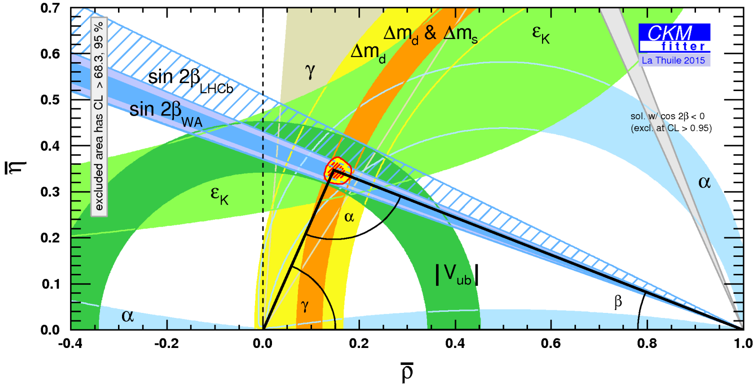

Example: unitarity constraints on physical parameters in quark mixing matrix

if Standard Model correct then measurements must reproduce unitarity triangle

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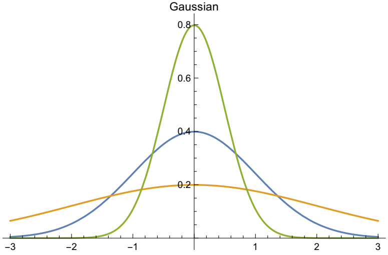

green: localized in coordinate space (x), delocalized in momentum space (p)
blue: mildly (de-)localized in coordinate and momentum space
orange: delocalized in coordinate space (x), localized in momentum space (p)

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  - Definition: (local) inequalities on the stress tensor $T_{\mu\nu}$
    e.g. Null Energy Condition (NEC)

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For instance: Penrose singularity theorem from Raychaudhuri eq.

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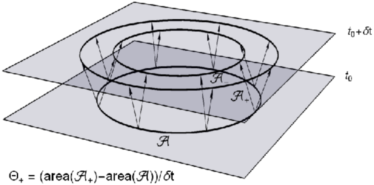
If $T_{kk} \geq 0$ (NEC) $\Rightarrow$ focussing!
(negative acceleration of area)

For experts:
$\frac{\mathrm{d}\,\text{area}}{\mathrm{d}k} = \theta$ is null expansion



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However: all classical energy inequalities violated by quantum effects!

NEC violated by Casimir energy, accelerated mirrors, Hawking radiation, ...

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Are there quantum energy conditions?
[How is $2^{\rm nd}$ law saved?]

Quantum energy conditions

- Definition: quantum energy condition = convexity condition for $\langle T_{\mu\nu}\rangle$ valid for any state and any (reasonable) quantum field theory (QFT)

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valid $\forall k^\mu (\text{with } k^\mu k_\mu = 0)$ and $\forall$ states $|\rangle$ in any (reasonable) QFT

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  Faulkner, Leigh, Parrikar and Wang 1605.08072
  Hartman, Kundu and Tajdini 1610.05308

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Is there a local quantum energy condition?

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Quantum null energy condition (QNEC)

Proposed by Bousso, Fisher, Leichenauer and Wall in 1506.02669

QNEC (in $D > 2$) is the following inequality

$$\langle T_{kk} \rangle \geq \frac{\hbar}{2\pi\sqrt{\gamma}}\, S''$$

Physical motivation from focussing properties and second law:
Replace area by area $+\, 4G$ (entanglement entropy)
Modified Raychaudhuri eq., schematically:

$$\frac{\mathrm{d}^2\text{area}}{\mathrm{d}k^2} + 4G\, S'' = -8\pi G\, T_{kk} + 4G\, S'' \overset{\text{QNEC}}{\leq} 0$$

requires for focussing property (= 2^{nd} law) QNEC

fineprint: above we set expansion to zero, $\frac{\mathrm{d\,area}}{\mathrm{d}k} = 0$, and shear to zero; we also set the area to unity, $\sqrt{\gamma} = 1$

thus, QNEC is implied from quantum focussing for special congruences

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Obvious observations:

- if r.h.s. vanishes: semi-classical version of NEC
- if r.h.s. negative: weaker condition than NEC
  (NEC can be violated while QNEC holds)
- if r.h.s. positive: stronger condition than NEC
  (if QNEC holds also NEC holds)

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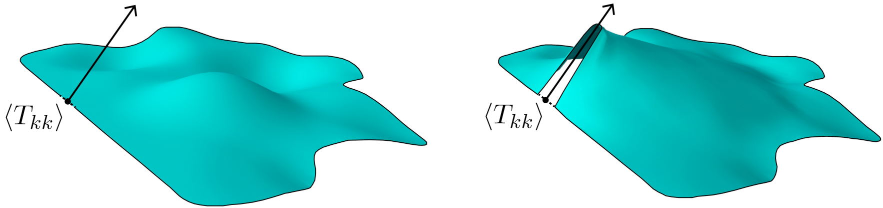

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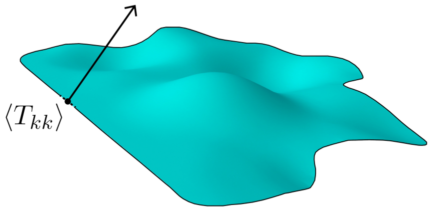

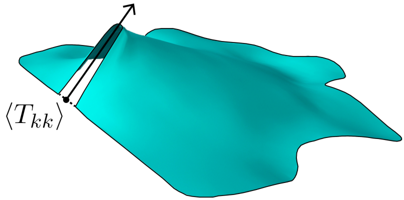

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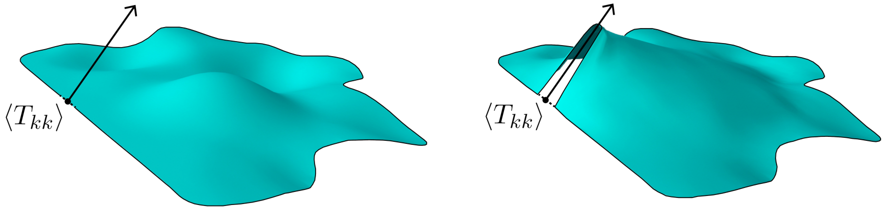

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- $\sqrt{\gamma}$: induced volume form of entangling region (black boundary curve)

Proofs $(D > 2)$

- For free QFTs: Bousso, Fisher, Koeller, Leichenauer and Wall, 1509.02542
- For holographic CFTs: Koeller and Leichenauer, 1512.06109
- For general CFTs: Balakrishnan, Faulkner, Khandker and Wang, 1706.09432

Proofs and non-saturation ($D = 2$)

Ongoing work with Ecker, Sheikh-Jabbari, Stanzer and van der Schee

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- QNEC not saturated in hol. CFT_2 with positive bulk energy fluxes
- QNEC can be violated in hol. CFT_2 with negative bulk energy fluxes

Calculating QNEC holographically

calculating CFT observable holographically = some gravity calculation

AdS/CFT:

Maldacena hep-th/9711200
Gubser, Klebanov and Polyakov hep-th/9802109
Witten hep-th/9802150

holographic stress tensor:

Henningson and Skenderis hep-th/9806087
Balasubramanian and Kraus hep-th/9902121
Emparan, Johnson and Myers hep-th/9903238
de Haro, Solodukhin and Skenderis hep-th/0002230

holographic entanglement entropy (HEE):

Ryu and Takayanagi hep-th/0603001
Hubeny, Rangamani and Takayanagi 0705.0016

Calculating QNEC holographically

calculating CFT observable holographically = some gravity calculation

- need holographic computation of $\langle T_{kk}\rangle$

  well-known AdS/CFT prescription: extract boundary stress tensor from bulk metric expanded near AdS boundary

  Example: $\text{AdS}_3/\text{CFT}_2$

  $$\mathrm{d}s^2 = \frac{\ell^2}{z^2}\left(\mathrm{d}z^2 + 2\,\mathrm{d}x^+\,\mathrm{d}x^-\right) + \langle T_{++}\rangle\left(\mathrm{d}x^+\right)^2 + \langle T_{--}\rangle\left(\mathrm{d}x^-\right)^2 + \mathcal{O}(z^2)$$

  AdS_3 boundary: $z \to 0$
  $\mathcal{O}(1)$ terms in metric: flux components of stress tensor $\langle T_{\pm\pm}\rangle$
  (trace vanishes, $\langle T_{+-}\rangle = 0$)
  ℓ: so-called AdS-radius (cosmological constant $\Lambda = -1/\ell^2$)

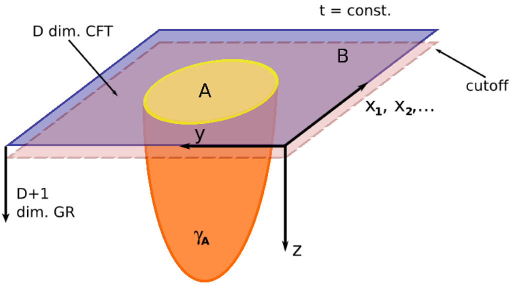
Calculating QNEC holographically

calculating CFT observable holographically = some gravity calculation

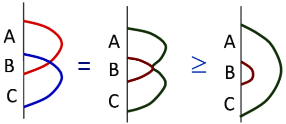
- need holographic computation of $\langle T_{kk} \rangle$
- need holographic computation of (deformations of) EE

HEE = area of extremal surface

simple to calculate!



also: simple proof of strong subadditivity inequalities



$$\Rightarrow \; S_{A\cup B} + S_{B\cup C} \geq S_{A\cup B\cup C} + S_B$$

Outline

Inequalities

QNEC

Holographic QNEC in 4d

Holographic QNEC in 2d

Thermal case

see work with Ecker, Stanzer and van der Schee 1710.09837

thermal states in CFT_4 = black holes in AdS_5

- paper-and-pencil calculation starts with Schwarzschild black brane

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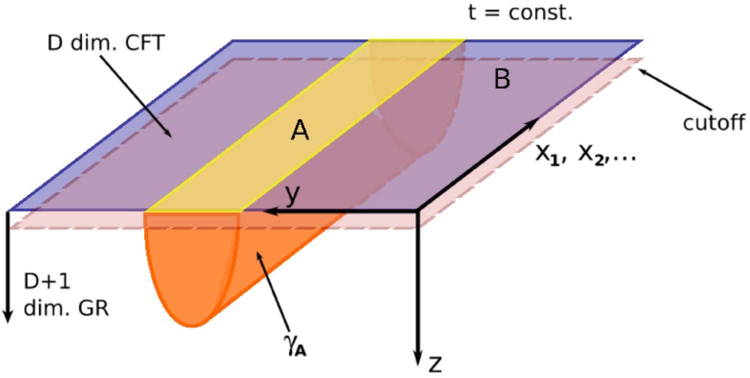
with $f(z) = 1 - \pi^4 T^4 z^4$



Thermal case

see work with Ecker, Stanzer and van der Schee 1710.09837

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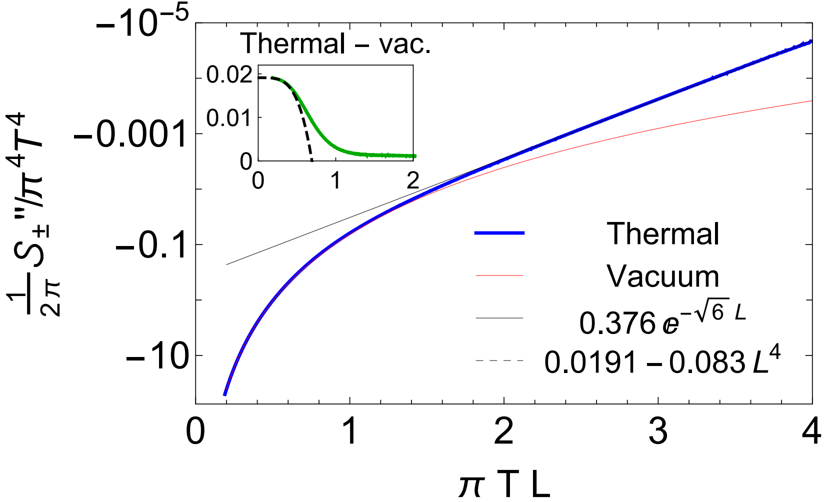

notational alert: L in the plot corresponds to width ℓ

Colliding gravitational shockwaves and QNEC saturation

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plasma formation in CFT_4 = colliding gravitational shock waves in AdS_5
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- check QNEC and its saturation, particularly in region of NEC violation

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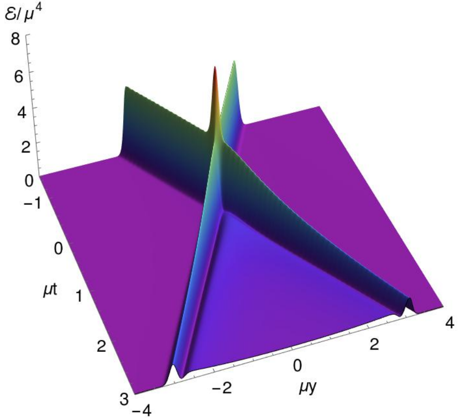

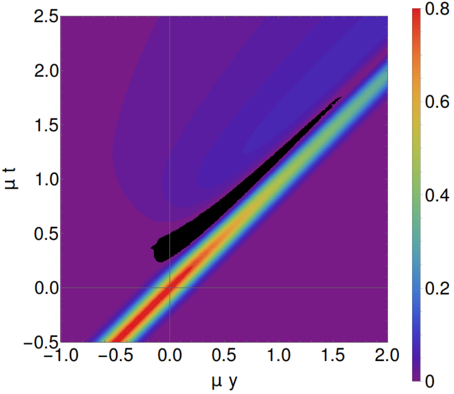

Left: energy density plot

Right: black region violates NEC

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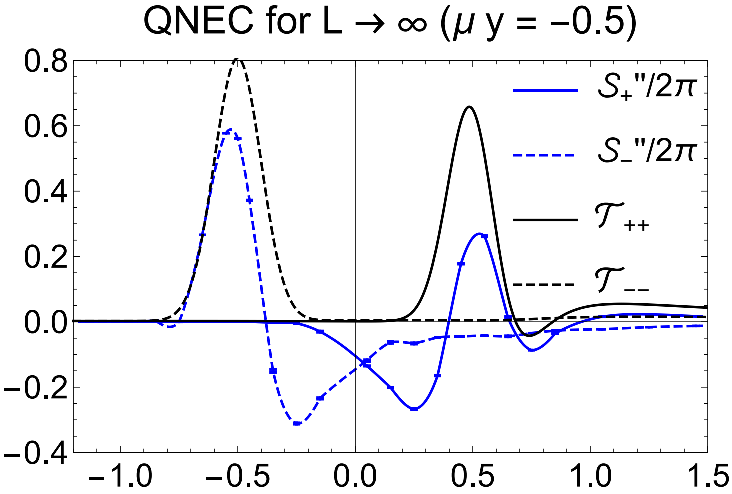

Outline

Inequalities

QNEC

Holographic QNEC in 4d

Holographic QNEC in 2d

Non-equilibrium and quantum equilibrium

Definition:

A state is in quantum equilibrium when QNEC saturates for all times and all entangling regions

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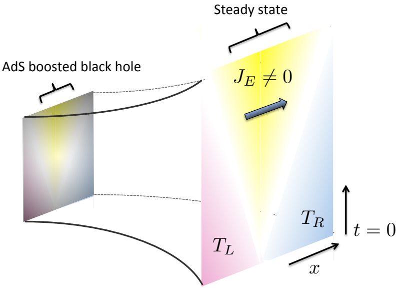
Figure 2 from `1311.3655` (Nature Phys.) Bhaseen, Doyon, Lucas, Schalm



- Far from equilibrium transport in strongly coupled CFT
- Long-time energy transport universally via steady-state
- In AdS_3/CFT_2: specific Bañados geometry with step function
- Our results imply QNEC saturation at all times

Definition:

A state is in quantum equilibrium when QNEC saturates for all times and all entangling regions

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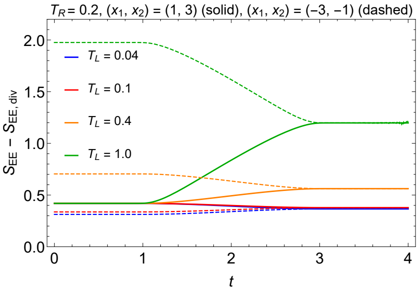

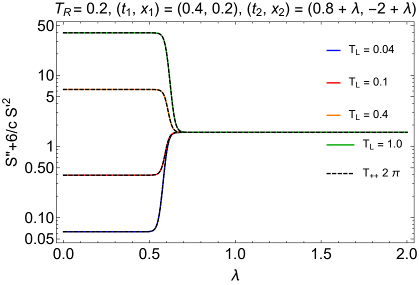

Left: HEE

Right: QNEC saturation

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Quantum equilibrium hopefully a useful notion

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- Vaidya $=$ simple model for bulk matter; mass function $M(t)$

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- If time is much larger than entangling region we find QNEC saturation

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Finite central charge corrections to QNEC

Consider global AdS_3 with massive scalar field and take into account quantum backreactions (Belin, Iqbal, Lokhande 1805.08782)

$$\mathrm{d}s^2 = -\big(r^2 + G_1(r)^2\big)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{r^2 + G_2(r)^2} + r^2\,\mathrm{d}\varphi^2$$

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  Interpretation for small interval, $\Delta\varphi \ll 1$:
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- From CFT perspective hard to understand where $h^{3/2}$ comes from

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QNEC non-saturation:

$$\langle T_{kk}\rangle - S'' - \frac{6}{c}\,(S')^2\big|_{\Delta\varphi=\pi,\,c\gg h\gg 1} = \frac{1}{4}\,h$$

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- QNEC in words: Expectation value of null projection of stress tensor bigger or equal than second derivative of entanglement entropy with respect to null deformations of entangling surface
- Only known local energy condition that could be universally true
- Various proofs of QNEC exist
- Provided first numerical studies of QNEC in AdS_5/CFT_4 context 1710.09837
- Shockwaves can saturate QNEC in far from equilibrium regime
- QNEC sharper in AdS_3/CFT_2

$$\langle T_{kk} \rangle \geq S'' + \frac{6}{c}\, S'^{\,2}$$

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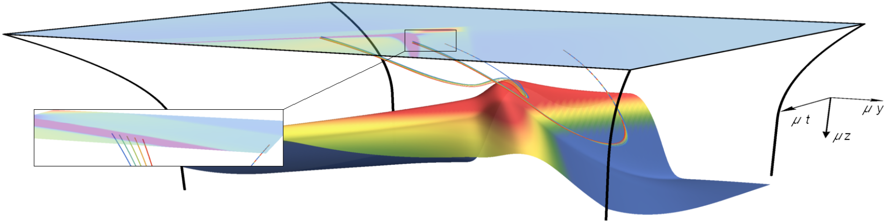
- Curious QNEC "half-saturation" in Vaidya
- $1/c$ corrections can spoil saturation and require to take into account bulk corrections to entanglement entropy

Much to be learned about QNEC and its potential applications



Thanks for your attention!