$\text{AdS}_3/\text{LCFT}_2$ correspondence

Massive gravity in three dimensions

Daniel Grumiller

Institute for Theoretical Physics
Vienna University of Technology

40^{th} International Symposium Ahrenshoop, August 2010

with: Sabine Ertl, Olaf Hohm, Roman Jackiw, Niklas Johansson, Ivo Sachs, Dima Vassilevich, Thomas Zojer

Outline

Motivation for 3D massive gravity and introduction to LCFTs

Topologically massive gravity

Logarithmic CFT conjecture

Consequences, Generalizations & Applications

Outline

Motivation for 3D massive gravity and introduction to LCFTs

Topologically massive gravity

Logarithmic CFT conjecture

Consequences, Generalizations & Applications

Motivations for studying gravity in three dimensions

- Quantum gravity
  - Address conceptual issues of quantum gravity
  - Black hole evaporation, information loss, black hole microstate counting, virtual black hole production, ...
  - Technically much simpler than 4D or higher D gravity
  - Integrable models: powerful tools in physics (Coulomb problem, Hydrogen atom, harmonic oscillator, ...)
  - Models should be as simple as possible, but not simpler
- Gauge/gravity duality
  - Deeper understanding of black hole holography
  - AdS_3/CFT_2 correspondence best understood
  - Quantum gravity via AdS/CFT? (Witten '07, Li, Song, Strominger '08)
  - Applications to 2D condensed matter systems?
  - Gauge gravity duality beyond standard AdS/CFT: warped AdS, asymptotic Schrödinger, non-relativistic CFTs, logarithmic CFTs, ...
- Physics
  - Cosmic strings (Deser, Jackiw, 't Hooft '84, '92)
  - Black hole analog systems in condensed matter physics (graphene, BEC, fluids, ...)

Motivations for studying gravity in three dimensions

- Gauge/gravity duality
  - Gauge gravity duality beyond standard AdS/CFT: warped AdS, asymptotic Schrödinger, non-relativistic CFTs, logarithmic CFTs, ...

Main motivation of this talk:
Gravity duals for logarithmic CFTs?

Motivations for studying gravity in three dimensions

Main motivation of this talk:
Gravity duals for logarithmic CFTs?

Theoretically interesting (for a review see Gaberdiel '01):

- logarithmic CFTs are non-unitary

Motivations for studying gravity in three dimensions

Main motivation of this talk:
Gravity duals for logarithmic CFTs?

Theoretically interesting (for a review see Gaberdiel '01):

- logarithmic CFTs are non-unitary
- not evident if/how gauge/gravity duality is supposed to work in this case — would need gravity theories reflecting non-unitarity

Motivations for studying gravity in three dimensions

Main motivation of this talk:
Gravity duals for logarithmic CFTs?

Theoretically interesting (for a review see Gaberdiel '01):

- logarithmic CFTs are non-unitary
- not evident if/how gauge/gravity duality is supposed to work in this case — would need gravity theories reflecting non-unitarity
- for specific logarithmic CFTs where energy momentum tensor acquires a logarithmic partner: would need some partner of the graviton on the gravity side

Motivations for studying gravity in three dimensions

Main motivation of this talk:
Gravity duals for logarithmic CFTs?

Theoretically interesting (for a review see Gaberdiel '01):

- logarithmic CFTs are non-unitary
- not evident if/how gauge/gravity duality is supposed to work in this case — would need gravity theories reflecting non-unitarity
- for specific logarithmic CFTs where energy momentum tensor acquires a logarithmic partner: would need some partner of the graviton on the gravity side

Phenomenologically interesting:

- logarithmic CFTs describe e.g. systems with quenched disorder
- examples: spin glasses, quenched random magnets, percolation, dilute self-avoiding polymers

Motivations for studying gravity in three dimensions

Main motivation of this talk:
Gravity duals for logarithmic CFTs?

Theoretically interesting (for a review see Gaberdiel '01):

- logarithmic CFTs are non-unitary
- not evident if/how gauge/gravity duality is supposed to work in this case — would need gravity theories reflecting non-unitarity
- for specific logarithmic CFTs where energy momentum tensor acquires a logarithmic partner: would need some partner of the graviton on the gravity side

Phenomenologically interesting:

- logarithmic CFTs describe e.g. systems with quenched disorder
- examples: spin glasses, quenched random magnets, percolation, dilute self-avoiding polymers
- in appropriate strong coupling limit: exploit AdS/LCFT correspondence to calculate observables on gravity side?

Logarithmic CFT in a nutshell

Reminder: energy-momentum tensor of CFTs

$$T_{zz} = \mathcal{O}^L(z) \qquad T_{\bar{z}\bar{z}} = \mathcal{O}^R(\bar{z})$$

Suppose that CFT has operator $\mathcal{O}^{\log}$ with same conformal weights as $\mathcal{O}^L$

Logarithmic CFT in a nutshell

Reminder: energy-momentum tensor of CFTs

$$T_{zz} = \mathcal{O}^L(z) \qquad T_{\bar{z}\bar{z}} = \mathcal{O}^R(\bar{z})$$

Suppose that CFT has operator $\mathcal{O}^{\log}$ with same conformal weights as $\mathcal{O}^L$
If Hamiltonian does not diagonalize we have a (specific) logarithmic CFT:

$$H \begin{pmatrix} \mathcal{O}^{\log} \\ \mathcal{O}^L \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \mathcal{O}^{\log} \\ \mathcal{O}^L \end{pmatrix}$$

Logarithmic CFT in a nutshell

Reminder: energy-momentum tensor of CFTs

$$T_{zz} = \mathcal{O}^L(z) \qquad T_{\bar{z}\bar{z}} = \mathcal{O}^R(\bar{z})$$

Suppose that CFT has operator $\mathcal{O}^{\log}$ with same conformal weights as $\mathcal{O}^L$
If Hamiltonian does not diagonalize we have a (specific) logarithmic CFT:

$$H \begin{pmatrix} \mathcal{O}^{\log} \\ \mathcal{O}^L \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \mathcal{O}^{\log} \\ \mathcal{O}^L \end{pmatrix}$$

Alternatively: suppose that CFT has operator $\mathcal{O}^{\mathrm{M}}$ with conformal weights

$$h = 2 + \varepsilon \qquad \bar{h} = \varepsilon \qquad \langle \mathcal{O}^M(z,\bar{z})\, \mathcal{O}^M(0,0) \rangle = \frac{\hat{B}}{z^{4+2\varepsilon}\, \bar{z}^{2\varepsilon}} + \dots$$

Logarithmic CFT in a nutshell

Reminder: energy-momentum tensor of CFTs

$$T_{zz} = \mathcal{O}^L(z) \qquad T_{\bar z\bar z} = \mathcal{O}^R(\bar z)$$

Suppose that CFT has operator $\mathcal{O}^{\log}$ with same conformal weights as $\mathcal{O}^L$
If Hamiltonian does not diagonalize we have a (specific) logarithmic CFT:

$$H \begin{pmatrix} \mathcal{O}^{\log} \\ \mathcal{O}^L \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \mathcal{O}^{\log} \\ \mathcal{O}^L \end{pmatrix}$$

Alternatively: suppose that CFT has operator $\mathcal{O}^{\mathrm{M}}$ with conformal weights

$$h = 2 + \varepsilon \qquad \bar h = \varepsilon \qquad \langle \mathcal{O}^M(z,\bar z)\, \mathcal{O}^M(0,0)\rangle = \frac{\hat B}{z^{4+2\varepsilon}\,\bar z^{2\varepsilon}} + \dots$$

Send simultaneously left central charge c_L and parameter ε to zero.
If these limits exist then get a logarithmic CFT:

$$b_L := \lim_{c_L\to 0} -\frac{c_L}{\varepsilon} \neq 0 \qquad B := \lim_{c_L\to 0} \big(\hat B + \frac{2}{c_L}\big)$$

Two-point correlators in LCFTs

Recapitulate some formulas from the last slide:

$$h = 2 + \varepsilon \qquad \bar{h} = \varepsilon \qquad \langle \mathcal{O}^M(z,\bar{z})\,\mathcal{O}^M(0,0)\rangle = \frac{\hat{B}}{z^{4+2\varepsilon}\,\bar{z}^{2\varepsilon}} + \dots$$

$$b_L := \lim_{c_L\to 0} -\frac{c_L}{\varepsilon} \neq 0 \qquad B := \lim_{c_L\to 0}\big(\hat{B} + \frac{2}{c_L}\big)$$

Define a new operator $\mathcal{O}^{\log}$ that linearly combines $\mathcal{O}^{L/M}$.

$$\mathcal{O}^{\log} = b_L\,\frac{\mathcal{O}^L}{c_L} + \frac{b_L}{2}\,\mathcal{O}^M$$

Two-point correlators in LCFTs

Recapitulate some formulas from the last slide:

$$h = 2 + \varepsilon \qquad \bar{h} = \varepsilon \qquad \langle \mathcal{O}^M(z,\bar{z})\, \mathcal{O}^M(0,0)\rangle = \frac{\hat{B}}{z^{4+2\varepsilon}\,\bar{z}^{2\varepsilon}} + \dots$$

$$b_L := \lim_{c_L\to 0} -\frac{c_L}{\varepsilon} \neq 0 \qquad B := \lim_{c_L\to 0} \big(\hat{B} + \frac{2}{c_L}\big)$$

Define a new operator $\mathcal{O}^{\log}$ that linearly combines $\mathcal{O}^{L/M}$.

$$\mathcal{O}^{\log} = b_L\, \frac{\mathcal{O}^L}{c_L} + \frac{b_L}{2}\, \mathcal{O}^M$$

Taking the limit $c_L \to 0$ leads to the following 2-point correlators:

$$\langle \mathcal{O}^L(z)\mathcal{O}^L(0,0)\rangle = 0$$

$$\langle \mathcal{O}^L(z)\mathcal{O}^{\log}(0,0)\rangle = \frac{b_L}{2z^4}$$

$$\langle \mathcal{O}^{\log}(z,\bar{z})\mathcal{O}^{\log}(0,0)\rangle = -\frac{b_L \ln(m_L^2|z|^2)}{z^4}$$

"New anomaly" b_L characterizes LCFT

Conformal Ward identities

Like in ordinary CFTs, conformal Ward identities determine essentially uniquely the form of 2- and 3-point correlators (set $m_L = 1$)

$$\langle \mathcal{O}^L(z)\mathcal{O}^L(0,0)\rangle = 0$$
$$\langle \mathcal{O}^L(z)\mathcal{O}^{\log}(0,0)\rangle = \frac{b_L}{2z^4}$$
$$\langle \mathcal{O}^{\log}(z,\bar{z})\mathcal{O}^{\log}(0,0)\rangle = -\frac{b_L \ln\left(|z|^2\right)}{z^4}$$

$$\langle \mathcal{O}^L(z,\bar{z})\mathcal{O}^L(z',\bar{z}')\mathcal{O}^{\log}(0,0)\rangle = \frac{b_L}{z^2 z'^2 (z-z')^2}$$
$$\langle \mathcal{O}^L(z,\bar{z})\mathcal{O}^{\log}(z',\bar{z}')\mathcal{O}^{\log}(0,0)\rangle = -\frac{2b_L \ln|z'|^2 + \frac{b_L}{2}}{z^2 z'^2 (z-z')^2}$$
$$\langle \mathcal{O}^{\log}(z,\bar{z})\mathcal{O}^{\log}(z',\bar{z}')\mathcal{O}^{\log}(0,0)\rangle = \frac{\text{lengthy}}{z^2 z'^2 (z-z')^2}$$

Requirements for gravity duals to LCFTs

Checks for purported gravity duals to logarithmic CFTs

- There exists some bulk mode corresponding to the operator $\mathcal{O}^{\log}$
- Weights of $\mathcal{O}^{\log}$ must degenerate with weights of $\mathcal{O}^{L}$
- Jordan cell structure of $H \sim L_0 + \bar{L}_0$ with respect to $\mathcal{O}^{L}$, $\mathcal{O}^{\log}$
- Central charges must be tunable to zero by some parameter
- Gravity theory should exhibit non-unitarity
- Conformal Ward identities must hold

Requirements for gravity duals to LCFTs

Checks for purported gravity duals to logarithmic CFTs

- There exists some bulk mode corresponding to the operator $\mathcal{O}^{\log}$
- Weights of $\mathcal{O}^{\log}$ must degenerate with weights of $\mathcal{O}^L$
- Jordan cell structure of $H \sim L_0 + \bar{L}_0$ with respect to $\mathcal{O}^L$, $\mathcal{O}^{\log}$
- Central charges must be tunable to zero by some parameter
- Gravity theory should exhibit non-unitarity
- Conformal Ward identities must hold

Cannot be Einstein gravity!

Requirements for gravity duals to LCFTs

Checks for purported gravity duals to logarithmic CFTs

- There exists some bulk mode corresponding to the operator $\mathcal{O}^{\log}$
- Weights of $\mathcal{O}^{\log}$ must degenerate with weights of $\mathcal{O}^{L}$
- Jordan cell structure of $H \sim L_0 + \bar{L}_0$ with respect to $\mathcal{O}^{L}$, $\mathcal{O}^{\log}$
- Central charges must be tunable to zero by some parameter
- Gravity theory should exhibit non-unitarity
- Conformal Ward identities must hold

Cannot be Einstein gravity!

Consider theories that naturally generalize Einstein gravity:

Massive gravity in three dimensions

Outline

Action and equations of motion of topologically massive gravity (TMG)

Consider the action (Deser, Jackiw & Templeton '82)

$$I_{\text{TMG}} = \frac{1}{16\pi\, G}\int \mathrm{d}^3x\sqrt{-g}\,\Big[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)\Big]$$

Action and equations of motion of topologically massive gravity (TMG)

Consider the action (Deser, Jackiw & Templeton '82)

$$I_{\text{TMG}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x\sqrt{-g}\,\Big[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)\Big]$$

Equations of motion:

$$R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}R - \frac{1}{\ell^2}\,g_{\mu\nu} + \frac{1}{\mu}\,C_{\mu\nu} = 0$$

with the Cotton tensor defined as

$$C_{\mu\nu} = \frac{1}{2}\,\varepsilon_\mu{}^{\alpha\beta}\nabla_\alpha R_{\beta\nu} + (\mu \leftrightarrow \nu)$$

Action and equations of motion of topologically massive gravity (TMG)

Consider the action (Deser, Jackiw & Templeton '82)

$$I_{\text{TMG}} = \frac{1}{16\pi\,G}\int \mathrm{d}^3x\sqrt{-g}\,\Big[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)\Big]$$

Equations of motion:

$$R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}R - \frac{1}{\ell^2}\,g_{\mu\nu} + \frac{1}{\mu}\,C_{\mu\nu} = 0$$

with the Cotton tensor defined as

$$C_{\mu\nu} = \frac{1}{2}\,\varepsilon_\mu{}^{\alpha\beta}\nabla_\alpha R_{\beta\nu} + (\mu \leftrightarrow \nu)$$

Some properties of TMG

- Massive gravitons and black holes
- AdS solutions and asymptotic AdS solutions
- warped AdS solutions and warped AdS black holes
- Schrödinger solutions and Schrödinger pp-waves

TMG at the chiral point

Definition: TMG at the chiral point is TMG with the tuning

$$\mu\,\ell = 1$$

between the cosmological constant and the Chern–Simons coupling.

TMG at the chiral point

Definition: TMG at the chiral point is TMG with the tuning

$$\mu \ell = 1$$

between the cosmological constant and the Chern–Simons coupling.
Why special? (Li, Song & Strominger '08)

TMG at the chiral point

Definition: TMG at the chiral point is TMG with the tuning

$$\mu\,\ell = 1$$

between the cosmological constant and the Chern–Simons coupling.
Why special? (Li, Song & Strominger '08)
Calculating the central charges of the dual boundary CFT yields

$$c_L = \frac{3\ell}{2G}\,\big(1 - \frac{1}{\mu\,\ell}\big) \qquad c_R = \frac{3\ell}{2G}\,\big(1 + \frac{1}{\mu\,\ell}\big)$$

Thus, at the chiral point we get

$$c_L = 0 \qquad c_R = \frac{3\ell}{G}$$

TMG at the chiral point

Definition: TMG at the chiral point is TMG with the tuning

$$\mu\,\ell = 1$$

between the cosmological constant and the Chern–Simons coupling.
Why special? (Li, Song & Strominger '08)
Calculating the central charges of the dual boundary CFT yields

$$c_L = \frac{3\ell}{2G}\,\big(1 - \frac{1}{\mu\,\ell}\big) \qquad c_R = \frac{3\ell}{2G}\,\big(1 + \frac{1}{\mu\,\ell}\big)$$

Thus, at the chiral point we get

$$c_L = 0 \qquad c_R = \frac{3\ell}{G}$$

- Dual CFT: chiral? (conjecture by Li, Song & Strominger '08)

TMG at the chiral point

Definition: TMG at the chiral point is TMG with the tuning

$$\mu\,\ell = 1$$

between the cosmological constant and the Chern–Simons coupling.
Why special? (Li, Song & Strominger '08)
Calculating the central charges of the dual boundary CFT yields

$$c_L = \frac{3\ell}{2G}\,\big(1 - \frac{1}{\mu\,\ell}\big) \qquad c_R = \frac{3\ell}{2G}\,\big(1 + \frac{1}{\mu\,\ell}\big)$$

Thus, at the chiral point we get

$$c_L = 0 \qquad c_R = \frac{3\ell}{G}$$

- Dual CFT: chiral? (conjecture by Li, Song & Strominger '08)
- Dual CFT: logarithmic? (conjecture by Grumiller & Johansson '08)

Gravitons around AdS_3 in TMG

Linearization around AdS background.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Gravitons around AdS_3 in TMG

Linearization around AdS background.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

leads to linearized EOM that are third order PDE

$$G^{(1)}_{\mu\nu} + \frac{1}{\mu}\,C^{(1)}_{\mu\nu} = (\mathcal{D}^R\mathcal{D}^L\mathcal{D}^M h)_{\mu\nu} = 0 \qquad (1)$$

with three mutually commuting first order operators

$$(\mathcal{D}^{L/R})_\mu{}^\nu = \delta_\mu^\nu \pm \ell\,\varepsilon_\mu{}^{\alpha\nu}\bar{\nabla}_\alpha \qquad (\mathcal{D}^M)_\mu{}^\nu = \delta_\mu^\nu + \frac{1}{\mu}\varepsilon_\mu{}^{\alpha\nu}\bar{\nabla}_\alpha$$

Gravitons around AdS_3 in TMG

Linearization around AdS background.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

leads to linearized EOM that are third order PDE

$$G^{(1)}_{\mu\nu} + \frac{1}{\mu} C^{(1)}_{\mu\nu} = (\mathcal{D}^R \mathcal{D}^L \mathcal{D}^M h)_{\mu\nu} = 0 \qquad (1)$$

with three mutually commuting first order operators

$$(\mathcal{D}^{L/R})_\mu{}^\nu = \delta^\nu_\mu \pm \ell\, \varepsilon_\mu{}^{\alpha\nu} \bar{\nabla}_\alpha \qquad (\mathcal{D}^M)_\mu{}^\nu = \delta^\nu_\mu + \frac{1}{\mu} \varepsilon_\mu{}^{\alpha\nu} \bar{\nabla}_\alpha$$

Three linearly independent solutions to (1):

$$\big(\mathcal{D}^L h^L\big)_{\mu\nu} = 0 \qquad \big(\mathcal{D}^R h^R\big)_{\mu\nu} = 0 \qquad \big(\mathcal{D}^M h^M\big)_{\mu\nu} = 0$$

Gravitons around AdS_3 in TMG

Linearization around AdS background.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

leads to linearized EOM that are third order PDE

$$G^{(1)}_{\mu\nu} + \frac{1}{\mu} C^{(1)}_{\mu\nu} = (\mathcal{D}^R \mathcal{D}^L \mathcal{D}^M h)_{\mu\nu} = 0 \tag{1}$$

with three mutually commuting first order operators

$$(\mathcal{D}^{L/R})_\mu{}^\nu = \delta^\nu_\mu \pm \ell\, \varepsilon_\mu{}^{\alpha\nu} \bar{\nabla}_\alpha \qquad (\mathcal{D}^M)_\mu{}^\nu = \delta^\nu_\mu + \frac{1}{\mu} \varepsilon_\mu{}^{\alpha\nu} \bar{\nabla}_\alpha$$

Three linearly independent solutions to (1):

$$\left(\mathcal{D}^L h^L\right)_{\mu\nu} = 0 \qquad \left(\mathcal{D}^R h^R\right)_{\mu\nu} = 0 \qquad \left(\mathcal{D}^M h^M\right)_{\mu\nu} = 0$$

At chiral point left (L) and massive (M) branches coincide!
First hint that logarithmic CFT could emerge!

The logarithmic graviton mode

Grumiller & Johansson '08

Standard construction:

$$h_{\mu\nu}^{\log} = \lim_{\mu\ell\to 1} \frac{h_{\mu\nu}^{M}(\mu\ell) - h_{\mu\nu}^{L}}{\mu\ell - 1}$$

with property

$$\big(\mathcal{D}^L h^{\log}\big)_{\mu\nu} = \big(\mathcal{D}^M h^{\log}\big)_{\mu\nu} \neq 0\,, \qquad \big((\mathcal{D}^L)^2 h^{\log}\big)_{\mu\nu} = 0$$

The logarithmic graviton mode

Grumiller & Johansson '08

Standard construction:

$$h_{\mu\nu}^{\log} = \lim_{\mu\ell\to 1} \frac{h_{\mu\nu}^M(\mu\ell) - h_{\mu\nu}^L}{\mu\ell - 1}$$

with property

$$\big(\mathcal{D}^L h^{\log}\big)_{\mu\nu} = \big(\mathcal{D}^M h^{\log}\big)_{\mu\nu} \neq 0\,, \qquad \big((\mathcal{D}^L)^2 h^{\log}\big)_{\mu\nu} = 0$$

Log mode leads to Jordan cell structure like in LCFT:

$$H \begin{pmatrix} h^{\log} \\ h^L \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} h^{\log} \\ h^L \end{pmatrix}$$

$H = L_0 + \bar{L}_0 \sim \partial_t$ is Hamilton operator
Motivates LCFT conjecture

Outline

Motivation for 3D massive gravity and introduction to LCFTs

Topologically massive gravity

Logarithmic CFT conjecture

Consequences, Generalizations & Applications

Early hints for legitimacy of conjecture

Properties of logarithmic mode:

- Perturbative solution of linearized EOM, but not pure gauge
- Energy of logarithmic mode is finite

$$E^{\log} = -\frac{47}{1152G\,\ell^3}$$

  and negative → instability! (Grumiller & Johansson '08)
- Logarithmic mode is asymptotically AdS

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + \left(\gamma_{ij}^{(0)} e^{2\rho/\ell} + \gamma_{ij}^{(1)} \rho + \gamma_{ij}^{(0)} + \gamma_{ij}^{(2)} e^{-2\rho/\ell} + \dots\right) \mathrm{d}x^i \, \mathrm{d}x^j$$

  but violates Brown–Henneaux boundary conditions! ($\gamma_{ij}^{(1)}\big|_{\mathrm{BH}} = 0$)
- Consistent log boundary conditions replacing Brown–Henneaux (Grumiller & Johansson '08, Martinez, Henneaux & Troncoso '09)
- Brown–York stress tensor is finite and traceless, but not chiral
- Log mode persists non-perturbatively, as shown by Hamilton analysis (Grumiller, Jackiw & Johansson '08, Carlip '08)

Reminder: Requirements for gravity duals to LCFTs

Checks for purported gravity duals to logarithmic CFTs

- There exists some bulk mode corresponding to the operator $\mathcal{O}^{\log}$ OK
- Weights of $\mathcal{O}^{\log}$ must degenerate with weights of $\mathcal{O}^{L}$ OK
- Jordan cell structure of $H \sim L_0 + \bar{L}_0$ with respect to $\mathcal{O}^{L}$, $\mathcal{O}^{\log}$ OK
- Central charges must be tunable to zero by some parameter OK
- Gravity theory should exhibit non-unitarity OK
- Conformal Ward identities must hold ???

Topologically massive gravity looks promising
as candidate for a gravity dual to a logarithmic CFT!

Reminder: Requirements for gravity duals to LCFTs

Checks for purported gravity duals to logarithmic CFTs

- There exists some bulk mode corresponding to the operator $\mathcal{O}^{\log}$ OK
- Weights of $\mathcal{O}^{\log}$ must degenerate with weights of $\mathcal{O}^{L}$ OK
- Jordan cell structure of $H \sim L_0 + \bar{L}_0$ with respect to $\mathcal{O}^{L}$, $\mathcal{O}^{\log}$ OK
- Central charges must be tunable to zero by some parameter OK
- Gravity theory should exhibit non-unitarity OK
- Conformal Ward identities must hold ???

Topologically massive gravity looks promising
as candidate for a gravity dual to a logarithmic CFT!

Relevant question at this stage:

Consistency of conformal Ward identities?

Check of logarithmic CFT conjecture for 2- and 3-point correlators

If LCFT conjecture is correct then following procedure must work:

Check of logarithmic CFT conjecture for 2- and 3-point correlators

If LCFT conjecture is correct then following procedure must work:

- Calculate non-normalizable modes for left, right and logarithmic branches by solving linearized EOM on gravity side

Check of logarithmic CFT conjecture for 2- and 3-point correlators

If LCFT conjecture is correct then following procedure must work:

- Calculate non-normalizable modes for left, right and logarithmic branches by solving linearized EOM on gravity side
- According to $\text{AdS}_3/\text{LCFT}_2$ dictionary these non-normalizable modes are sources for corresponding operators in the dual CFT

Check of logarithmic CFT conjecture for 2- and 3-point correlators

If LCFT conjecture is correct then following procedure must work:

- Calculate non-normalizable modes for left, right and logarithmic branches by solving linearized EOM on gravity side
- According to $\text{AdS}_3/\text{LCFT}_2$ dictionary these non-normalizable modes are sources for corresponding operators in the dual CFT
- Calculate 2- and 3-point correlators on the gravity side, e.g. by plugging non-normalizable modes into second and third variation of the on-shell action

Check of logarithmic CFT conjecture for 2- and 3-point correlators

If LCFT conjecture is correct then following procedure must work:

- Calculate non-normalizable modes for left, right and logarithmic branches by solving linearized EOM on gravity side
- According to $AdS_3/LCFT_2$ dictionary these non-normalizable modes are sources for corresponding operators in the dual CFT
- Calculate 2- and 3-point correlators on the gravity side, e.g. by plugging non-normalizable modes into second and third variation of the on-shell action
- These correlators must coinicde with the ones of a logarithmic CFT

Check of logarithmic CFT conjecture for 2- and 3-point correlators

If LCFT conjecture is correct then following procedure must work:

- Calculate non-normalizable modes for left, right and logarithmic branches by solving linearized EOM on gravity side
- According to $\mathrm{AdS}_3/\mathrm{LCFT}_2$ dictionary these non-normalizable modes are sources for corresponding operators in the dual CFT
- Calculate 2- and 3-point correlators on the gravity side, e.g. by plugging non-normalizable modes into second and third variation of the on-shell action
- These correlators must coinicde with the ones of a logarithmic CFT

Except for value of new anomaly b_L no freedom in this procedure. Either it works or it does not work.

Check of logarithmic CFT conjecture for 2- and 3-point correlators

If LCFT conjecture is correct then following procedure must work:

- Calculate non-normalizable modes for left, right and logarithmic branches by solving linearized EOM on gravity side
- According to $AdS_3/LCFT_2$ dictionary these non-normalizable modes are sources for corresponding operators in the dual CFT
- Calculate 2- and 3-point correlators on the gravity side, e.g. by plugging non-normalizable modes into second and third variation of the on-shell action
- These correlators must coinicde with the ones of a logarithmic CFT

> Except for value of new anomaly b_L no freedom in this procedure.
> Either it works or it does not work.

- Works at level of 2-point correlators (Skenderis, Taylor & van Rees '09, Grumiller & Sachs '09)
- Works at level of 3-point correlators (Grumiller & Sachs '09)
- Value of new anomaly: $b_L = -c_R = -3\ell/G$

Alternative calculation of new anomaly b_L

As final consistency check perform the following short-cut.

Alternative calculation of new anomaly b_L

As final consistency check perform the following short-cut.

- Consider small but non-vanising central charge c_L

Alternative calculation of new anomaly b_L

As final consistency check perform the following short-cut.

- Consider small but non-vanising central charge c_L
- Then weights $h = 2 + \varepsilon$ and $\bar{h} = \varepsilon$ of massive modes differ infinitesimally from weights 2 and 0 of left mode

Alternative calculation of new anomaly b_L

As final consistency check perform the following short-cut.

- Consider small but non-vanising central charge c_L
- Then weights $h = 2 + \varepsilon$ and $\bar{h} = \varepsilon$ of massive modes differ infinitesimally from weights 2 and 0 of left mode
- The new anomaly is given by the ratio of these two small quantities

$$b_L = \lim_{\varepsilon \to 0} -\frac{c_L}{\varepsilon}$$

Alternative calculation of new anomaly b_L

As final consistency check perform the following short-cut.

- Consider small but non-vanising central charge c_L
- Then weights $h = 2 + \varepsilon$ and $\bar{h} = \varepsilon$ of massive modes differ infinitesimally from weights 2 and 0 of left mode
- The new anomaly is given by the ratio of these two small quantities

$$b_L = \lim_{\varepsilon \to 0} -\frac{c_L}{\varepsilon}$$

- Result obtained in this way must coincide with result for b_L from the 2- and 3-point correlators

Alternative calculation of new anomaly b_L

As final consistency check perform the following short-cut.

- Consider small but non-vanising central charge c_L
- Then weights $h = 2 + \varepsilon$ and $\bar{h} = \varepsilon$ of massive modes differ infinitesimally from weights 2 and 0 of left mode
- The new anomaly is given by the ratio of these two small quantities

$$b_L = \lim_{\varepsilon \to 0} -\frac{c_L}{\varepsilon}$$

- Result obtained in this way must coincide with result for b_L from the 2- and 3-point correlators

Recover the result (Grumiller & Hohm '09, Grumiller, Johansson & Zojer, to appear)

$$b_L = -\frac{3\ell}{G}$$

Alternative calculation of new anomaly b_L

As final consistency check perform the following short-cut.

- Consider small but non-vanising central charge c_L
- Then weights $h = 2 + \varepsilon$ and $\bar{h} = \varepsilon$ of massive modes differ infinitesimally from weights 2 and 0 of left mode
- The new anomaly is given by the ratio of these two small quantities

$$b_L = \lim_{\varepsilon \to 0} -\frac{c_L}{\varepsilon}$$

- Result obtained in this way must coincide with result for b_L from the 2- and 3-point correlators

Recover the result (Grumiller & Hohm '09, Grumiller, Johansson & Zojer, to appear)

$$b_L = -\frac{3\ell}{G}$$

Conclusion: all consistency tests show validity of LCFT conjecture!

Outline

Motivation for 3D massive gravity and introduction to LCFTs

Topologically massive gravity

Logarithmic CFT conjecture

Consequences, Generalizations & Applications

TMG at the chiral/logarithmic point $\mu\ell = 1$:

- 3D gravity theory with black holes and massive graviton excitations

Summary and comments

TMG at the chiral/logarithmic point $\mu\ell = 1$:

- 3D gravity theory with black holes and massive graviton excitations
- Conjectured to be dual to logarithmic CFT

Summary and comments

TMG at the chiral/logarithmic point $\mu\ell = 1$:

- 3D gravity theory with black holes and massive graviton excitations
- Conjectured to be dual to logarithmic CFT
- Conjecture passed several independent consistency tests

Summary and comments

TMG at the chiral/logarithmic point $\mu\ell = 1$:

- 3D gravity theory with black holes and massive graviton excitations
- Conjectured to be dual to logarithmic CFT
- Conjecture passed several independent consistency tests
- Non-trivial Jordan cell structure on gravity side, like in LCFT

Summary and comments

TMG at the chiral/logarithmic point $\mu\ell = 1$:

- 3D gravity theory with black holes and massive graviton excitations
- Conjectured to be dual to logarithmic CFT
- Conjecture passed several independent consistency tests
- Non-trivial Jordan cell structure on gravity side, like in LCFT
- Operator degenerates with energy-momentum tensor at the point where central charge vanishes $\rightarrow$ good indication for a LCFT

Summary and comments

TMG at the chiral/logarithmic point $\mu\ell = 1$:

- 3D gravity theory with black holes and massive graviton excitations
- Conjectured to be dual to logarithmic CFT
- Conjecture passed several independent consistency tests
- Non-trivial Jordan cell structure on gravity side, like in LCFT
- Operator degenerates with energy-momentum tensor at the point where central charge vanishes $\rightarrow$ good indication for a LCFT
- Correlators on gravity side match precisely those of LCFT

Summary and comments

TMG at the chiral/logarithmic point $\mu\ell = 1$:

- 3D gravity theory with black holes and massive graviton excitations
- Conjectured to be dual to logarithmic CFT
- Conjecture passed several independent consistency tests
- Non-trivial Jordan cell structure on gravity side, like in LCFT
- Operator degenerates with energy-momentum tensor at the point where central charge vanishes $\rightarrow$ good indication for a LCFT
- Correlators on gravity side match precisely those of LCFT
- Central charges: $c_L = 0$, $c_R = 3\ell/G$, new anomaly: $b_L = -3\ell/G$

Summary and comments

TMG at the chiral/logarithmic point $\mu\ell = 1$:

- 3D gravity theory with black holes and massive graviton excitations
- Conjectured to be dual to logarithmic CFT
- Conjecture passed several independent consistency tests
- Non-trivial Jordan cell structure on gravity side, like in LCFT
- Operator degenerates with energy-momentum tensor at the point where central charge vanishes $\rightarrow$ good indication for a LCFT
- Correlators on gravity side match precisely those of LCFT
- Central charges: $c_L = 0$, $c_R = 3\ell/G$, new anomaly: $b_L = -3\ell/G$
- LCFTs non-unitary $\leftrightarrow$ bulk gravitons negative energy

Summary and comments

TMG at the chiral/logarithmic point $\mu\ell = 1$:

- 3D gravity theory with black holes and massive graviton excitations
- Conjectured to be dual to logarithmic CFT
- Conjecture passed several independent consistency tests
- Non-trivial Jordan cell structure on gravity side, like in LCFT
- Operator degenerates with energy-momentum tensor at the point where central charge vanishes $\rightarrow$ good indication for a LCFT
- Correlators on gravity side match precisely those of LCFT
- Central charges: $c_L = 0$, $c_R = 3\ell/G$, new anomaly: $b_L = -3\ell/G$
- LCFTs non-unitary $\leftrightarrow$ bulk gravitons negative energy
- LCFTs cannot be chiral $\leftrightarrow$ Brown–York stress tensor not chiral

Summary and comments

TMG at the chiral/logarithmic point $\mu\ell = 1$:

- 3D gravity theory with black holes and massive graviton excitations
- Conjectured to be dual to logarithmic CFT
- Conjecture passed several independent consistency tests
- Non-trivial Jordan cell structure on gravity side, like in LCFT
- Operator degenerates with energy-momentum tensor at the point where central charge vanishes $\to$ good indication for a LCFT
- Correlators on gravity side match precisely those of LCFT
- Central charges: $c_L = 0$, $c_R = 3\ell/G$, new anomaly: $b_L = -3\ell/G$
- LCFTs non-unitary $\leftrightarrow$ bulk gravitons negative energy
- LCFTs cannot be chiral $\leftrightarrow$ Brown–York stress tensor not chiral

If conjecture true: first example of $AdS_3/LCFT_2$ correspondence!

Note: 1-loop calculations with Vassilevich '10 provide further indication

Generalizations to new massive gravity and generalized massive gravity

New massive gravity (Bergshoeff, Hohm & Townsend '09):

$$I_{\text{NMG}} = \frac{1}{16\pi G} \int \mathrm{d}^3x\sqrt{-g}\,\Big[\sigma R + \frac{1}{m^2}\,\Big(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}\,R^2\Big) - 2\lambda m^2\Big]$$

Similar story (Grumiller & Hohm '09, Alishahiha & Naseh '10):

- Linearized EOM around AdS_3 ($g = \bar{g} + h$)

$$\big(\mathcal{D}^R\mathcal{D}^L\mathcal{D}^M\mathcal{D}^{\bar{M}}h\big)_{\mu\nu} = 0$$

- Logarithmic point for $\lambda = 3$: $c_L = c_R = 0$
- Massive modes degenerate with left and right boundary gravitons
- 2-point correlators on gravity side match precisely those of a LCFT
- New anomalies: $b_L = b_R = -\sigma 12\ell/G$

Generalizations to new massive gravity and generalized massive gravity

New massive gravity (Bergshoeff, Hohm & Townsend '09):

$$I_{\text{NMG}} = \frac{1}{16\pi G} \int \mathrm{d}^3x\sqrt{-g}\left[\sigma R + \frac{1}{m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}R^2\right) - 2\lambda m^2\right]$$

Similar story (Grumiller & Hohm '09, Alishahiha & Naseh '10):

- Linearized EOM around AdS_3 ($g = \bar{g} + h$)

$$\left(\mathcal{D}^R\mathcal{D}^L\mathcal{D}^M\mathcal{D}^{\bar{M}}h\right)_{\mu\nu} = 0$$

- Logarithmic point for $\lambda = 3$: $c_L = c_R = 0$
- Massive modes degenerate with left and right boundary gravitons
- 2-point correlators on gravity side match precisely those of a LCFT
- New anomalies: $b_L = b_R = -\sigma 12\ell/G$

Further generalizations: Higher derivative theories (Sinha '10, Paulos '10): similar story seems likely (but potentially with higher order Jordan cells)

Potential applications in condensed matter physics

LCFTs arise in systems with quenched disorder.

Potential applications in condensed matter physics

LCFTs arise in systems with quenched disorder.

- Quenched disorder: systems with random variable that does not evolve in time

Potential applications in condensed matter physics

LCFTs arise in systems with quenched disorder.

- Quenched disorder: systems with random variable that does not evolve in time
- Examples: spin glasses, quenched random magnets, ...

Potential applications in condensed matter physics

LCFTs arise in systems with quenched disorder.

- Quenched disorder: systems with random variable that does not evolve in time
- Examples: spin glasses, quenched random magnets, ...
- For sufficient amount of disorder perturbation theory breaks down — random critical point

Potential applications in condensed matter physics

LCFTs arise in systems with quenched disorder.

- Quenched disorder: systems with random variable that does not evolve in time
- Examples: spin glasses, quenched random magnets, ...
- For sufficient amount of disorder perturbation theory breaks down — random critical point
- Infamous denominator in correlators:

$$\overline{\langle\mathcal{O}(z)\,\mathcal{O}(0)\rangle} = \int\!\mathcal{D}V\,P[V]\,\frac{\int\!\mathcal{D}\phi\,\exp\big(-I[\phi]-\int\!\mathrm{d}^2z'V(z')\mathcal{O}(z')\big)\,\mathcal{O}(z)\,\mathcal{O}(0)}{\int\!\mathcal{D}\phi\,\exp\big(-I[\phi]-\int\!\mathrm{d}^2z'V(z')\mathcal{O}(z')\big)}$$

Potential applications in condensed matter physics

LCFTs arise in systems with quenched disorder.

- Quenched disorder: systems with random variable that does not evolve in time
- Examples: spin glasses, quenched random magnets, ...
- For sufficient amount of disorder perturbation theory breaks down — random critical point
- Infamous denominator in correlators:

$$\overline{\langle\mathcal{O}(z)\,\mathcal{O}(0)\rangle} = \int\mathcal{D}V\,P[V]\,\frac{\int\mathcal{D}\phi\,\exp\big(-I[\phi]-\int\mathrm{d}^2z' V(z')\mathcal{O}(z')\big)\,\mathcal{O}(z)\,\mathcal{O}(0)}{\int\mathcal{D}\phi\,\exp\big(-I[\phi]-\int\mathrm{d}^2z' V(z')\mathcal{O}(z')\big)}$$

- Different ways to deal with denominator (replica trick, SUSY)

Potential applications in condensed matter physics

LCFTs arise in systems with quenched disorder.

- Quenched disorder: systems with random variable that does not evolve in time
- Examples: spin glasses, quenched random magnets, ...
- For sufficient amount of disorder perturbation theory breaks down — random critical point
- Infamous denominator in correlators:

$$\overline{\langle\mathcal{O}(z)\,\mathcal{O}(0)\rangle} = \int\!\mathcal{D}V\,P[V]\,\frac{\int\mathcal{D}\phi\,\exp\big(-I[\phi]-\int\mathrm{d}^2z'V(z')\mathcal{O}(z')\big)\,\mathcal{O}(z)\,\mathcal{O}(0)}{\int\mathcal{D}\phi\,\exp\big(-I[\phi]-\int\mathrm{d}^2z'V(z')\mathcal{O}(z')\big)}$$

- Different ways to deal with denominator (replica trick, SUSY)
- Result: operators degenerate and correlators acquire logarithmic behavior, exactly as in LCFT (Cardy '99)

Potential applications in condensed matter physics

LCFTs arise in systems with quenched disorder.

- Quenched disorder: systems with random variable that does not evolve in time
- Examples: spin glasses, quenched random magnets, ...
- For sufficient amount of disorder perturbation theory breaks down — random critical point
- Infamous denominator in correlators:

$$\overline{\langle\mathcal{O}(z)\,\mathcal{O}(0)\rangle} = \int\!\mathcal{D}V\,P[V]\,\frac{\int\mathcal{D}\phi\,\exp\big(-I[\phi]-\int\mathrm{d}^2z'V(z')\mathcal{O}(z')\big)\,\mathcal{O}(z)\,\mathcal{O}(0)}{\int\mathcal{D}\phi\,\exp\big(-I[\phi]-\int\mathrm{d}^2z'V(z')\mathcal{O}(z')\big)}$$

- Different ways to deal with denominator (replica trick, SUSY)
- Result: operators degenerate and correlators acquire logarithmic behavior, exactly as in LCFT (Cardy '99)
- Exploit LCFTs to compute correlators of quenched random systems

Potential applications in condensed matter physics

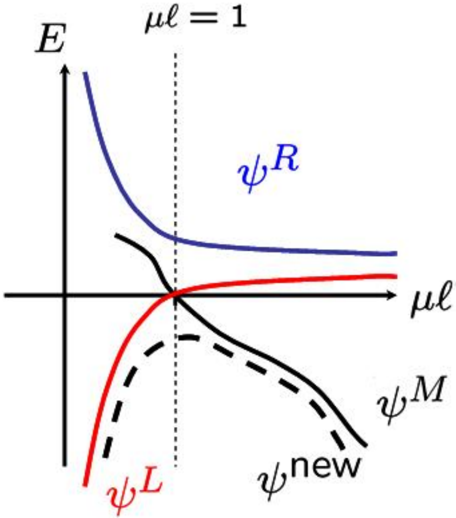
LCFTs arise in systems with quenched disorder.

- Quenched disorder: systems with random variable that does not evolve in time
- Examples: spin glasses, quenched random magnets, ...
- For sufficient amount of disorder perturbation theory breaks down — random critical point
- Infamous denominator in correlators:

$$\overline{\langle\mathcal{O}(z)\,\mathcal{O}(0)\rangle} = \int\mathcal{D}V\,P[V]\,\frac{\int\mathcal{D}\phi\,\exp\big(-I[\phi]-\int\mathrm{d}^2z'V(z')\mathcal{O}(z')\big)\,\mathcal{O}(z)\,\mathcal{O}(0)}{\int\mathcal{D}\phi\,\exp\big(-I[\phi]-\int\mathrm{d}^2z'V(z')\mathcal{O}(z')\big)}$$

- Different ways to deal with denominator (replica trick, SUSY)
- Result: operators degenerate and correlators acquire logarithmic behavior, exactly as in LCFT (Cardy '99)
- Exploit LCFTs to compute correlators of quenched random systems
- Apply $AdS_3/LCFT_2$ to describe strongly coupled LCFTs!

Thanks for your attention!



Some literature

- M. R. Gaberdiel, "An algebraic approach to logarithmic conformal field theory," Int. J. Mod. Phys. **A18** (2003) 4593 hep-th/0111260.
- D. Grumiller and N. Johansson, "Gravity duals for logarithmic conformal field theories," 1001.0002. See also Refs. therein.
- W. Li, W. Song and A. Strominger, "Chiral Gravity in Three Dimensions," JHEP **0804** (2008) 082, 0801.4566.
- D. Grumiller and N. Johansson, "Instability in cosmological topologically massive gravity at the chiral point," JHEP **0807** (2008) 134, 0805.2610.
- K. Skenderis, M. Taylor and B. C. van Rees, "Topologically Massive Gravity and the AdS/CFT Correspondence," JHEP **0909** (2009) 045 0906.4926.
- D. Grumiller and I. Sachs, "$AdS_3/LCFT_2$ – Correlators in Cosmological Topologically Massive Gravity," JHEP **1003** (2010) 012 0910.5241.