Holograms of conformal Chern–Simons gravity

Niklas Johansson

Vienna University of Technology

Uppsala, June 30, 2011

Work done with H. Afshar, B. Cvetković, S. Ertl and D. Grumiller

We study...

$$S_{\mathrm{CS}} = \frac{k}{4\pi} \int_{M^3} (\Gamma \wedge \mathrm{d}\Gamma + \frac{2}{3}\Gamma^3)$$

[Deser, Jackiw & Templeton, '82], [Horne & Witten, '89]

We study...

$$S_{\mathrm{CS}} = \frac{k}{4\pi} \int_{M^3} (\Gamma \wedge \mathrm{d}\Gamma + \frac{2}{3}\Gamma^3)$$

[Deser, Jackiw & Templeton, '82], [Horne & Witten, '89]

- Topological. (Gauge symmetries: diffeos + Weyl.)

We study...

$$S_{\mathrm{CS}} = \frac{k}{4\pi} \int_{M^3} (\Gamma \wedge \mathrm{d}\Gamma + \frac{2}{3}\Gamma^3)$$

[Deser, Jackiw & Templeton, '82], [Horne & Witten, '89]

- Topological. (Gauge symmetries: diffeos + Weyl.)
- $\partial M^3 \neq \emptyset \implies$ non-trivial dynamics.

We study...

$$S_{\mathrm{CS}} = \frac{k}{4\pi} \int_{M^3} (\Gamma \wedge \mathrm{d}\Gamma + \frac{2}{3}\Gamma^3)$$

[Deser, Jackiw & Templeton, '82], [Horne & Witten, '89]

- Topological. (Gauge symmetries: diffeos + Weyl.)
- $\partial M^3 \neq \emptyset \implies$ non-trivial dynamics.
- Holographic description: Partially massless gravitons, Brown–York responses, correlators... [arXiv:1107.xxxx]

We study...

$$S_{\mathrm{CS}} = \frac{k}{4\pi}\int_{M^3}(\Gamma \wedge \mathrm{d}\Gamma + \frac{2}{3}\Gamma^3)$$

[Deser, Jackiw & Templeton, '82], [Horne & Witten, '89]

- Topological. (Gauge symmetries: diffeos + Weyl.)
- $\partial M^3 \neq \emptyset \implies$ non-trivial dynamics.
- Holographic description: Partially massless gravitons, Brown–York responses, correlators... [arXiv:1107.xxxx]

- This talk: What does the Weyl symmetry give rise to at the boundary?

Warm-up: (2+1)-dimensional EH gravity

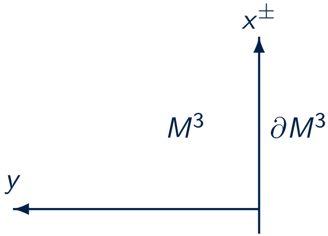
$S_{\mathrm{EH}} = \int_{M^3} \sqrt{-g}(R - 2\Lambda)$ Gauge symmetry: $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)}$

Warm-up: (2+1)-dimensional EH gravity

$S_{\mathrm{EH}} = \int_{M^3} \sqrt{-g}(R - 2\Lambda)$ Gauge symmetry: $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)}$

Boundary conditions: [Brown, Henneaux '86]

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu} = g_{\mu\nu}^{\mathrm{AdS}} + \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(y) \\ & \mathcal{O}(1) & \mathcal{O}(y) \\ & & \mathcal{O}(1) \end{pmatrix}_{\mu\nu}$$



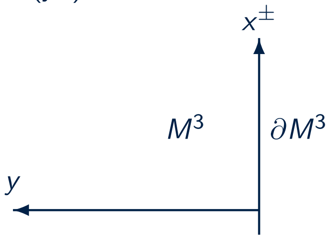
Warm-up: (2+1)-dimensional EH gravity

$S_{\mathrm{EH}} = \int_{M^3} \sqrt{-g}(R - 2\Lambda)$ Gauge symmetry: $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)}$

Boundary conditions: [Brown, Henneaux '86]

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu} = g_{\mu\nu}^{\mathrm{AdS}} + \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(y) \\ & \mathcal{O}(1) & \mathcal{O}(y) \\ & & \mathcal{O}(1) \end{pmatrix}_{\mu\nu}$$

Diffeos that preserve the BCs: $\xi^{\pm} = \epsilon^{\pm}(x^{\pm}) + \mathcal{O}(y^2)$



Warm-up: (2+1)-dimensional EH gravity

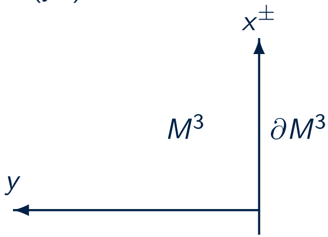
$S_{\mathrm{EH}} = \int_{M^3} \sqrt{-g}(R - 2\Lambda)$ Gauge symmetry: $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)}$

Boundary conditions: [Brown, Henneaux '86]

$$g_{\mu\nu} = g^{\mathrm{AdS}}_{\mu\nu} + h_{\mu\nu} = g^{\mathrm{AdS}}_{\mu\nu} + \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(y) \\ & \mathcal{O}(1) & \mathcal{O}(y) \\ & & \mathcal{O}(1) \end{pmatrix}_{\mu\nu}$$

Diffeos that preserve the BCs: $\xi^{\pm} = \epsilon^{\pm}(x^{\pm}) + \mathcal{O}(y^2)$

- Canonical realization $\Longrightarrow$ boundary charge!



Warm-up: (2+1)-dimensional EH gravity

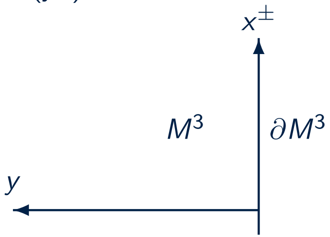
$S_{\mathrm{EH}} = \int_{M^3} \sqrt{-g}(R - 2\Lambda)$ Gauge symmetry: $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)}$

Boundary conditions: [Brown, Henneaux '86]

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu} = g_{\mu\nu}^{\mathrm{AdS}} + \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(y) \\ & \mathcal{O}(1) & \mathcal{O}(y) \\ & & \mathcal{O}(1) \end{pmatrix}_{\mu\nu}$$

Diffeos that preserve the BCs: $\xi^{\pm} = \epsilon^{\pm}(x^{\pm}) + \mathcal{O}(y^2)$

- Canonical realization $\Longrightarrow$ boundary charge!
- Central extension of the algebra!



Warm-up: (2+1)-dimensional EH gravity

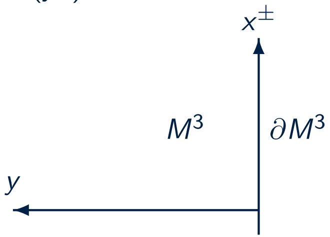
$S_{\mathrm{EH}} = \int_{M^3} \sqrt{-g}(R - 2\Lambda)$ Gauge symmetry: $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)}$

Boundary conditions: [Brown, Henneaux '86]

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu} = g_{\mu\nu}^{\mathrm{AdS}} + \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(y) \\ & \mathcal{O}(1) & \mathcal{O}(y) \\ & & \mathcal{O}(1) \end{pmatrix}_{\mu\nu}$$

Diffeos that preserve the BCs: $\xi^{\pm} = \epsilon^{\pm}(x^{\pm}) + \mathcal{O}(y^2)$

- Canonical realization $\Longrightarrow$ boundary charge!
- Central extension of the algebra!
- Gauge symmetry $\rightarrow$ global symmetry!



Warm-up: (2+1)-dimensional EH gravity

$S_{\mathrm{EH}} = \int_{M^3} \sqrt{-g}(R - 2\Lambda)$ Gauge symmetry: $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)}$

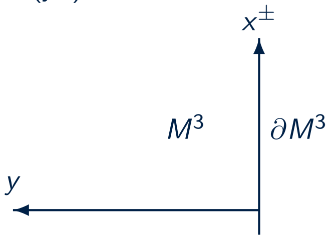
Boundary conditions: [Brown, Henneaux '86]

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu} = g_{\mu\nu}^{\mathrm{AdS}} + \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(y) \\ & \mathcal{O}(1) & \mathcal{O}(y) \\ & & \mathcal{O}(1) \end{pmatrix}_{\mu\nu}$$

Diffeos that preserve the BCs: $\xi^{\pm} = \epsilon^{\pm}(x^{\pm}) + \mathcal{O}(y^2)$

- Canonical realization $\implies$ boundary charge!
- Central extension of the algebra!
- Gauge symmetry $\to$ global symmetry!

$$i\{L_n, L_m\} = (n-m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m}$$



Warm-up: (2+1)-dimensional EH gravity

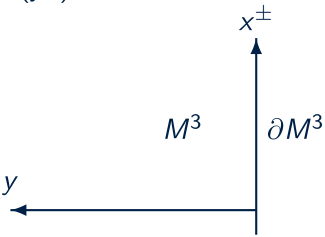
$S_{\mathrm{EH}} = \int_{M^3} \sqrt{-g}(R - 2\Lambda)$ Gauge symmetry: $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)}$

Boundary conditions: [Brown, Henneaux '86]

$$g_{\mu\nu} = g^{\mathrm{AdS}}_{\mu\nu} + h_{\mu\nu} = g^{\mathrm{AdS}}_{\mu\nu} + \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(y) \\ & \mathcal{O}(1) & \mathcal{O}(y) \\ & & \mathcal{O}(1) \end{pmatrix}_{\mu\nu}$$

Diffeos that preserve the BCs: $\xi^{\pm} = \epsilon^{\pm}(x^{\pm}) + \mathcal{O}(y^2)$

- Canonical realization $\Longrightarrow$ boundary charge!
- Central extension of the algebra!
- Gauge symmetry $\rightarrow$ global symmetry!



$$i\{\bar{L}_n, \bar{L}_m\} = (n-m)\bar{L}_{n+m} + \frac{\bar{c}}{12}(n^3 - n)\delta_{n+m}$$

Conformal Chern-Simons gravity

Gauge sym: $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)}$, $\delta g_{\mu\nu} = 2\Omega(x) g_{\mu\nu}$

Conformal Chern-Simons gravity

Gauge sym: $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)}$, $\delta g_{\mu\nu} = 2\Omega(x) g_{\mu\nu}$

Boundary conditions: [arXiv:1107.xxxx].

$$g_{\mu\nu} = e^{\phi(x,y)} \left(g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu} \right)$$

Conformal Chern-Simons gravity

Gauge sym: $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)}$, $\delta g_{\mu\nu} = 2\Omega(x) g_{\mu\nu}$

Boundary conditions: [arXiv:1107.xxxx].

$$g_{\mu\nu} = e^{\phi(x,y)} \left(g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu} \right)$$

Gauge trafo's that preserve the BCs: depends on BC on ϕ.

Conformal Chern-Simons gravity

$$g_{\mu\nu} = e^{\phi(x,y)} \left(g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu}\right)$$

Conformal Chern-Simons gravity

$g_{\mu\nu} = e^{\phi(x,y)} \left(g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu} \right)$

$$\phi \equiv 0 : \qquad \textit{diffeo} \quad \rightarrow \quad Vir^2, c = -\bar{c}.$$
$$\textit{Weyl} \quad \rightarrow \quad \text{trivial}$$

Conformal Chern-Simons gravity

$g_{\mu\nu} = e^{\phi(x,y)} \left(g^{\mathrm{AdS}}_{\mu\nu} + h_{\mu\nu} \right)$

$$
\begin{aligned}
\phi \equiv 0: &\quad \textit{diffeo} \quad \rightarrow \quad Vir^2, c = -\bar{c}. \\
&\quad \textit{Weyl} \quad \rightarrow \quad \text{trivial} \\
\phi \equiv f_{\text{fixed}}(x) + \ldots: &\quad \textit{diffeo} + \textit{Weyl} \quad \rightarrow \quad Vir^2, c = -\bar{c} \\
&\quad \textit{Weyl} \rightarrow \quad \text{trivial}
\end{aligned}
$$

Conformal Chern-Simons gravity

$g_{\mu\nu} = e^{\phi(x,y)} \left(g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu}\right)$

$$
\begin{array}{lll}
\phi \equiv 0 : & \textit{diffeo} \quad \rightarrow \quad Vir^2, c = -\bar{c}. \\
 & \textit{Weyl} \quad \rightarrow \quad \text{trivial} \\
\phi \equiv f_{\mathrm{fixed}}(x) + \ldots : & \textit{diffeo} + \textit{Weyl} \quad \rightarrow \quad Vir^2, c = -\bar{c} \\
 & \textit{Weyl} \rightarrow \quad \text{trivial} \\
\phi = f_{\mathrm{free}}(x) + \ldots : & \textit{diffeo} + \textit{Weyl} \quad \rightarrow \quad Vir^2, c = -\bar{c} \\
 & \textit{Weyl} \quad \rightarrow \quad \text{nontrivial charge}
\end{array}
$$

Conformal Chern-Simons gravity

$g_{\mu\nu} = e^{\phi(x,y)} \left(g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu} \right)$

$$
\begin{aligned}
\phi \equiv 0 : &\quad \textit{diffeo} \quad \rightarrow \quad Vir^2, c = -\bar{c}. \\
&\quad \textit{Weyl} \quad \rightarrow \quad \text{trivial} \\
\phi \equiv f_{\mathrm{fixed}}(x) + \ldots : &\quad \textit{diffeo} + \textit{Weyl} \quad \rightarrow \quad Vir^2, c = -\bar{c} \\
&\quad \textit{Weyl} \rightarrow \quad \text{trivial} \\
\phi = f_{\mathrm{free}}(x) + \ldots : &\quad \textit{diffeo} + \textit{Weyl} \quad \rightarrow \quad Vir^2, c = -\bar{c} \\
&\quad \textit{Weyl} \quad \rightarrow \quad \text{nontrivial charge}
\end{aligned}
$$

Conservation of the Weyl charge $\implies$ consistency conditions.

Conformal Chern-Simons gravity

$g_{\mu\nu} = e^{\phi(x,y)} \left(g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu} \right)$

Conservation of the Weyl charge $\implies$ consistency conditions.

Simplest case: $\phi = \phi(x^+)$ and $\Omega = \Omega(x^+)$

Conformal Chern-Simons gravity

$g_{\mu\nu} = e^{\phi(x,y)} \left(g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu} \right)$

Conservation of the Weyl charge $\implies$ consistency conditions.

Simplest case: $\phi = \phi(x^+)$ and $\Omega = \Omega(x^+)$

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c}{12}\,(n^3 - n)\,\delta_{n+m}$$
$$[\bar{L}_n,\, \bar{L}_m] = (n-m)\, \bar{L}_{n+m} + \frac{\bar{c}}{12}\,(n^3 - n)\,\delta_{n+m}$$
$$[\mathcal{J}_n,\, \mathcal{J}_m] = 2k\, n\, \delta_{n+m}$$

Conformal Chern-Simons gravity

$g_{\mu\nu} = e^{\phi(x,y)} \left(g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu} \right)$

Conservation of the Weyl charge $\implies$ consistency conditions.

Simplest case: $\phi = \phi(x^+)$ and $\Omega = \Omega(x^+)$

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c+1}{12}\,(n^3 - n)\,\delta_{n+m}$$
$$[\bar{L}_n,\, \bar{L}_m] = (n-m)\, \bar{L}_{n+m} + \frac{\bar{c}}{12}\,(n^3 - n)\,\delta_{n+m}$$
$$[\mathcal{J}_n,\, \mathcal{J}_m] = 2k\, n\, \delta_{n+m}$$
$$[L_n,\, \mathcal{J}_m] = -m\, \mathcal{J}_{n+m}$$

Pure diffeos $\implies$ Sugawara shift $L_n \to L_n + \sum_k \mathcal{J}_k \mathcal{J}_{n-k}$.

Conformal Chern-Simons gravity

$g_{\mu\nu} = e^{\phi(x,y)} \left(g_{\mu\nu}^{\mathrm{AdS}} + h_{\mu\nu} \right)$

Conservation of the Weyl charge $\implies$ consistency conditions.

Simplest case: $\phi = \phi(x^+)$ and $\Omega = \Omega(x^+)$

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c+1}{12}\,(n^3 - n)\,\delta_{n+m}$$
$$[\bar{L}_n,\, \bar{L}_m] = (n-m)\, \bar{L}_{n+m} + \frac{\bar{c}}{12}\,(n^3 - n)\,\delta_{n+m}$$
$$[\mathcal{J}_n,\, \mathcal{J}_m] = 2k\, n\, \delta_{n+m}$$
$$[L_n,\, \mathcal{J}_m] = -m\, \mathcal{J}_{n+m}$$

Pure diffeos $\implies$ Sugawara shift $L_n \to L_n + \sum_k \mathcal{J}_k \mathcal{J}_{n-k}$.

Thank you!