Rindler Holography

Daniel Grumiller

Institute for Theoretical Physics
TU Wien

Workshop on Topics in Three Dimensional Gravity
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FWF

based on work w. H. Afshar, S. Detournay, W. Merbis, (B. Oblak), A. Perez, D. Tempo, R. Troncoso

Outline

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Motivation

Near horizon boundary conditions

Soft Heisenberg hair

Soft hairy black hole entropy

Concluding comments

There is a well-known system with many microstates studied for a long time (recently with help of computers)

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Go: $\approx 10^{172}$ microstates ($S_{\text{Go}} \approx 396$) $\to$ black holes more complicated!

Black hole microstates

Bekenstein–Hawking

$$S_{\rm BH} = \frac{A}{4G_N}[\text{for } M_\odot : e^{S_{\rm BH}} \sim \mathcal{O}(e^{10^{76}}) \sim e^{\text{chess microstates}}]$$

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- Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...) $\rightarrow$ see talk by Stephane Detournay!

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- Main idea: consider near horizon symmetries for non-extremal horizons
- Near horizon line-element with Rindler acceleration a:

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

Meaning of coordinates:

- ρ: radial direction ($\rho = 0$ is horizon)
- $\varphi \sim \varphi + 2\pi$: angular direction
- v: (advanced) time

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of Rindler metric

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  We make this choice in this talk!

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- Work in 3d Einstein gravity in Chern–Simons formulation (see talk by Jorge Zanelli!)

$$I_{\text{CS}} = \pm \sum_{\pm} \frac{k}{4\pi} \int \langle A^\pm \wedge \mathrm{d}A^\pm + \tfrac{2}{3} A^\pm \wedge A^\pm \wedge A^\pm \rangle$$

  with $sl(2)$ connections $A^\pm$ and $k = \ell/(4G_N)$ with AdS radius $\ell = 1$

Outline

Diagonal gauge

Standard trick: partially fix gauge

$$A^{\pm} = b_{\pm}^{-1}(\rho)\,\big(\mathrm{d}+\mathfrak{a}_{\pm}(x^0,\,x^1)\big)b_{\pm}(\rho)$$

with some group element $b \in SL(2)$ depending on radius ρ

Drop $\pm$ decorations in most of talk

Manifold topologically a cylinder or torus, with radial coordinate ρ and boundary coordinates $(x^0, x^1) \sim (v, \varphi)$

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- Standard AdS_3 approach: highest weight gauge

$$\mathfrak{a} \sim L_+ + \mathcal{L}(x^0,\, x^1) L_- \qquad b(\rho) = \exp\left(\rho L_0\right)$$

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- Precise boundary conditions (ζ: chemical potential):

  $$\mathfrak{a} = \left(\mathcal{J}\ \mathrm{d}\varphi + \zeta\ \mathrm{d}v\right)\ L_0$$

  and $b = \exp\left(\frac{1}{\zeta}\, L_1\right) \cdot \exp\left(\frac{\rho}{2}\, L_{-1}\right)$. (assume constant ζ for simplicity)

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$$\mathrm{d}s^2 = -2a\rho f\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho - 2\omega a^{-1}\,\mathrm{d}\varphi\,\mathrm{d}\rho + 4\omega\rho f\,\mathrm{d}v\,\mathrm{d}\varphi + \left[\gamma^2 + \frac{2\rho}{a}f(\gamma^2 - \omega^2)\right]\mathrm{d}\varphi^2$$

state-dependent functions $\mathcal{J}^\pm = \gamma \pm \omega$, chemical potentials $\zeta^\pm = -a \pm \Omega$

For simplicity set $\Omega = 0$ and $a = \text{const.}$ in metric above

EOM imply $\partial_v \mathcal{J}^\pm = \pm\partial_\varphi \zeta^\pm$; in this case $\partial_v \mathcal{J}^\pm = 0$

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Neglecting rotation terms ($\omega = 0$) yields Rindler plus higher order terms:

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- Recover desired near horizon metric

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Comments:

- Recover desired near horizon metric
- Rindler acceleration a indeed state-independent
- Two state-dependent functions (γ, ω) as usual in 3d gravity

Canonical boundary charges

- Canonical boundary charges non-zero for large trafos that preserve boundary conditions
- Zero mode charges: mass and angular momentum

For covariant approach to boundary charges see e.g. talks by Kamal Hajian, Ali Seraj, Hossein Yavartanoo

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Background independent result for Chern–Simons yields

$$Q[\eta] = \frac{k}{4\pi} \oint \mathrm{d}\varphi \, \eta(\varphi) \, \mathcal{J}(\varphi)$$

- Finite
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- Conserved
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Meaningful near horizon boundary conditions and non-trivial theory!

Outline

Near horizon symmetry algebra

- Near horizon symmetry algebra = all near horizon boundary conditions preserving trafos, modulo trivial gauge trafos

  Most general trafo

  $$\delta_\epsilon \mathfrak{a} = \mathrm{d}\epsilon + [\mathfrak{a},\, \epsilon] = \mathcal{O}(\delta\mathfrak{a})$$

  that preserves our boundary conditions for constant ζ given by

  $$\epsilon = \epsilon^+ L_+ + \eta L_0 + \epsilon^- L_-$$

  with

  $$\partial_v \eta = 0$$

  implying

  $$\delta_\epsilon \mathcal{J} = \partial_\varphi \eta$$

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- Expand charges in Fourier modes

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- Map

$$P_0 = J_0^+ + J_0^- \qquad P_n = \tfrac{i}{kn} (J_{-n}^+ + J_{-n}^-) \text{ if } n \neq 0 \qquad X_n = J_n^+ - J_n^-$$

  yields Heisenberg algebra (with Casimirs X_0, P_0)

$$[X_n, \, X_m] = [P_n, \, P_m] = [X_0, \, P_n] = [P_0, \, X_n] = 0$$
$$[X_n, \, P_m] = i\delta_{n,m} \quad \text{if } n \neq 0$$

Soft hair

- Vacuum descendants $|\psi(q)\rangle$

$$|\psi(q)\rangle \sim \prod (J^+_{-n_i^+})^{m_i^+} \prod (J^-_{-n_i^-})^{m_i^-} |0\rangle$$

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commutes with all generators of algebra

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Soft hair = zero energy excitations on horizon

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$$S = 2\pi(J_0^+ + J_0^-) = \frac{A}{4G_N}$$

calculated directly in Chern–Simons formulation

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Before addressing microstates consider map to aymptotic variables

Map to asymptotic variables

- Usual asymptotic AdS_3 connection with chemical potential μ:

$$\hat{A} = \hat{b}^{-1}\big(\,\mathrm{d}+\hat{\mathfrak{a}}\big)\hat{b} \qquad \hat{\mathfrak{a}}_\varphi = L_1 - \tfrac{1}{2}\,\mathcal{L}\,L_{-1}$$

$$\hat{b} = e^{\rho L_0} \quad \hat{\mathfrak{a}}_t = \mu L_1 - \mu' L_0 + \big(\tfrac{1}{2}\,\mu'' - \tfrac{1}{2}\,\mathcal{L}\mu\big)\,L_{-1}$$

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- Get Virasoro with non-zero central charge $\delta\mathcal{L} = 2\mathcal{L}\varepsilon' + \mathcal{L}'\varepsilon - \varepsilon'''$

Remarks on asymptotic and near horizon variables

- Asymptotic spin-2 currents fulfill Virasoro algebra, but charges obey still Heisenberg algebra

$$\delta Q = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\varepsilon\,\delta\mathcal{L} = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\eta\,\delta\mathcal{J}$$

Reason: asymptotic "chemical potentials" μ depend on near horizon charges $\mathcal{J}$ and chemical potentials ζ

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$$\delta Q = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\, \varepsilon\, \delta\mathcal{L} = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\, \eta\, \delta\mathcal{J}$$

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  Solved automatically from map to asymptotic observables; reminder:

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Near horizon boundary conditions natural for near horizon observer

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Precise numerical factor in twist term crucial for correct results

Warped CFT counting

See talk by Stephane Detournay

- Map near horizon algebra $J_n^{\pm} = \frac{1}{2}(J_n \pm K_n)$

$$Y_n \sim \sum J_{n-p} K_p \qquad T_n \sim J_n$$

to centerless warped conformal algebra

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- Assuming $J^{\text{vac}} = 0$ yields

$$S = \beta H = S_{\text{BH}}$$

  Hamiltonian H is product of BH entropy and Unruh temperature

Outline

Comparison to related approaches

- Brown, Henneaux '86

  Our boundary conditions differ from Brown–Henneaux — their chemical potentials depend on our charges and chemical potentials!

> Virasoro composite in terms of Heisenberg algebra

Comparison to related approaches

- Brown, Henneaux '86
- Donnay, Giribet, González, Pino 1511.08687 — see talk by Miguel Pino!
  - Observed already $H = TS_{\text{BH}}$
  - Changing our bc's to

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho - 2\omega a^{-1}\,\mathrm{d}\varphi\,\mathrm{d}\rho + 4\omega\rho\,\mathrm{d}v\,\mathrm{d}\varphi + \big[\gamma^2 + \tfrac{2\rho}{a}(\gamma^2 - \omega^2)\big]\,\mathrm{d}\varphi^2 + \mathcal{O}(\rho^2)$$

yields AKVs

$$\xi = T(\varphi)\partial_v + Y(\varphi)\partial_\varphi + \mathcal{O}(\rho^3)$$

  - Up to subleading terms same AKVs as DGGP

But: T and Y state-dependent for our boundary conditions!

Comment: map to Brown–Henneaux variables requires second chemical potential, not just Rindler acceleration!

Warped CFT algebra composite in terms of Heisenberg algebra

Comparison to related approaches

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- Afshar, Detournay, DG, Oblak 1512.08233 — see talk by Hamid Afshar!

> Rindler acceleration state-dependent in that approach

> Twisted warped CFT algebra composite in terms of Heisenberg algebra

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- Afshar, Detournay, DG, Oblak 1512.08233 — see talk by Hamid Afshar!
- Hawking, Perry, Strominger 1601.00921 — see also talk by Geoffrey Compère!
    - We constructed explicitly gravitational soft hair
    - We find no soft hair contribution to black hole entropy*
    - BMS_3 follows from Sugawara-like construction from Heisenberg algebra

> BMS algebra (supertranslations + superrotation) composite in terms of near horizon Heisenberg algebra

* See comment by Jan de Boer on Tuesday!

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- Comment on complementarity:
    - Asymptotic Virasoro algebra composite from near horizon perspective
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    - In particular, asymptotic chemical potentials depend on near horizon charges and chemical potentials

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- Li, Lucietti 1312.2626 — 3d black holes and descendants

Elaborations and generalizations

- More on dual field theory — to be done
- Flat space
  - Similar story works!
  - Get centerless BMS_3 as composite algebra from Heisenberg algebra!
  - Soft hairy flat space cosmologies
  - Asymptotic chemical potentials again depend on near horizon charges and chemical potentials
  - Obtain again Bekenstein–Hawking entropy with no soft hair contribution

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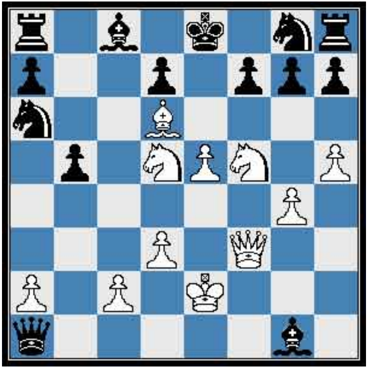
- 4d — Does it work? Is there soft Heisenberg hair? Is BMS_4 composite? What are near horizon symmetries?

> Near horizon symmetries shed new light on soft hair, microstate counting and complementarity

Thanks for your attention!



H. Afshar, S. Detournay, D. Grumiller, W. Merbis, A. Perez, D. Tempo and R. Troncoso
"Soft Heisenberg hair on black holes in three dimensions,"
1603.04824

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