Holography in three dimensions

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Outline

Motivations

Holography basics

Applications

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Holography basics

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General motivations

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  - Address conceptual issues of quantum gravity

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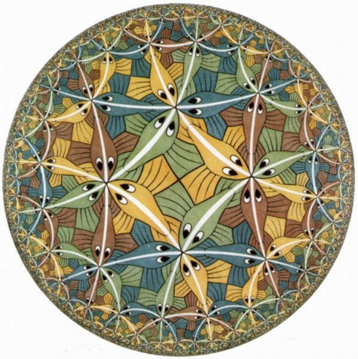
  - Holographic principle realized in Nature? (yes/no)



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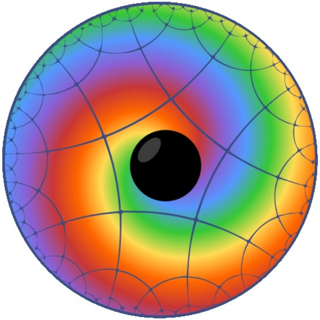

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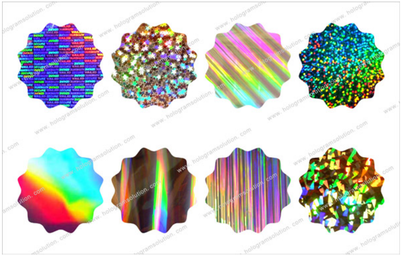

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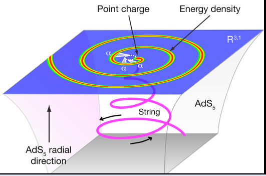
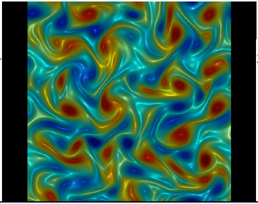
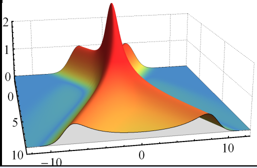
- Applications
  - Gauge gravity correspondence (plasmas, condensed matter, ...)







Specific motivation for 3D

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Address these issues in 3D!

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Interesting dichotomy:

- Either dual field theory exists $\rightarrow$ useful toy model for quantum gravity
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This talk:

- Remain agnostic about dichotomy
- Focus on generic features of dual field theories that do not require string theory embedding

Gravity in 3D
AdS_3 gravity

- Lowest dimension with black holes and (off-shell) gravitons

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- Einstein gravity: no on-shell gravitons

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- Dual field theory (if it exists): 2D

Interesting generic constraints from CFT_2!
e.g. Hellerman '09, Hartman, Keller, Stoica '14

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Caveat: while there are many string compactifications with AdS_3 factor, applying holography just to AdS_3 factor does not capture everything!

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle

   Example: bulk theory = EH

   $$I \sim \int \mathrm{d}^3x \sqrt{|g|}\,\big(R + 2/\ell^2\big)$$

   use Dirichlet boundary value problem (keep fixed δg at boundary)

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions

   Example: asymptotically AdS background with Brown–Henneaux boundary conditions

$$g \sim \begin{pmatrix} g_{++} = \mathcal{O}(1) & g_{+-} = e^{2\rho/\ell} + \mathcal{O}(1) & g_{+\rho} = \mathcal{O}(e^{-2\rho/\ell}) \\ & g_{--} = \mathcal{O}(1) & g_{-\rho} = \mathcal{O}(e^{-2\rho/\ell}) \\ & & g_{\rho\rho} = 1 + \mathcal{O}(e^{-2\rho/\ell}) \end{pmatrix}$$

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's

    - Find and classify all constraints
    - Construct canonical gauge generators
    - Add boundary terms and get (variation of) canonical charges
    - Check integrability of canonical charges
    - Check finiteness of canonical charges
    - Check conservation (in time) of canonical charges
    - Calculate Dirac bracket algebra of canonical charges

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
4. Derive (classical) asymptotic symmetry algebra and central charges

Example:

$$i\{L_n,\, L_m\}_{D.b.} = (n-m)\, L_{n+m} + \frac{c}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

with

$$c = \frac{3\ell}{2G}$$

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Trivial example:

$$i\{\,,\,\}_{D.b.} \to [\,,\,]$$

Less trivial example: Polyakov Bershadsky algebra in spin-3 gravity (finite quantum shifts of structure functions at finite central charge c, e.g. $c \to c + 22/5$ in W_3)

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6. Study unitary representations of quantum ASA

Example: unitary highest weight representations of Virasoro algebra

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   Example: it must be a CFT with central charge $c = \frac{3\ell}{2G}$

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Many examples!

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In this talk:

Apply algorithm above to 3D (higher spin) gravity in Chern–Simons formulation

Bulk theory and variational principle

Chern–Simons theory with some gauge algebra that contains either $sl(2) \times sl(2)$ or $isl(2)$

$$I = \frac{k}{4\pi} \int_{\mathcal{M}} \mathsf{Tr}(A \wedge \mathrm{d}A + \tfrac{2}{3} A \wedge A \wedge A) + B[A]$$

with boundary term $B[A] = 0$ or

$$B[A] = \frac{k}{4\pi} \int_{\partial\mathcal{M}} \mathsf{Tr}(A_+ \, \mathrm{d}x^+ \, A_- \, \mathrm{d}x^-)$$

Variational principle consistent for Dirichlet, Neumann or more general boundary conditions (assume topology of cylinder or torus).

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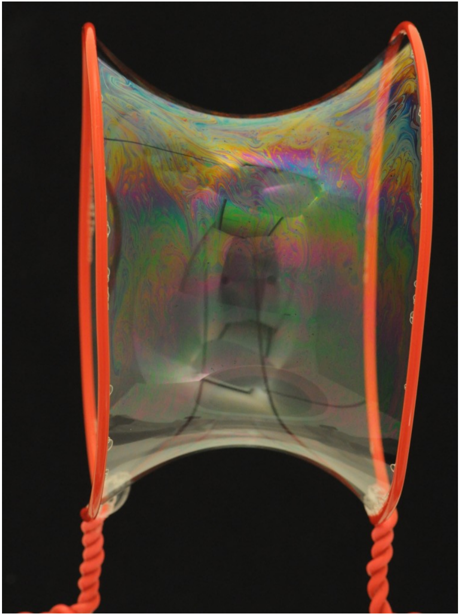
Field equations:

$$F = \mathrm{d}A + [A, \, A] = 0$$

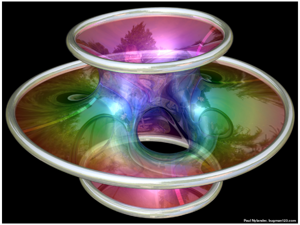
A locally pure gauge $\Rightarrow$ physics largely defined by boundary behavior!

Picturesque analogy: soap films



Both soap films and Chern–Simons theories have

- essentially no bulk dynamics
- highly non-trivial boundary dynamics
- most of the physics determined by boundary conditions
- esthetic appeal (at least for me)



Examples

- Einstein gravity in AdS_3
  Brown, Henneaux '86
  Bañados '99

Examples

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  Afshar, Cvetkovic, Ertl, DG, Johansson '11

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- Flat space Einstein gravity
  Barnich, Compere '06
  Barnich, Gonzalez '13

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  Bagchi, Detournay, DG '12
  Afshar '13

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- Higher spin gravity in AdS_3
  Henneaux, Rey '10
  Campoleoni, Fredenhagen, Pfenninger, Theisen '10

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- Flat space chiral gravity
- Higher spin gravity in AdS_3
- Non-AdS higher spin gravity
  Gary, DG, Rashkov '12
  Afshar, Gary, DG, Rashkov, Riegler '12

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- Flat space chiral gravity
- Higher spin gravity in AdS_3
- Non-AdS higher spin gravity
- Lobachevsky holography
  Bertin, Ertl, Ghorbani, DG, Johansson, Vassilevich '12
  Afshar, Gary, DG, Rashkov, Riegler '12

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- Higher spin gravity in AdS_3
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- Lobachevsky holography
- Higher spin Lifshitz holography
  Gutperle, Hijano, Samani '13
  Gary, DG, Rashkov, Rey '14

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- ... and many more (Schrödinger, warped AdS, more general backgrounds with anisotropic scale invariance, less symmetric asymptotic backgrounds, to be discovered)

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Background and fluctuations

Take suitable group element b (often: $b = e^{\rho L_0}$) and make Ansatz for connection

$$A = b^{-1}\left(\mathrm{d} + \hat{a}^{(0)} + a^{(0)} + a^{(1)}\right) b$$

- $\hat{a}^{(0)} \sim \mathcal{O}(1)$: determines asymptotic background
- $a^{(0)} \sim \mathcal{O}(1)$: determines state-dependent fluctuations
- $a^{(1)} \sim o(1)$: sub-leading fluctuations

Boundary-condition preserving gauge transformations generated by ϵ

$$\epsilon = b^{-1}\left(\epsilon^{(0)} + \epsilon^{(1)}\right) b$$

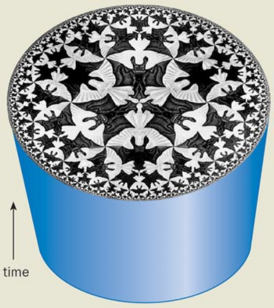
with $\epsilon^{(0)} \sim \mathcal{O}(1)$ (subject to constraints) and $\epsilon^{(1)} \sim o(1)$
Metric is then determined from

$$g_{\mu\nu} = \frac{1}{2}\,\mathrm{Tr}\,\left[A^e_\mu\, A^e_\nu\right]$$

where A^e is a suitable projection of A identified with the (zu-)vielbein

Example: AdS holography in Einstein gravity

Cartoon of AdS_3:



Asymptotic AdS background:

$$\mathrm{d}s^2 \sim \mathrm{d}\rho^2 + e^{2\rho}\,2\,\mathrm{d}x^+\,\mathrm{d}x^-$$

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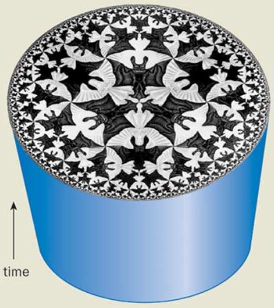

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Connection decomposed into two $sl(2)$ parts,
$A = b^{-1}(\mathrm{d}+\hat{a}^{(0)} + a^{(0)})b$ and similarly for $\bar{A}$:

$$\underbrace{b = e^{\rho L_0}}_{\text{group element}} \qquad \underbrace{\hat{A} = b^{-1}(\mathrm{d}+\hat{a}^{(0)})b}_{\text{background}} \qquad \underbrace{\Delta A = b^{-1} a^{(0)} b}_{\text{state dependent}}$$

Neglect trivial pure gauge contribution from $a^{(1)}$

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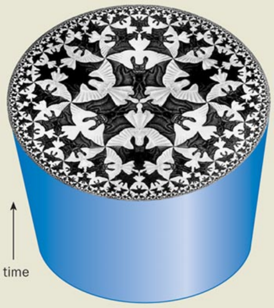

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$$\hat{a}_\rho^{(0)} = 0 \qquad \Rightarrow \qquad \hat{A}_\rho = L_0$$

$$\hat{a}_+^{(0)} = L_1 \qquad \Rightarrow \qquad \hat{A}_+ = e^\rho L_1$$

$$\hat{a}_-^{(0)} = 0 \qquad \Rightarrow \qquad \hat{A}_- = 0$$

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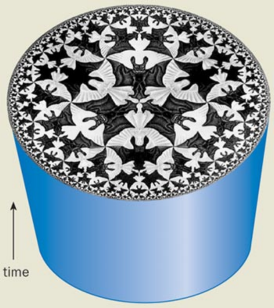

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$$\begin{aligned} \hat{a}^{(0)}_\rho &= 0 & \Rightarrow \quad & \hat{A}_\rho = L_0 \\ \hat{a}^{(0)}_+ &= L_1 & \Rightarrow \quad & \hat{A}_+ = e^\rho L_1 \\ \hat{a}^{(0)}_- &= 0 & \Rightarrow \quad & \hat{A}_- = 0 \end{aligned}$$

State-dependent contribution $A = \hat{A} + \Delta A$:

$$a^{(0)}_+ = \mathcal{L}(x^+)\, L_{-1} \qquad \Rightarrow \qquad \Delta A_+ = e^{-\rho}\, \mathcal{L}(x^+)\, L_{-1}$$

Metric:

$$g_{\mu\nu} = \frac{1}{2}\,\mathrm{Tr}\left[(A_\mu - \bar{A}_\mu)(A_\nu - \bar{A}_\nu)\right]$$

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Background independent result:

$$\delta Q[\epsilon] = \frac{k}{2\pi} \oint \mathrm{Tr}\left(\epsilon^{(0)}\, \delta a_\varphi^{(0)}\, \mathrm{d}\varphi\right)$$

- Manifestly finite! ($|\delta Q| < \infty$)
- Non-trivial? (δQ state-dependent?)
- Integrable? ($\delta Q \to Q$?)
- Conserved? ($\partial_t Q = 0$?)

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If any of these is answered with 'no' then back to square one in algorithm!

Example: AdS holography in Einstein gravity

Consider again only the A-sector ($\bar{A}$-sector is analogous)

Split gauge parameter into components:

$$\epsilon^{(0)} = \epsilon_1 \, L_1 + \epsilon_0 \, L_0 + \epsilon_{-1} \, L_{-1}$$

Solve constraint that local gauge trafos generated by $\epsilon^{(0)}$ preserve boundary conditions

$$\partial_\mu \epsilon^{(0)\,a} + f^a{}_{bc} \left(\hat{a}_\mu^{(0)} + a_\mu^{(0)}\right)^b \epsilon^{(0)\,c} = \mathcal{O}(a_\mu^{(0)})^a$$

Example: AdS holography in Einstein gravity

Consider again only the A-sector ($\bar{A}$-sector is analogous)

Split gauge parameter into components:

$$\epsilon^{(0)} = \epsilon_1 \, L_1 + \epsilon_0 \, L_0 + \epsilon_{-1} \, L_{-1}$$

Solve constraint that local gauge trafos generated by $\epsilon^{(0)}$ preserve boundary conditions

$$\partial_\mu \epsilon^{(0)\,a} + f^a{}_{bc} \left(\hat{a}_\mu^{(0)} + a_\mu^{(0)} \right)^b \epsilon^{(0)\,c} = \mathcal{O}(a_\mu^{(0)})^a$$

Result for components of $\epsilon^{(0)}$:

$$\epsilon_1 = \epsilon(x^+) \qquad \epsilon_0 = \epsilon'(x^+) \qquad \epsilon_{-1} = \epsilon''(x^+) + \mathcal{L}(x^+)\epsilon(x^+)$$

Canonical charges:

$$Q[\epsilon^{(0)}] = \frac{k}{2\pi} \oint \mathrm{d}\varphi \, \mathcal{L}(x^+)\epsilon(x^+)$$

Fourier modes:

$$\mathcal{L}(x^+) \sim \sum_n L_n e^{inx^+}$$

Holographic algorithm from gravity point of view

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Classical asymptotic symmetry algebra

Dirac bracket algebra of canonical boundary charges:

$$\{Q[\epsilon_1],\, Q[\epsilon_2]\} = \delta_{\epsilon_2}\, Q[\epsilon_1]$$

- Either evaluate left hand side directly (Dirac brackets)
- Or evaluate right hand side (usually easier)

Exactly like in seminal Brown–Henneaux work!

Example: AdS holography in Einstein gravity

- Variation of state-dependent function:

$$\delta_\varepsilon \mathcal{L} = \mathcal{L}'\,\varepsilon + 2\mathcal{L}\,\varepsilon' + \frac{k}{2\pi}\,\varepsilon'''$$

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- Alternatively: Dirac bracket algebra of canonical boundary charges:

$$\{\mathcal{L}(x^+), \mathcal{L}(\bar{x}^+)\} = \mathcal{L}'\delta(x^+ - \bar{x}^+) + 2\mathcal{L}\delta'(x^+ - \bar{x}^+) + \frac{k}{2\pi}\delta'''(x^+ - \bar{x}^+)$$

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- Converting $i\{\,,\,\} \to [\,,\,]$ and introducing Fourier modes yields

$$[L_n,\, L_m] = (n-m)\,L_{n+m} + \frac{c}{12}\,(n^3 - n)\,\delta_{n+m,\,0}$$

- Again, the bar-sector is completely analogous

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> Quantum violations of Jacobi-identities possible!

- Resolution: deform suitable structure constants/functions and demand validity of Jacobi identities
- Result is quantum asymptotic symmetry algebra, valid also at finite Chern–Simons level k

Example: Lobachevsky holography in spin-3 gravity

see Afshar, Gary, DG, Rashkov, Riegler '12 for details

Solving Jacobi identities yields (quantum) Polyakov–Bershadsky algebra

$$[J_n, J_m] = \frac{2\hat{k}+3}{3} n\delta_{n+m,0}$$

$$[J_n, \hat{L}_m] = nJ_{n+m}$$

$$[J_n, \hat{G}^{\pm}_m] = \pm G^{\pm}_{m+n}$$

$$[\hat{L}_n, \hat{L}_m] = (n-m)\hat{L}_{m+n} + \frac{\hat{c}}{12} n(n^2-1)\delta_{n+m,0}$$

$$[\hat{L}_n, \hat{G}^{\pm}_m] = \left(\frac{n}{2} - m\right) \hat{G}^{\pm}_{n+m}$$

$$[\hat{G}^{+}_n, \hat{G}^{-}_m] = -(\hat{k}+3)\hat{L}_{m+n} + \frac{3}{2}(\hat{k}+1)(n-m)J_{m+n} + 3\sum_{p\in\mathbb{Z}} :J_{m+n-p}J_p: + \frac{(\hat{k}+1)(2\hat{k}+3)}{2}(n^2 - \frac{1}{4})\delta_{m+n,0}$$

with central charge $\hat{c} = -(2\hat{k}+3)(3\hat{k}+1)/(\hat{k}+3) = -6\hat{k} + \mathcal{O}(1)$

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Unitary representations of quantum asymptotic symmetry algebra

Standard questions:

- Is current algebra level non-negative?
- Is central charge non-negative?
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To be decided case-by-case!

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Example: AdS holography in Einstein gravity

- ASA: two copies of Virasoro with central charge $c = \frac{3\ell}{2G}$
- Minimal requirement: $\ell/G \geq 0$
- Usual analysis of unitary representations of Virasoro

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Dual field theory (if it exists) must be a CFT_2

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Key open issue at this stage:

Identify precisely dual CFT
or show its (non-)existence

Outline

Motivations

Holography basics

Applications

Non-unitary holography

Quoted from workshop webpage "Bits, Branes, Black Holes - Black Holes and Information" (KITP Santa Barbara 2012):

1. How general is holography?

> To what extent do (previous) lessons rely on the particular constructions used to date? Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?

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Specific question addressed here:

> Does holography apply only to unitary theories?

Short answer: no

Example: critical topologically massive gravity (review: DG, Riedler, Rosseel, Zojer '13)

- Action (Deser, Jackiw, Templeton '82):

$$I_{\text{TMG}} = \frac{1}{16\pi\,G}\,\int \mathrm{d}^3x\sqrt{-g}\,\Big[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)\Big]$$

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- Critical tuning: $\mu\ell = 1$ (Li, Song, Strominger '08)

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- Holography logically independent from unitarity

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if holography is true $\Rightarrow$ must work in flat space

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- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\,(n^3 - n)\,\delta_{n+m,\,0}$$

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- This is nothing but the BMS$_3$ algebra (or GCA$_2$, URCA$_2$, CCA$_2$)!
- Example where it does not work easily: boundary conditions!
- Example where it does not work at all: highest weight conditions!

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Interesting example:

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Many open issues in flat space holography!

Next few slides: mention a couple of recent results

Flat space chiral gravity

Bagchi, Detournay, DG '12

Conjecture:

> Conformal Chern–Simons gravity at level $k = 1 \simeq$ chiral extremal CFT with central charge $c = 24$

$$I_{CSG} = \frac{k}{4\pi} \int \left(\Gamma \wedge \mathrm{d}\Gamma + \tfrac{2}{3}\Gamma \wedge \Gamma \wedge \Gamma\right) + \text{flat space bc's}$$

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- Gaps in spectra match

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- No issues with logarithmic modes/log CFTs

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Bagchi, Detournay, DG '12

Conjecture:

Conformal Chern–Simons gravity at level $k = 1 \simeq$ chiral extremal CFT with central charge $c = 24$

$$I_{CSG} = \frac{k}{4\pi} \int \left(\Gamma \wedge \mathrm{d}\Gamma + \tfrac{2}{3}\Gamma \wedge \Gamma \wedge \Gamma\right) + \text{flat space bc's}$$

- Symmetries match (Brown–Henneaux type of analysis)
- Trace and gravitational anomalies match
- Perturbative states match (Virasoro descendants of vacuum)
- Gaps in spectra match
- Microscopic counting of S_{FSC} reproduced by chiral Cardy formula
- No issues with logarithmic modes/log CFTs

Missing: full partition function on gravity side

$$Z(q) = J(q) = \frac{1}{q} + 196884\, q + \mathcal{O}(q^2)$$

Entanglement entropy of Galilean CFTs and flat space holography

Bagchi, Basu, DG, Riegler '14

Using methods similar to CFT:

$$S_{\mathrm{EE}}^{\mathrm{GCFT}} = \underbrace{\frac{c_L}{6}\,\ln\frac{\ell_x}{a}}_{\text{like CFT}} + \underbrace{\frac{c_M}{6}\,\frac{\ell_y}{\ell_x}}_{\text{like grav anomaly}}$$

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with

$$[L_n,\, L_m] = (n-m)\,L_{n+m} + \frac{c_L}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

$$[L_n,\, M_m] = (n-m)\,M_{n+m} + \frac{c_M}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

and

- ℓ_x: spatial distance
- ℓ_y: temporal distance
- a: UV cutoff (lattice size)

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Same results obtained holographically!

- Using methods similar to Ammon, Castro Iqbal '13, de Boer, Jottar '13, Castro, Detournay, Iqbal, Perlmutter '14
- geodesics $\Rightarrow$ Wilson lines

Flat space higher spin gravity

Afshar, Bagchi, Fareghbal, DG, Rosseel '13, Gonzalez, Matulich, Pino, Troncoso '13

- AdS gravity in CS formulation: spin 2 $\to$ spin 3 $\sim$ sl$(2) \to$ sl(3)

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$$S_{\text{CS}}^{\text{flat}} = \frac{k}{4\pi} \int \text{CS}(\mathcal{A})$$

with isl(3) connection ($e^a =$ "zuvielbein")

$$\mathcal{A} = e^a T_a + \omega^a J_a \qquad T_a = (M_n,\, V_m) \quad J_a = (L_n,\, U_m)$$

isl(3) algebra (spin 3 extension of global part of BMS/GCA algebra)

$$\begin{aligned}
[L_n,\, L_m] &= (n-m)L_{n+m} \\
[L_n,\, M_m] &= (n-m)M_{n+m} \\
[L_n,\, U_m] &= (2n-m)U_{n+m} \\
[M_n,\, U_m] = [L_n,\, V_m] &= (2n-m)V_{n+m} \\
[U_n,\, U_m] &= (n-m)(2n^2+2m^2-nm-8)L_{n+m} \\
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$$\mathcal{A}(r,\, t,\, \varphi) = b^{-1}(r)\, \big(\, \mathrm{d}{+}a(t,\, \varphi) + o(1)\big)\, b(r)$$

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- Spin 3 charges:

$$Q[\varepsilon_M,\,\varepsilon_L,\,\varepsilon_V,\,\varepsilon_U] \sim \oint \big(\varepsilon_M(\varphi)M(\varphi){+}\varepsilon_L(\varphi)L(\varphi){+}\varepsilon_V(\varphi)V(\varphi){+}\varepsilon_U(\varphi)U(\varphi)\big)$$

Flat space higher spin gravity

Asymptotic symmetry algebra at finite level k Afshar, Bagchi, Fareghbal, DG, Rosseel '13

- Do either Brown–Henneaux type of analysis or İnönü–Wigner contraction of two copies of quantum W_3-algebra

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- Do either Brown–Henneaux type of analysis or İnönü–Wigner contraction of two copies of quantum W_3-algebra
- Obtain new type of W-algebra as extension of BMS ("BMW")

$$[L_n,\, L_m] = (n-m)L_{n+m} + \frac{c_L}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

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$$[U_n,\, U_m] = (n-m)(2n^2+2m^2-nm-8)L_{n+m} + \frac{192}{c_M}(n-m)\Lambda_{n+m}$$
$$- \frac{96\big(c_L+\frac{44}{5}\big)}{c_M^2}(n-m)\Theta_{n+m} + \frac{c_L}{12}\,n(n^2-1)(n^2-4)\,\delta_{n+m,\,0}$$

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$$\Lambda_n = \sum_p :L_p\,M_{n-p}: \; - \tfrac{3}{10}\,(n+2)(n+3)M_n \qquad \Theta_n = \sum_p M_p\,M_{n-p}$$

other commutators as in isl(3) with $n \in \mathbb{Z}$

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- Note quantum shift and poles in central terms!

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- Note quantum shift and poles in central terms!
- Analysis generalizes to flat space contractions of other W-algebras

Unitarity in flat space

Unitarity leads to further contraction DG, Riegler, Rosseel '14

Facts:

- Unitarity in GCA requires $c_M = 0$ (see paper for caveats!)

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Limit $c_M \to 0$ requires further contraction: $U_n \to c_M \, U_n$

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Higher spin states decouple and become null states!

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Generic flat space W-algebras DG, Riegler, Rosseel '14

1. NO–GO:

Generically (see paper) you can have only two out of three:

- Unitarity
- Flat space
- Non-trivial higher spin states

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Example:

Flat space chiral gravity

Bagchi, Detournay, DG, 1208.1658

Unitarity in flat space

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Example:

Minimal model holography

Gaberdiel, Gopakumar, 1011.2986, 1207.6697

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Flat space higher spin gravity (Galilean W_3 algebra)

Afshar, Bagchi, Fareghbal, DG and Rosseel, 1307.4768

Gonzalez, Matulich, Pino and Troncoso, 1307.5651

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Compatible with "spirit" of various no-go results in higher dimensions!

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2. YES–GO:

There is (at least) one counter-example, namely a Vasiliev-type of theory, where you can have all three properties!

Unitary, non-trivial flat space higher spin algebra exists!
Vasiliev-type flat space chiral higher spin gravity?

Unitarity in flat space

Flat space W_∞-algebra compatible with unitarity DG, Riegler, Rosseel '14

- We do not know if flat space chiral higher spin gravity exists...

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- If it exists, this must be its asymptotic symmetry algebra:

$$[\mathcal{V}^i_m, \mathcal{V}^j_n] = \sum_{r=0}^{\lfloor \frac{i+j}{2} \rfloor} g^{ij}_{2r}(m,n)\, \mathcal{V}^{i+j-2r}_{m+n} + c^i_{\mathcal{V}}(m)\, \delta^{ij}\, \delta_{m+n,0}$$

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- Vacuum descendants $\mathcal{W}^i_m|0\rangle$ are null states for all i and m!
- AdS parent theory: no trace anomaly, but gravitational anomaly (Like in conformal Chern–Simons gravity $\to$ Vasiliev type analogue?)

Selected open issues

We have answered an ϵ of the open questions.

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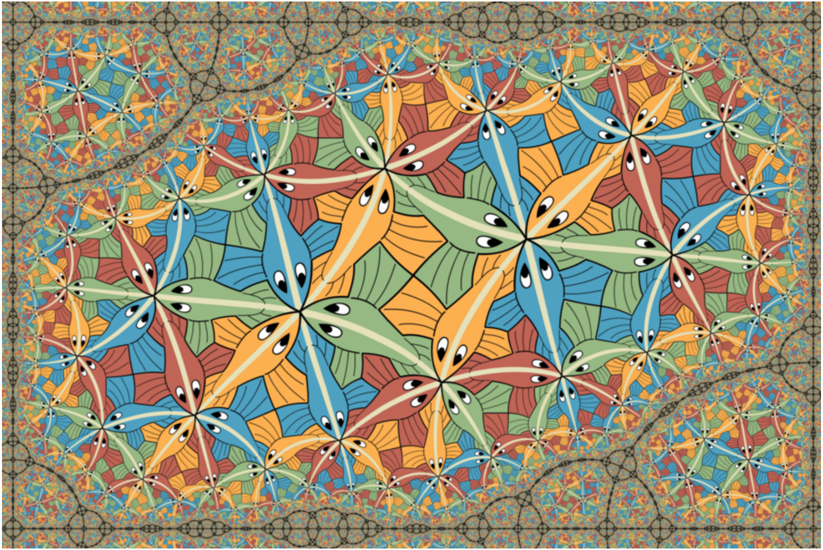
- other non-AdS holography examples?

Still missing: comprehensive family of simple models such that

- dual (conformal) field theory identified
- exists for $c \sim \mathcal{O}(1)$ (ultra-quantum limit)
- exists for $c \to \infty$ (semi-classical limit)

... or prove that no such model $\exists$, unless UV-completed to string theory!

Thanks for your attention!



Vladimir Bulatov, M.C.Escher Circle Limit III in a rectangle