Gravity in lower dimensions

Daniel Grumiller

Institute for Theoretical Physics
Vienna University of Technology

IPM, Teheran, January 2012

Outline

Why lower-dimensional gravity?

Which 2D theory?

Holographic renormalization

Which 3D theory?

Outline

Motivation for studying gravity in 2 and 3 dimensions

- Quantum gravity
  - Address conceptual issues of quantum gravity
  - Black hole evaporation, information loss, black hole microstate counting, virtual black hole production, ...
  - Technically much simpler than 4D or higher D gravity
  - Integrable models: powerful tools in physics (Coulomb problem, Hydrogen atom, harmonic oscillator, ...)
  - Models should be as simple as possible, but not simpler

Motivation for studying gravity in 2 and 3 dimensions

- Quantum gravity
  - Address conceptual issues of quantum gravity
  - Black hole evaporation, information loss, black hole microstate counting, virtual black hole production, ...
  - Technically much simpler than 4D or higher D gravity
  - Integrable models: powerful tools in physics (Coulomb problem, Hydrogen atom, harmonic oscillator, ...)
  - Models should be as simple as possible, but not simpler
- Gauge/gravity duality + indirect physics applications
  - Deeper understanding of black hole holography
  - $\text{AdS}_3/\text{CFT}_2$ correspondence best understood
  - Quantum gravity via AdS/CFT? (Witten '07, Li, Song, Strominger '08)
  - Applications to 2D condensed matter systems?
  - Gauge gravity duality beyond standard AdS/CFT: warped AdS, asymptotic Lifshitz, non-relativistic CFTs, logarithmic CFTs, ...

Motivation for studying gravity in 2 and 3 dimensions

- Quantum gravity
  - Address conceptual issues of quantum gravity
  - Black hole evaporation, information loss, black hole microstate counting, virtual black hole production, ...
  - Technically much simpler than 4D or higher D gravity
  - Integrable models: powerful tools in physics (Coulomb problem, Hydrogen atom, harmonic oscillator, ...)
  - Models should be as simple as possible, but not simpler
- Gauge/gravity duality + indirect physics applications
  - Deeper understanding of black hole holography
  - $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence best understood
  - Quantum gravity via AdS/CFT? (Witten '07, Li, Song, Strominger '08)
  - Applications to 2D condensed matter systems?
  - Gauge gravity duality beyond standard AdS/CFT: warped AdS, asymptotic Lifshitz, non-relativistic CFTs, logarithmic CFTs, ...
- Direct physics applications
  - Cosmic strings (Deser, Jackiw, 't Hooft '84, '92)
  - Black hole analog systems in condensed matter physics (graphene, BEC, fluids, ...)

Gravity in lower dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
- 5D: 50 (35 Weyl and 15 Ricci)
- 4D: 20 (10 Weyl and 10 Ricci)

Gravity in lower dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
- 5D: 50 (35 Weyl and 15 Ricci)
- 4D: 20 (10 Weyl and 10 Ricci)
- 3D: 6 (Ricci)
- 2D: 1 (Ricci scalar)

Gravity in lower dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
- 5D: 50 (35 Weyl and 15 Ricci)
- 4D: 20 (10 Weyl and 10 Ricci)
- 3D: 6 (Ricci)
- 2D: 1 (Ricci scalar)

> - 2D: lowest dimension exhibiting black holes (BHs)
> - Simplest gravitational theories with BHs in 2D

Gravity in lower dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
- 5D: 50 (35 Weyl and 15 Ricci)
- 4D: 20 (10 Weyl and 10 Ricci)
- 3D: 6 (Ricci)
- 2D: 1 (Ricci scalar)

- 2D: lowest dimension exhibiting black holes (BHs)
- Simplest gravitational theories with BHs in 2D

- 3D: lowest dimension exhibiting BHs and gravitons
- Simplest gravitational theories with BHs and gravitons in 3D

Outline

Attempt 1: Einstein–Hilbert in and near two dimensions

Let us start with the simplest attempt. `Einstein-Hilbert` action in 2 dimensions:

$$I_{\text{EH}} = \frac{1}{16\pi\, G} \int \mathrm{d}^2x \sqrt{|g|}\, R = \frac{1}{2G}\,(1-\gamma)$$

- Action is topological
- No equations of motion
- Formal counting of number of gravitons: -1

Attempt 1: Einstein–Hilbert in and near two dimensions

Let us continue with the next simplest attempt. Einstein-Hilbert action in $2+\epsilon$ dimensions:

$$I_{\text{EH}}{}^{\epsilon} = \frac{1}{16\pi G} \int \mathrm{d}^{2+\epsilon}x \sqrt{|g|}\, R$$

- Weinberg: theory is asymptotically safe
- Mann: limit $\epsilon \to 0$ should be possible and lead to 2D dilaton gravity
- DG, Jackiw: limit $\epsilon \to 0$ yields Liouville gravity

$$\lim_{\epsilon\to 0} I_{EH}{}^{\epsilon} = \frac{1}{16\pi G_2} \int \mathrm{d}^2x \sqrt{|g|}\, \left[XR - (\nabla X)^2 + \lambda e^{-2X}\right]$$

Attempt 1: Einstein–Hilbert in and near two dimensions

Let us continue with the next simplest attempt. Einstein-Hilbert action in $2+\epsilon$ dimensions:

$$I_{\text{EH}}{}^{\epsilon} = \frac{1}{16\pi\, G} \int \mathrm{d}^{2+\epsilon}x \sqrt{|g|}\, R$$

- Weinberg: theory is asymptotically safe
- Mann: limit $\epsilon \to 0$ should be possible and lead to 2D dilaton gravity
- DG, Jackiw: limit $\epsilon \to 0$ yields Liouville gravity

$$\lim_{\epsilon\to 0} I_{EH}{}^{\epsilon} = \frac{1}{16\pi\, G_2} \int \mathrm{d}^2x \sqrt{|g|}\, \left[XR - (\nabla X)^2 + \lambda e^{-2X}\right]$$

Result of attempt 1:

A specific 2D dilaton gravity model

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS_2 gauge theory

$$[P_a, P_b] = \Lambda\, \epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS$_2$ gauge theory

$$[P_a, P_b] = \Lambda\, \epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

- Start with $SO(1,2)$ connection $A = e^a P_a + \omega J$

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS$_2$ gauge theory

$$[P_a, P_b] = \Lambda\, \epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

- Start with $SO(1,2)$ connection $A = e^a P_a + \omega J$
- Take field strength $F = \mathrm{d}A + \frac{1}{2}[A, A]$ and coadjoint scalar X

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS$_2$ gauge theory

$$[P_a, P_b] = \Lambda\,\epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

- Start with $SO(1,2)$ connection $A = e^a P_a + \omega J$
- Take field strength $F = \mathrm{d}A + \frac{1}{2}[A, A]$ and coadjoint scalar X
- Construct non-abelian BF theory

$$I = \int X_A F^A = \int \Big[X_a(\mathrm{d}e^a + \epsilon^a{}_b \omega \wedge e^b) + X\,\mathrm{d}\omega + \epsilon_{ab} e^a \wedge e^b\,\Lambda X\Big]$$

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS$_2$ gauge theory

$$[P_a, P_b] = \Lambda\,\epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

- Start with $SO(1,2)$ connection $A = e^a P_a + \omega J$
- Take field strength $F = \mathrm{d}A + \frac{1}{2}[A, A]$ and coadjoint scalar X
- Construct non-abelian BF theory

$$I = \int X_A F^A = \int \Big[X_a(\mathrm{d}e^a + \epsilon^a{}_b \omega \wedge e^b) + X\,\mathrm{d}\omega + \epsilon_{ab} e^a \wedge e^b\,\Lambda X\Big]$$

- Eliminate X_a (Torsion constraint) and ω (Levi-Civita connection)

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS_2 gauge theory

$$[P_a, P_b] = \Lambda\, \epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

- Start with $SO(1,2)$ connection $A = e^a P_a + \omega J$
- Take field strength $F = \mathrm{d}A + \frac{1}{2}[A, A]$ and coadjoint scalar X
- Construct non-abelian BF theory

$$I = \int X_A F^A = \int \Big[X_a(\mathrm{d}e^a + \epsilon^a{}_b \omega \wedge e^b) + X\, \mathrm{d}\omega + \epsilon_{ab} e^a \wedge e^b\, \Lambda X\Big]$$

- Eliminate X_a (Torsion constraint) and ω (Levi-Civita connection)
- Obtain the second order action

$$I = \frac{1}{16\pi\, G_2} \int \mathrm{d}^2x \sqrt{-g}\, X\, [R - \Lambda]$$

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS_2 gauge theory

$$[P_a, P_b] = \Lambda\, \epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

- Start with $SO(1,2)$ connection $A = e^a P_a + \omega J$
- Take field strength $F = \mathrm{d}A + \frac{1}{2}[A, A]$ and coadjoint scalar X
- Construct non-abelian BF theory

$$I = \int X_A F^A = \int \Big[X_a (\mathrm{d}e^a + \epsilon^a{}_b \omega \wedge e^b) + X\,\mathrm{d}\omega + \epsilon_{ab} e^a \wedge e^b\, \Lambda X \Big]$$

- Eliminate X_a (Torsion constraint) and ω (Levi-Civita connection)
- Obtain the second order action

$$I = \frac{1}{16\pi\, G_2} \int \mathrm{d}^2x \sqrt{-g}\, X\, [R - \Lambda]$$

Result of attempt 2:

A specific 2D dilaton gravity model

Attempt 3: Dimensional reduction

For example: spherical reduction from D dimensions

Line element adapted to spherical symmetry:

$$\mathrm{d}s^2 = \underbrace{g_{\mu\nu}^{(D)}}_{\text{full metric}} \, \mathrm{d}x^\mu \, \mathrm{d}x^\nu = \underbrace{g_{\alpha\beta}(x^\gamma)}_{2D \text{ metric}} \, \mathrm{d}x^\alpha \, \mathrm{d}x^\beta - \underbrace{\phi^2(x^\alpha)}_{\text{surface area}} \, \mathrm{d}\Omega^2_{S_{D-2}} \,,$$

Attempt 3: Dimensional reduction

For example: spherical reduction from D dimensions

Line element adapted to spherical symmetry:

$$\mathrm{d}s^2 = \underbrace{g_{\mu\nu}^{(D)}}_{\text{full metric}} \mathrm{d}x^\mu \, \mathrm{d}x^\nu = \underbrace{g_{\alpha\beta}(x^\gamma)}_{2D \text{ metric}} \mathrm{d}x^\alpha \, \mathrm{d}x^\beta - \underbrace{\phi^2(x^\alpha)}_{\text{surface area}} \mathrm{d}\Omega^2_{S_{D-2}} \,,$$

Insert into D-dimensional EH action $I_{EH} = \kappa \int \mathrm{d}^D x \sqrt{-g^{(D)}} R^{(D)}$:

$$I_{EH} = \kappa \frac{2\pi^{(D-1)/2}}{\Gamma(\frac{D-1}{2})} \int \mathrm{d}^2 x \sqrt{-g} \, \phi^{D-2} \Big[R + \frac{(D-2)(D-3)}{\phi^2} \big((\nabla\phi)^2 - 1 \big) \Big]$$

Attempt 3: Dimensional reduction

For example: spherical reduction from D dimensions

Line element adapted to spherical symmetry:

$$\mathrm{d}s^2 = \underbrace{g_{\mu\nu}^{(D)}}_{\text{full metric}} \,\mathrm{d}x^\mu \,\mathrm{d}x^\nu = \underbrace{g_{\alpha\beta}(x^\gamma)}_{2D \text{ metric}} \,\mathrm{d}x^\alpha \,\mathrm{d}x^\beta - \underbrace{\phi^2(x^\alpha)}_{\text{surface area}} \,\mathrm{d}\Omega^2_{S_{D-2}}\,,$$

Insert into D-dimensional EH action $I_{EH} = \kappa \int \mathrm{d}^D x \sqrt{-g^{(D)}} R^{(D)}$:

$$I_{EH} = \kappa \frac{2\pi^{(D-1)/2}}{\Gamma(\frac{D-1}{2})} \int \mathrm{d}^2 x \sqrt{-g}\, \phi^{D-2} \Big[R + \frac{(D-2)(D-3)}{\phi^2} \left((\nabla\phi)^2 - 1 \right) \Big]$$

Cosmetic redefinition $X \propto (\lambda\phi)^{D-2}$:

$$I_{EH} = \frac{1}{16\pi\, G_2} \int \mathrm{d}^2 x \sqrt{-g} \Big[X R + \frac{D-3}{(D-2)X} (\nabla X)^2 - \lambda^2 X^{(D-4)/(D-2)} \Big]$$

Result of attempt 3:

A specific class of 2D dilaton gravity models

Attempt 4: Poincare gauge theory and higher power curvature theories

Basic idea: since EH is trivial consider $f(R)$ theories or/and theories with torsion or/and theories with non-metricity

Attempt 4: Poincare gauge theory and higher power curvature theories

Basic idea: since EH is trivial consider $f(R)$ theories or/and theories with torsion or/and theories with non-metricity

- Example: `Katanaev-Volovich` model (Poincare gauge theory)

$$I_{\text{KV}} \sim \int \mathrm{d}^2x\sqrt{-g}\,\left[\alpha T^2 + \beta R^2\right]$$

- `Kummer, Schwarz`: bring into first order form:

$$I_{\text{KV}} \sim \int \Big[X_a(\mathrm{d}e^a + \epsilon^a{}_b\omega \wedge e^b) + X\,\mathrm{d}\omega + \epsilon_{ab}e^a \wedge e^b\,(\alpha X^a X_a + \beta X^2)\Big]$$

- Use same algorithm as before to convert into second order action:

$$I_{\text{KV}} = \frac{1}{16\pi\,G_2}\int \mathrm{d}^2x\sqrt{-g}\Big[XR + \alpha(\nabla X)^2 + \beta X^2\Big]$$

Attempt 4: Poincare gauge theory and higher power curvature theories

Basic idea: since EH is trivial consider $f(R)$ theories or/and theories with torsion or/and theories with non-metricity

- Example: Katanaev-Volovich model (Poincare gauge theory)

$$I_{\text{KV}} \sim \int \mathrm{d}^2x\sqrt{-g}\,\left[\alpha T^2 + \beta R^2\right]$$

- Kummer, Schwarz: bring into first order form:

$$I_{\text{KV}} \sim \int \Big[X_a(\mathrm{d}e^a + \epsilon^a{}_b\omega\wedge e^b) + X\,\mathrm{d}\omega + \epsilon_{ab}e^a\wedge e^b\,(\alpha X^a X_a + \beta X^2)\Big]$$

- Use same algorithm as before to convert into second order action:

$$I_{\text{KV}} = \frac{1}{16\pi\,G_2}\int \mathrm{d}^2x\sqrt{-g}\Big[XR + \alpha(\nabla X)^2 + \beta X^2\Big]$$

Result of attempt 4:

A specific 2D dilaton gravity model

Attempt 5: Strings in two dimensions

Conformal invariance of the σ model

$$I_\sigma \propto \int \mathrm{d}^2\xi \sqrt{|h|}\,\left[g_{\mu\nu} h^{ij}\partial_i x^\mu \partial_j x^\nu + \alpha' \phi \mathcal{R} + \dots\right]$$

requires vanishing of β-functions

$$\beta^\phi \propto -4b^2 - 4(\nabla\phi)^2 + 4\Box\phi + R + \dots$$
$$\beta^g_{\mu\nu} \propto R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi + \dots$$

Conditions $\beta^\phi = \beta^g_{\mu\nu} = 0$ follow from target space action

$$I_{\text{target}} = \frac{1}{16\pi\, G_2}\int \mathrm{d}^2x\sqrt{-g}\Big[XR + \frac{1}{X}\,(\nabla X)^2 - 4b^2\Big]$$

where $X = e^{-2\phi}$

Attempt 5: Strings in two dimensions

Conformal invariance of the σ model

$$I_\sigma \propto \int \mathrm{d}^2\xi \sqrt{|h|}\left[g_{\mu\nu}h^{ij}\partial_i x^\mu \partial_j x^\nu + \alpha'\phi\mathcal{R} + \dots\right]$$

requires vanishing of β-functions

$$\beta^\phi \propto -4b^2 - 4(\nabla\phi)^2 + 4\Box\phi + R + \dots$$
$$\beta^g_{\mu\nu} \propto R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi + \dots$$

Conditions $\beta^\phi = \beta^g_{\mu\nu} = 0$ follow from target space action

$$I_{\text{target}} = \frac{1}{16\pi\, G_2}\int \mathrm{d}^2x\sqrt{-g}\Big[XR + \frac{1}{X}\,(\nabla X)^2 - 4b^2\Big]$$

where $X = e^{-2\phi}$

Result of attempt 5:

A specific 2D dilaton gravity model

Selected List of Models

Black holes in $(A)dS$, asymptotically flat or arbitrary spaces (Wheeler property)

Model	$U(X)$	$V(X)$
1. Schwarzschild (1916)	$-\frac{1}{2X}$	$-\lambda^2$
2. Jackiw-Teitelboim (1984)	0	ΛX
3. Witten Black Hole (1991)	$-\frac{1}{X}$	$-2b^2 X$
4. CGHS (1992)	0	$-2b^2$
5. $(A)dS_2$ ground state (1994)	$-\frac{a}{X}$	BX
6. Rindler ground state (1996)	$-\frac{a}{X}$	BX^a
7. Black Hole attractor (2003)	0	BX^{-1}
8. Spherically reduced gravity ($N > 3$)	$-\frac{N-3}{(N-2)X}$	$-\lambda^2 X^{(N-4)/(N-2)}$
9. All above: ab-family (1997)	$-\frac{a}{X}$	BX^{a+b}
10. Liouville gravity	a	$be^{\alpha X}$
11. Reissner-Nordström (1916)	$-\frac{1}{2X}$	$-\lambda^2 + \frac{Q^2}{X}$
12. Schwarzschild-$(A)dS$	$-\frac{1}{2X}$	$-\lambda^2 - \ell X$
13. Katanaev-Volovich (1986)	α	$\beta X^2 - \Lambda$
14. BTZ/Achucarro-Ortiz (1993)	0	$\frac{Q^2}{X} - \frac{J}{4X^3} - \Lambda X$
15. KK reduced CS (2003)	0	$\frac{1}{2}X(c - X^2)$
16. KK red. conf. flat (2006)	$-\frac{1}{2}\tanh(X/2)$	$A \sinh X$
17. 2D type 0A string Black Hole	$-\frac{1}{X}$	$-2b^2 X + \frac{b^2 q^2}{8\pi}$
18. exact string Black Hole (2005)	lengthy	lengthy

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \big[XR - U(X)(\nabla X)^2 - V(X)\big]$$
$$- \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|}\ \ [XK - S(X)] + I^{(m)}$$

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \big[XR - U(X)(\nabla X)^2 - V(X)\big] - \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|}\ \big[XK - S(X)\big] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \big[XR - U(X)(\nabla X)^2 - V(X)\big]$$
$$- \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|}\ \ [XK - S(X)] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \big[XR - U(X)(\nabla X)^2 - V(X)\big]$$
$$- \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|}\ \ [XK - S(X)] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$
- Self-interaction potential $V(X)$ leads to non-trivial geometries

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\big[XR - U(X)(\nabla X)^2 - V(X)\big] - \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x\sqrt{|\gamma|}\;\;[XK - S(X)] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$
- Self-interaction potential $V(X)$ leads to non-trivial geometries
- Gibbons–Hawking–York boundary term guarantees Dirichlet boundary problem for metric

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \big[XR - U(X)(\nabla X)^2 - V(X)\big]$$
$$- \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|}\;\; [XK - S(X)] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$
- Self-interaction potential $V(X)$ leads to non-trivial geometries
- Gibbons–Hawking–York boundary term guarantees Dirichlet boundary problem for metric
- Hamilton–Jacobi counterterm contains superpotential $S(X)$

$$S(X)^2 = e^{-\int^X U(y)\,\mathrm{d}y} \int^X V(y) e^{\int^y U(z)\,\mathrm{d}z}\, \mathrm{d}y$$

and guarantees well-defined variational principle $\delta I = 0$

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \big[XR - U(X)(\nabla X)^2 - V(X)\big]$$
$$- \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|}\; \big[XK - S(X)\big] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$
- Self-interaction potential $V(X)$ leads to non-trivial geometries
- Gibbons–Hawking–York boundary term guarantees Dirichlet boundary problem for metric
- Hamilton–Jacobi counterterm contains superpotential $S(X)$

$$S(X)^2 = e^{-\int^X U(y)\,\mathrm{d}y} \int^X V(y) e^{\int^y U(z)\,\mathrm{d}z}\, \mathrm{d}y$$

  and guarantees well-defined variational principle $\delta I = 0$
- Interesting option: couple 2D dilaton gravity to matter

Acknowledgments

List of collaborators on 2D classical and quantum gravity:

- Wolfgang Kummer (VUT, 1935–2007)
- Dima Vassilevich (ABC Sao Paulo)
- Luzi Bergamin
- Herbert Balasin (VUT)
- Rene Meyer (Crete U.)
- Alfredo Iorio (Charles U. Prague)
- Carlos Nuñez (Swansea U.)
- Roman Jackiw (MIT)
- Robert McNees (Loyola U. Chicago)
- Muzaffer Adak (Pamukkale U.)
- Alejandra Castro (McGill U.)
- Finn Larsen (Michigan U.)
- Peter van Nieuwenhuizen (YITP, Stony Brook)
- Steve Carlip (UC Davis)
- ...

Outline

Why lower-dimensional gravity?

Which 2D theory?

Holographic renormalization

Which 3D theory?

Why do we need holographic renormalization?

What is holographic renormalization?

Holographic renormalization is the subtraction of appropriate boundary terms from the action.

Without holographic renormalization:

Why do we need holographic renormalization?

What is holographic renormalization?

Holographic renormalization is the subtraction of appropriate boundary terms from the action.

Without holographic renormalization:

- Wrong black hole thermodynamics

Why do we need holographic renormalization?

What is holographic renormalization?

Holographic renormalization is the subtraction of appropriate boundary terms from the action.

Without holographic renormalization:

- Wrong black hole thermodynamics
- Wrong (typically divergent) boundary stress tensor

Why do we need holographic renormalization?

What is holographic renormalization?

Holographic renormalization is the subtraction of appropriate boundary terms from the action.

Without holographic renormalization:

- Wrong black hole thermodynamics
- Wrong (typically divergent) boundary stress tensor
- Inconsistent theory (no classical limit)

Why do we need holographic renormalization?

What is holographic renormalization?

Holographic renormalization is the subtraction of appropriate boundary terms from the action.

Without holographic renormalization:

- Wrong black hole thermodynamics
- Wrong (typically divergent) boundary stress tensor
- Inconsistent theory (no classical limit)
- Unphysical divergences and finite parts of observables can be wrong

Why do we need holographic renormalization?

What is holographic renormalization?

Holographic renormalization is the subtraction of appropriate boundary terms from the action.

Without holographic renormalization:

- Wrong black hole thermodynamics
- Wrong (typically divergent) boundary stress tensor
- Inconsistent theory (no classical limit)
- Unphysical divergences and finite parts of observables can be wrong
- `Susskind, Witten '98`: in field theory: field theory UV divergences (which need to be renormalized) correspond to IR divergences on the gravity side if gauge/gravity duality exists

Why do we need holographic renormalization?

What is holographic renormalization?

Holographic renormalization is the subtraction of appropriate boundary terms from the action.

Without holographic renormalization:

- Wrong black hole thermodynamics
- Wrong (typically divergent) boundary stress tensor
- Inconsistent theory (no classical limit)
- Unphysical divergences and finite parts of observables can be wrong
- `Susskind, Witten '98`: in field theory: field theory UV divergences (which need to be renormalized) correspond to IR divergences on the gravity side if gauge/gravity duality exists
- `DG, van Nieuwenhuizen '09`: SUSY at boundary requires unique holographic counterterm, at least in 2 and 3 dimensions

Why do we need holographic renormalization?

What is holographic renormalization?

Holographic renormalization is the subtraction of appropriate boundary terms from the action.

Without holographic renormalization:

- Wrong black hole thermodynamics
- Wrong (typically divergent) boundary stress tensor
- Inconsistent theory (no classical limit)
- Unphysical divergences and finite parts of observables can be wrong
- `Susskind, Witten '98`: in field theory: field theory UV divergences (which need to be renormalized) correspond to IR divergences on the gravity side if gauge/gravity duality exists
- `DG, van Nieuwenhuizen '09`: SUSY at boundary requires unique holographic counterterm, at least in 2 and 3 dimensions
- Variational principle ill-defined

AdS_2

… the simplest gravity model where the need for holographic renormalization arises!

Bulk action:

$$I_B = -\frac{1}{2}\int_{\mathcal{M}} \mathrm{d}^2 x \sqrt{g}\,\Big[X\,\big(R + \frac{2}{\ell^2}\big)\Big]$$

AdS_2

... the simplest gravity model where the need for holographic renormalization arises!

Bulk action:

$$I_B = -\frac{1}{2}\int_{\mathcal{M}} \mathrm{d}^2x\sqrt{g}\,\Big[X\,\big(R+\frac{2}{\ell^2}\big)\Big]$$

Variation with respect to scalar field X yields

$$R = -\frac{2}{\ell^2}$$

This means curvature is constant and negative, i.e., AdS_2.

AdS_2

... the simplest gravity model where the need for holographic renormalization arises!

Bulk action:

$$I_B = -\frac{1}{2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{g} \left[X \left(R + \frac{2}{\ell^2} \right) \right]$$

Variation with respect to scalar field X yields

$$R = -\frac{2}{\ell^2}$$

This means curvature is constant and negative, i.e., AdS_2.

Variation with respect to metric g yields

$$\nabla_\mu \nabla_\nu X - g_{\mu\nu} \Box X + g_{\mu\nu} \frac{X}{\ell^2} = 0$$

AdS_2

... the simplest gravity model where the need for holographic renormalization arises!

Bulk action:

$$I_B = -\frac{1}{2}\int_{\mathcal{M}} \mathrm{d}^2x\sqrt{g}\,\Big[X\,\big(R+\frac{2}{\ell^2}\big)\Big]$$

Variation with respect to scalar field X yields

$$R = -\frac{2}{\ell^2}$$

This means curvature is constant and negative, i.e., AdS_2.

Variation with respect to metric g yields

$$\nabla_\mu\nabla_\nu X - g_{\mu\nu}\Box X + g_{\mu\nu}\frac{X}{\ell^2} = 0$$

Equations of motion above solved by

$$X = r\,, \qquad g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \big(\frac{r^2}{\ell^2}-M\big)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\frac{r^2}{\ell^2}-M}$$

AdS$_2$

… the simplest gravity model where the need for holographic renormalization arises!

Bulk action:

$$I_B = -\frac{1}{2}\int_{\mathcal{M}} \mathrm{d}^2x\sqrt{g}\,\Big[X\,\big(R+\frac{2}{\ell^2}\big)\Big]$$

Variation with respect to scalar field X yields

$$R = -\frac{2}{\ell^2}$$

This means curvature is constant and negative, i.e., AdS$_2$.
Variation with respect to metric g yields

$$\nabla_\mu\nabla_\nu X - g_{\mu\nu}\Box X + g_{\mu\nu}\frac{X}{\ell^2} = 0$$

Equations of motion above solved by

$$X = r\,, \qquad g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \Big(\frac{r^2}{\ell^2} - M\Big)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\frac{r^2}{\ell^2} - M}$$

There is an important catch, however: Boundary terms tricky!

Gibbons–Hawking–York boundary terms: quantum mechanical toy model

Let us start with an bulk Hamiltonian action

$$I_B = \int_{t_i}^{t_f} \mathrm{d}t \, [-\dot{p}q - H(q,p)]$$

Boundary terms, Part I

Gibbons–Hawking–York boundary terms: quantum mechanical toy model

Let us start with an bulk Hamiltonian action

$$I_B = \int_{t_i}^{t_f} \mathrm{d}t \left[-\dot{p}q - H(q,p) \right]$$

Want to set up a Dirichlet boundary value problem $q = \text{fixed at } t_i, t_f$

Boundary terms, Part I

Gibbons–Hawking–York boundary terms: quantum mechanical toy model

Let us start with an bulk Hamiltonian action

$$I_B = \int_{t_i}^{t_f} \mathrm{d}t \, [-\dot{p}q - H(q,p)]$$

Want to set up a Dirichlet boundary value problem $q = \text{fixed at } t_i, t_f$
Problem:

$$\delta I_B = 0 \text{ requires } q\,\delta p = 0 \text{ at boundary}$$

Boundary terms, Part I

Gibbons–Hawking–York boundary terms: quantum mechanical toy model

Let us start with an bulk Hamiltonian action

$$I_B = \int_{t_i}^{t_f} \mathrm{d}t \, [-\dot{p}q - H(q,p)]$$

Want to set up a Dirichlet boundary value problem $q = \text{fixed at } t_i, t_f$

Problem:

$$\delta I_B = 0 \text{ requires } q\,\delta p = 0 \text{ at boundary}$$

Solution: add "Gibbons–Hawking–York" boundary term

$$I_E = I_B + I_{GHY}\,, \qquad I_{GHY} = pq\big|_{t_i}^{t_f}$$

Boundary terms, Part I
Gibbons–Hawking–York boundary terms: quantum mechanical toy model

Let us start with an bulk Hamiltonian action

$$I_B = \int_{t_i}^{t_f} \mathrm{d}t \, [-\dot{p}q - H(q,p)]$$

Want to set up a Dirichlet boundary value problem $q = \text{fixed at } t_i, t_f$
Problem:

$$\delta I_B = 0 \text{ requires } q \, \delta p = 0 \text{ at boundary}$$

Solution: add "Gibbons–Hawking–York" boundary term

$$I_E = I_B + I_{GHY} \,, \qquad I_{GHY} = pq\big|_{t_i}^{t_f}$$

As expected $I_E = \int_{t_i}^{t_f} [p\dot{q} - H(q,p)]$ is standard Hamiltonian action

Boundary terms, Part II

Gibbons–Hawking–York boundary terms in gravity — something still missing!

That was easy! In gravity the result is

$$I_{GHY} = -\int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{\gamma}\, X\, K$$

where γ (K) is determinant (trace) of first (second) fundamental form. Euclidean action with correct boundary value problem is

$$I_E = I_B + I_{GHY}$$

The boundary lies at $r = r_0$, with $r_0 \to \infty$. Are we done?

Boundary terms, Part II

Gibbons–Hawking–York boundary terms in gravity — something still missing!

That was easy! In gravity the result is

$$I_{GHY} = -\int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{\gamma}\, X\, K$$

where γ (K) is determinant (trace) of first (second) fundamental form.
Euclidean action with correct boundary value problem is

$$I_E = I_B + I_{GHY}$$

The boundary lies at $r = r_0$, with $r_0 \to \infty$. Are we done?
No! Serious Problem! Variation of I_E yields

$$\delta I_E \sim \mathrm{EOM} + \delta X (\mathrm{boundary-term}) - \lim_{r_0\to\infty} \int_{\partial\mathcal{M}} \mathrm{d}t\, \delta\gamma$$

That was easy! In gravity the result is

$$I_{GHY} = -\int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{\gamma}\, X\, K$$

where γ (K) is determinant (trace) of first (second) fundamental form.
Euclidean action with correct boundary value problem is

$$I_E = I_B + I_{GHY}$$

The boundary lies at $r = r_0$, with $r_0 \to \infty$. Are we done?
No! Serious Problem! Variation of I_E yields

$$\delta I_E \sim \mathrm{EOM} + \delta X (\mathrm{boundary-term}) - \lim_{r_0\to\infty} \int_{\partial\mathcal{M}} \mathrm{d}t\, \delta\gamma$$

Asymptotic metric: $\gamma = r^2/\ell^2 + \mathcal{O}(1)$. Thus, $\delta\gamma$ may be *finite!*

Gibbons–Hawking–York boundary terms in gravity — something still missing!

That was easy! In gravity the result is

$$I_{GHY} = -\int_{\partial\mathcal{M}} \mathrm{d}x\sqrt{\gamma}\, X\, K$$

where γ (K) is determinant (trace) of first (second) fundamental form. Euclidean action with correct boundary value problem is

$$I_E = I_B + I_{GHY}$$

The boundary lies at $r = r_0$, with $r_0 \to \infty$. Are we done?
No! Serious Problem! Variation of I_E yields

$$\delta I_E \sim \mathrm{EOM} + \delta X (\mathrm{boundary-term}) - \lim_{r_0\to\infty}\int_{\partial\mathcal{M}} \mathrm{d}t\, \delta\gamma$$

Asymptotic metric: $\gamma = r^2/\ell^2 + \mathcal{O}(1)$. Thus, $\delta\gamma$ may be *finite!*

$\delta I_E \neq 0$ for some variations that preserve boundary conditions!!!

Boundary terms, Part III

Holographic renormalization: quantum mechanical toy model

Key observation: Dirichlet boundary problem not changed under

$$I_E \to \Gamma = I_E - I_{CT} = I_{EH} + I_{GHY} - I_{CT}$$

with

$$I_{CT} = S(q,t)\big|^{t_f}$$

Key observation: Dirichlet boundary problem not changed under

$$I_E \to \Gamma = I_E - I_{CT} = I_{EH} + I_{GHY} - I_{CT}$$

with

$$I_{CT} = S(q,t)\big|^{t_f}$$

Improved action:

$$\Gamma = \int_{t_i}^{t_f} \mathrm{d}t \left[-\dot{p}q - H(q,p)\right] + pq\big|_{t_i}^{t_f} - S(q,t)\big|^{t_f}$$

Boundary terms, Part III

Holographic renormalization: quantum mechanical toy model

Key observation: Dirichlet boundary problem not changed under

$$I_E \to \Gamma = I_E - I_{CT} = I_{EH} + I_{GHY} - I_{CT}$$

with

$$I_{CT} = S(q,t)\big|^{t_f}$$

Improved action:

$$\Gamma = \int_{t_i}^{t_f} \mathrm{d}t \left[-\dot{p}q - H(q,p)\right] + pq\big|_{t_i}^{t_f} - S(q,t)\big|^{t_f}$$

First variation (assuming $p = \partial H / \partial p$):

$$\delta\Gamma = \left(p - \frac{\partial S(q,t)}{\partial q}\right) \delta q \bigg|^{t_f} = 0?$$

Boundary terms, Part III

Holographic renormalization: quantum mechanical toy model

Key observation: Dirichlet boundary problem not changed under

$$I_E \to \Gamma = I_E - I_{CT} = I_{EH} + I_{GHY} - I_{CT}$$

with

$$I_{CT} = S(q,t)\big|^{t_f}$$

Improved action:

$$\Gamma = \int_{t_i}^{t_f} \mathrm{d}t \, [-\dot{p}q - H(q,p)] + pq\big|_{t_i}^{t_f} - S(q,t)\big|^{t_f}$$

First variation (assuming $p = \partial H / \partial p$):

$$\delta\Gamma = \left(p - \frac{\partial S(q,t)}{\partial q} \right) \delta q \bigg|^{t_f} = 0?$$

Works if $S(q,t)$ is Hamilton's principal function!

Boundary terms, Part IV

Holographic renormalization in AdS_2 gravity

Hamilton's principle function

- Solves the Hamilton–Jacobi equation

Boundary terms, Part IV

Holographic renormalization in AdS_2 gravity

Hamilton's principle function

- Solves the Hamilton–Jacobi equation
- Does not change boundary value problem when added to action

Hamilton's principle function

- Solves the Hamilton–Jacobi equation
- Does not change boundary value problem when added to action
- Is capable to render $\delta\Gamma = 0$ even when $\delta I_E \neq 0$

Boundary terms, Part IV

Holographic renormalization in AdS_2 gravity

Hamilton's principle function

- Solves the Hamilton–Jacobi equation
- Does not change boundary value problem when added to action
- Is capable to render $\delta\Gamma = 0$ even when $\delta I_E \neq 0$
- Reasonable Ansatz: Holographic counterterm = Solution of Hamilton–Jacobi equation!

Hamilton's principle function

- Solves the Hamilton–Jacobi equation
- Does not change boundary value problem when added to action
- Is capable to render $\delta\Gamma = 0$ even when $\delta I_E \neq 0$
- Reasonable Ansatz: Holographic counterterm = Solution of Hamilton–Jacobi equation!

In case of AdS_2 gravity this Ansatz yields

$$I_{\mathrm{CT}} = -\int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{\gamma}\,\frac{X}{\ell}$$

Hamilton's principle function

- Solves the Hamilton–Jacobi equation
- Does not change boundary value problem when added to action
- Is capable to render $\delta\Gamma = 0$ even when $\delta I_E \neq 0$
- Reasonable Ansatz: Holographic counterterm = Solution of Hamilton–Jacobi equation!

In case of AdS_2 gravity this Ansatz yields

$$I_{\mathrm{CT}} = -\int_{\partial\mathcal{M}} \mathrm{d}x\sqrt{\gamma}\,\frac{X}{\ell}$$

Action consistent with boundary value problem and variational principle:

$$\Gamma = -\frac{1}{2}\int_{\mathcal{M}} \mathrm{d}^2x\sqrt{g}\,\Big[X\,\big(R + \frac{2}{\ell^2}\big)\Big] - \int_{\partial\mathcal{M}} \mathrm{d}x\sqrt{\gamma}\,X\,K + \int_{\partial\mathcal{M}} \mathrm{d}x\sqrt{\gamma}\,\frac{X}{\ell}$$

Hamilton's principle function

- Solves the Hamilton–Jacobi equation
- Does not change boundary value problem when added to action
- Is capable to render $\delta\Gamma = 0$ even when $\delta I_E \neq 0$
- Reasonable Ansatz: Holographic counterterm = Solution of Hamilton–Jacobi equation!

In case of AdS_2 gravity this Ansatz yields

$$I_{\text{CT}} = -\int_{\partial\mathcal{M}} \mathrm{d}x\sqrt{\gamma}\,\frac{X}{\ell}$$

Action consistent with boundary value problem and variational principle:

$$\Gamma = -\frac{1}{2}\int_{\mathcal{M}} \mathrm{d}^2x\sqrt{g}\left[X\left(R + \frac{2}{\ell^2}\right)\right] - \int_{\partial\mathcal{M}} \mathrm{d}x\sqrt{\gamma}\,X\,K + \int_{\partial\mathcal{M}} \mathrm{d}x\sqrt{\gamma}\,\frac{X}{\ell}$$

$\delta\Gamma = 0$ for all variations that preserve the boundary conditions!

Thermodynamics of Black Holes as a Simple Application

Consider small perturbation around classical solution

$$I_E[g_{cl} + \delta g, X_{cl} + \delta X] = I_E[g_{cl}, X_{cl}] + \delta I_E + \dots$$

Thermodynamics of Black Holes as a Simple Application

Consider small perturbation around classical solution

$$I_E[g_{cl}+\delta g, X_{cl}+\delta X] = I_E[g_{cl}, X_{cl}] + \delta I_E + \dots$$

- The leading term is the ‘on-shell’ action.

Thermodynamics of Black Holes as a Simple Application

Consider small perturbation around classical solution

$$I_E[g_{cl} + \delta g, X_{cl} + \delta X] = I_E[g_{cl}, X_{cl}] + \delta I_E + \dots$$

- The leading term is the 'on-shell' action.
- The linear term should vanish on solutions g_{cl} and X_{cl}.

Thermodynamics of Black Holes as a Simple Application

Consider small perturbation around classical solution

$$I_E[g_{cl}+\delta g, X_{cl}+\delta X] = I_E[g_{cl}, X_{cl}] + \delta I_E + \dots$$

- The leading term is the 'on-shell' action.
- The linear term should vanish on solutions g_{cl} and X_{cl}.

If nothing goes wrong get partition function

$$\boxed{\mathcal{Z} \sim \exp\Big(-I_E[g_{cl}, X_{cl}]\Big) \times \dots}$$

Thermodynamics of Black Holes as a Simple Application

Consider small perturbation around classical solution

$$I_E[g_{cl} + \delta g, X_{cl} + \delta X] = I_E[g_{cl}, X_{cl}] + \delta I_E + \dots$$

- The leading term is the 'on-shell' action.
- The linear term should vanish on solutions g_{cl} and X_{cl}.

If nothing goes wrong get partition function

$$\boxed{\mathcal{Z} \sim \exp\Big(- I_E[g_{cl}, X_{cl}]\Big) \times \dots}$$

Accessibility of the semi-classical approximation requires

1. $I_E[g_{cl}, X_{cl}] > -\infty$
2. $\delta I_E[g_{cl}, X_{cl}; \delta g, \delta X] = 0$

Thermodynamics of Black Holes as a Simple Application

Consider small perturbation around classical solution

$$I_E[g_{cl} + \delta g, X_{cl} + \delta X] = I_E[g_{cl}, X_{cl}] + \delta I_E + \dots$$

- The leading term is the 'on-shell' action.
- The linear term should vanish on solutions g_{cl} and X_{cl}.

If nothing goes wrong get partition function

$$\boxed{\mathcal{Z} \sim \exp\Big(- I_E[g_{cl}, X_{cl}]\Big) \times \dots}$$

Accessibility of the semi-classical approximation requires

1. $I_E[g_{cl}, X_{cl}] \to -\infty \quad \to$ violated in AdS gravity!
2. $\delta I_E[g_{cl}, X_{cl}; \delta g, \delta X] = 0$

Thermodynamics of Black Holes as a Simple Application

Consider small perturbation around classical solution

$$I_E[g_{cl} + \delta g, X_{cl} + \delta X] = I_E[g_{cl}, X_{cl}] + \delta I_E + \dots$$

- The leading term is the 'on-shell' action.
- The linear term should vanish on solutions g_{cl} and X_{cl}.

If nothing goes wrong get partition function

$$\boxed{\mathcal{Z} \sim \exp\Big(-I_E[g_{cl}, X_{cl}]\Big) \times \dots}$$

Accessibility of the semi-classical approximation requires

1. $I_E[g_{cl}, X_{cl}] \to -\infty \quad \to$ violated in AdS gravity!
2. $\delta I_E[g_{cl}, X_{cl}; \delta g, \delta X] \neq 0 \quad \to$ violated in AdS gravity!

Thermodynamics of Black Holes as a Simple Application

Consider small perturbation around classical solution

$$I_E[g_{cl} + \delta g, X_{cl} + \delta X] = I_E[g_{cl}, X_{cl}] + \delta I_E + \dots$$

- The leading term is the 'on-shell' action.
- The linear term should vanish on solutions g_{cl} and X_{cl}.

If nothing goes wrong get partition function

$$\boxed{\mathcal{Z} \sim \exp\Big(- I_E[g_{cl}, X_{cl}]\Big) \times \dots}$$

Accessibility of the semi-classical approximation requires

1. $I_E[g_{cl}, X_{cl}] \to -\infty \quad \to$ violated in AdS gravity!
2. $\delta I_E[g_{cl}, X_{cl}; \delta g, \delta X] \neq 0 \quad \to$ violated in AdS gravity!

Everything goes wrong with I_E!

In particular, do not get correct free energy $F = T I_E = -\infty$ or entropy

$$S = \infty$$

Thermodynamics of Black Holes as a Simple Application

Consider small perturbation around classical solution

$$\Gamma[g_{cl} + \delta g, X_{cl} + \delta X] = \Gamma[g_{cl}, X_{cl}] + \delta\Gamma + \dots$$

- The leading term is the 'on-shell' action.
- The linear term should vanish on solutions g_{cl} and X_{cl}.

If nothing goes wrong get partition function

$$\boxed{\mathcal{Z} \sim \exp\Big(- \Gamma[g_{cl}, X_{cl}]\Big) \times \dots}$$

Accessibility of the semi-classical approximation requires

1. $\Gamma[g_{cl}, X_{cl}] > -\infty \quad \rightarrow$ ok in AdS gravity!
2. $\delta\Gamma[g_{cl}, X_{cl}; \delta g, \delta X] = 0 \quad \rightarrow$ ok in AdS gravity!

Everything works with Γ!

In particular, do get correct free energy $F = TI_E = M - TS$ and entropy

$$S = 2\pi X\big|_{\text{horizon}} = \text{Area}/4$$

Summary and algorithm of holographic renormalization

In any dimension, for any asymptotics — may arise also in quantum field theory!

- Start with bulk action I_B

Summary and algorithm of holographic renormalization

In any dimension, for any asymptotics — may arise also in quantum field theory!

- Start with bulk action I_B
- Check consistency of boundary value problem

Summary and algorithm of holographic renormalization

In any dimension, for any asymptotics — may arise also in quantum field theory!

- Start with bulk action I_B
- Check consistency of boundary value problem
- If necessary, add boundary term I_{GHY}

Summary and algorithm of holographic renormalization

In any dimension, for any asymptotics — may arise also in quantum field theory!

- Start with bulk action I_B
- Check consistency of boundary value problem
- If necessary, add boundary term I_{GHY}
- Check consistency of variational principle

Summary and algorithm of holographic renormalization

In any dimension, for any asymptotics — may arise also in quantum field theory!

- Start with bulk action I_B
- Check consistency of boundary value problem
- If necessary, add boundary term I_{GHY}
- Check consistency of variational principle
- If necessary, subtract holographic counterterm I_{CT}

Summary and algorithm of holographic renormalization

In any dimension, for any asymptotics — may arise also in quantum field theory!

- Start with bulk action I_B
- Check consistency of boundary value problem
- If necessary, add boundary term I_{GHY}
- Check consistency of variational principle
- If necessary, subtract holographic counterterm I_{CT}
- Use improved action

$$\Gamma = I_B + I_{GHY} - I_{CT}$$

for applications!

Summary and algorithm of holographic renormalization

In any dimension, for any asymptotics — may arise also in quantum field theory!

- Start with bulk action I_B
- Check consistency of boundary value problem
- If necessary, add boundary term I_{GHY}
- Check consistency of variational principle
- If necessary, subtract holographic counterterm I_{CT}
- Use improved action

  $$\Gamma = I_B + I_{GHY} - I_{CT}$$

  for applications!
- Applications include thermodynamics from Euclidean path integral and calculation of holographic stress tensor in AdS/CFT

Summary and algorithm of holographic renormalization

In any dimension, for any asymptotics — may arise also in quantum field theory!

- Start with bulk action I_B
- Check consistency of boundary value problem
- If necessary, add boundary term I_{GHY}
- Check consistency of variational principle
- If necessary, subtract holographic counterterm I_{CT}
- Use improved action

  $$\Gamma = I_B + I_{GHY} - I_{CT}$$

  for applications!
- Applications include thermodynamics from Euclidean path integral and calculation of holographic stress tensor in AdS/CFT
- Straightforward applications in quantum field theory?

Summary and algorithm of holographic renormalization

In any dimension, for any asymptotics — may arise also in quantum field theory!

- Start with bulk action I_B
- Check consistency of boundary value problem
- If necessary, add boundary term I_{GHY}
- Check consistency of variational principle
- If necessary, subtract holographic counterterm I_{CT}
- Use improved action

$$\Gamma = I_B + I_{GHY} - I_{CT}$$

  for applications!
- Applications include thermodynamics from Euclidean path integral and calculation of holographic stress tensor in AdS/CFT
- Straightforward applications in quantum field theory? Possibly!

Holographic renormalization seems ubiquitous!
Dilaton gravity in two dimensions simplest gravity models where need for holographic renormalization arises

Outline

Why lower-dimensional gravity?

Which 2D theory?

Holographic renormalization

Which 3D theory?

Attempt 1: Einstein–Hilbert

As simple as possible... but not simpler!

Let us start with the simplest attempt. `Einstein-Hilbert` action:

$$I_{\text{EH}} = \frac{1}{16\pi G} \int \mathrm{d}^3x \sqrt{-g}\, R$$

Equations of motion:

$$R_{\mu\nu} = 0$$

Ricci-flat and therefore Riemann-flat – locally trivial!

Attempt 1: Einstein–Hilbert

As simple as possible... but not simpler!

Let us start with the simplest attempt. `Einstein-Hilbert` action:

$$I_{\text{EH}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x\sqrt{-g}\, R$$

Equations of motion:

$$R_{\mu\nu} = 0$$

Ricci-flat and therefore Riemann-flat – locally trivial!

Properties of `Einstein-Hilbert`

- No gravitons (recall: in D dimensions $D(D-3)/2$ gravitons)
- No BHs
- `Einstein-Hilbert` in 3D is too simple for us!

Attempt 2: Topologically massive gravity

Deser, Jackiw and Templeton found a way to introduce gravitons!

Let us now add a gravitational Chern–Simons term. TMG action:

$$I_{\rm TMG} = I_{\rm EH} + \frac{1}{16\pi\, G}\int \mathrm{d}^3x\sqrt{-g}\,\frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)$$

Equations of motion:

$$R_{\mu\nu} + \frac{1}{\mu}\,C_{\mu\nu} = 0$$

with the Cotton tensor defined as

$$C_{\mu\nu} = \frac{1}{2}\,\varepsilon_\mu{}^{\alpha\beta}\nabla_\alpha R_{\beta\nu} + (\mu\leftrightarrow\nu)$$

Attempt 2: Topologically massive gravity

Deser, Jackiw and Templeton found a way to introduce gravitons!

Let us now add a gravitational Chern–Simons term. TMG action:

$$I_{\text{TMG}} = I_{\text{EH}} + \frac{1}{16\pi G} \int \mathrm{d}^3x \sqrt{-g}\, \frac{1}{2\mu}\, \varepsilon^{\lambda\mu\nu}\, \Gamma^{\rho}{}_{\lambda\sigma} \big(\partial_\mu \Gamma^{\sigma}{}_{\nu\rho} + \frac{2}{3}\, \Gamma^{\sigma}{}_{\mu\tau} \Gamma^{\tau}{}_{\nu\rho}\big)$$

Equations of motion:

$$R_{\mu\nu} + \frac{1}{\mu}\, C_{\mu\nu} = 0$$

with the Cotton tensor defined as

$$C_{\mu\nu} = \frac{1}{2}\, \varepsilon_{\mu}{}^{\alpha\beta} \nabla_\alpha R_{\beta\nu} + (\mu \leftrightarrow \nu)$$

Properties of TMG

- Gravitons! Reason: third derivatives in Cotton tensor!
- No BHs
- TMG is slightly too simple for us!

Attempt 3: Einstein–Hilbert–AdS

Bañados, Teitelboim and Zanelli (and Henneaux) taught us how to get 3D BHs

Add negative cosmological constant to `Einstein-Hilbert` action:

$$I_{\Lambda\text{EH}} = \frac{1}{16\pi\, G}\,\int \mathrm{d}^3x\sqrt{-g}\,\big(R+\frac{2}{\ell^2}\big)$$

Equations of motion:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}R - \frac{1}{\ell^2}\,g_{\mu\nu} = 0$$

Particular solutions: BTZ BH with line-element

$$\mathrm{d}s^2_{\text{BTZ}} = -\frac{(r^2-r_+^2)(r^2-r_-^2)}{\ell^2r^2}\,\mathrm{d}t^2 + \frac{\ell^2r^2}{(r^2-r_+^2)(r^2-r_-^2)}\,\mathrm{d}r^2 + r^2\,\big(\mathrm{d}\phi - \frac{r_+r_-}{\ell r^2}\,\mathrm{d}t\big)^2$$

Attempt 3: Einstein–Hilbert–AdS

Bañados, Teitelboim and Zanelli (and Henneaux) taught us how to get 3D BHs

Add negative cosmological constant to `Einstein-Hilbert` action:

$$I_{\Lambda \mathrm{EH}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x \sqrt{-g}\, \big(R + \frac{2}{\ell^2}\big)$$

Equations of motion:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R - \frac{1}{\ell^2}\, g_{\mu\nu} = 0$$

Particular solutions: BTZ BH with line-element

$$\mathrm{d}s^2_{\mathrm{BTZ}} = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{\ell^2 r^2}\, \mathrm{d}t^2 + \frac{\ell^2 r^2}{(r^2 - r_+^2)(r^2 - r_-^2)}\, \mathrm{d}r^2 + r^2 \big(\mathrm{d}\phi - \frac{r_+ r_-}{\ell r^2}\, \mathrm{d}t\big)^2$$

Properties of `Einstein-Hilbert-AdS`

- No gravitons
- Rotating BH solutions that asymptote to AdS_3!
- Adding a negative cosmological constant produces BH solutions!

Cosmological topologically massive gravity

CTMG is a 3D theory with BHs and gravitons!

We want a 3D theory with gravitons and BHs and therefore take CTMG action

$$I_{\text{CTMG}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x \sqrt{-g} \left[R + \frac{2}{\ell^2} + \frac{1}{2\mu} \, \varepsilon^{\lambda\mu\nu} \, \Gamma^\rho{}_{\lambda\sigma} \left(\partial_\mu \Gamma^\sigma{}_{\nu\rho} + \frac{2}{3} \, \Gamma^\sigma{}_{\mu\tau} \Gamma^\tau{}_{\nu\rho} \right) \right]$$

Equations of motion:

$$G_{\mu\nu} + \frac{1}{\mu} \, C_{\mu\nu} = 0$$

Cosmological topologically massive gravity

CTMG is a 3D theory with BHs and gravitons!

We want a 3D theory with gravitons and BHs and therefore take CTMG action

$$I_{\rm CTMG} = \frac{1}{16\pi\,G}\,\int \mathrm{d}^3x\sqrt{-g}\,\Big[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)\Big]$$

Equations of motion:

$$G_{\mu\nu} + \frac{1}{\mu}\,C_{\mu\nu} = 0$$

Properties of CTMG

- Gravitons!
- BHs!
- CTMG is just perfect for us. Study this theory!

Cosmological topologically massive gravity

CTMG is a 3D theory with BHs and gravitons!

We want a 3D theory with gravitons and BHs and therefore take CTMG action

$$I_{\text{CTMG}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x\sqrt{-g}\left[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)\right]$$

Equations of motion:

$$G_{\mu\nu} + \frac{1}{\mu}\,C_{\mu\nu} = 0$$

Properties of CTMG

- Gravitons!
- BHs!
- CTMG is just perfect for us. Study this theory!
- ...see the talk on Wednesday!

Acknowledgments

List of collaborators on 3D classical and quantum gravity:

- Roman Jackiw (MIT)
- Niklas Johansson (VUT)
- Peter van Nieuwenhuizen (YITP, Stony Brook)
- Dima Vassilevich (ABC Sao Paulo)
- Ivo Sachs (LMU Munich)
- Olaf Hohm (MIT)
- Sabine Ertl (VUT)
- Matthias Gaberdiel (ETH Zurich)
- Thomas Zojer (Groningen U.)
- Mario Bertin (ABC Sao Paulo)
- Hamid Afshar (IPM Tehran & Sharif U. of Tech. & VUT)
- Branislav Cvetkovic (Belgrade U.)
- Michael Gary (VUT)
- Radoslav Rashkov (Sofia U. & VUT)
- ...

Some literature

- D. Grumiller, W. Kummer, and D. Vassilevich, "Dilaton gravity in two dimensions," *Phys. Rept.* **369** (2002) 327–429, `hep-th/0204253`.
- D. Grumiller and R. McNees, "Thermodynamics of black holes in two (and higher) dimensions," *JHEP* **0704**, 074 (2007) `hep-th/0703230`.
- E. Witten, `0706.3359`.
- W. Li, W. Song and A. Strominger, *JHEP* **0804** (2008) 082, `0801.4566`.
- S. Carlip, S. Deser, A. Waldron and D. Wise, *Phys.Lett.* **B666** (2008) 272, `0807.0486, 0803.3998`
- D. Grumiller and N. Johansson, *JHEP* **0807** (2008) 134, `0805.2610`.
- H. Afshar, B. Cvetkovic, S. Ertl, D. Grumiller and N. Johansson, *Phys. Rev.* **D** (2012) *in print*, `1110.5644`.

Thanks to Bob McNees for providing the LaTeX beamerclass!

Thank you for your attention!

Recent example: AdS$_2$ holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS$_2$ factor
- funnily, AdS$_3$ holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\text{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x\sqrt{-g}\,\Big[e^{-2\phi}\,\Big(R+\frac{8}{L^2}\Big)-\frac{L^2}{4}\,F^2\Big]$$

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\text{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x \sqrt{-g} \left[e^{-2\phi} \left(R + \frac{8}{L^2} \right) - \frac{L^2}{4} F^2 \right]$$

- Metric g has signature $-, +$ and Ricci-scalar $R < 0$ for AdS

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\mathrm{JT}} = \frac{\alpha}{2\pi}\int \mathrm{d}^2x\sqrt{-g}\,\left[e^{-2\phi}\left(R+\frac{8}{L^2}\right)-\frac{L^2}{4}\,F^2\right]$$

- Metric g has signature $-,+$ and Ricci-scalar $R < 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\,\varepsilon_{\mu\nu}$ dual to electric field E

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\text{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x \sqrt{-g}\, \left[e^{-2\phi}\, \left(R + \frac{8}{L^2}\right) - \frac{L^2}{4}\, F^2\right]$$

- Metric g has signature $-, +$ and Ricci-scalar $R < 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\, \varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\mathrm{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x\sqrt{-g}\,\left[e^{-2\phi}\,\left(R + \frac{8}{L^2}\right) - \frac{L^2}{4}\,F^2\right]$$

- Metric g has signature $-,+$ and Ricci-scalar $R < 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\,\varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field
- Cosmological constant $\Lambda = -\frac{8}{L^2}$ parameterized by AdS radius L

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\text{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x \sqrt{-g}\, \left[e^{-2\phi} \left(R + \frac{8}{L^2}\right) - \frac{L^2}{4}\, F^2\right]$$

- Metric g has signature $-, +$ and Ricci-scalar $R < 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\, \varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field
- Cosmological constant $\Lambda = -\frac{8}{L^2}$ parameterized by AdS radius L
- Coupling constant α usually is positive

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\text{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x \sqrt{-g} \left[e^{-2\phi} \left(R + \frac{8}{L^2} \right) - \frac{L^2}{4} F^2 \right]$$

- Metric g has signature $-, +$ and Ricci-scalar $R < 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\, \varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field
- Cosmological constant $\Lambda = -\frac{8}{L^2}$ parameterized by AdS radius L
- Coupling constant α usually is positive
- $\delta\phi$ EOM: $R = -\frac{8}{L^2} \qquad \Rightarrow \qquad$ AdS_2!

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\text{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x\sqrt{-g}\,\Big[e^{-2\phi}\,\Big(R + \frac{8}{L^2}\Big) - \frac{L^2}{4}\,F^2\Big]$$

- Metric g has signature $-,+$ and Ricci-scalar $R< 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\,\varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field
- Cosmological constant $\Lambda = -\frac{8}{L^2}$ parameterized by AdS radius L
- Coupling constant α usually is positive
- $\delta\phi$ EOM: $R = -\frac{8}{L^2}$ $\quad\Rightarrow\quad$ AdS_2!
- δA EOM: $\nabla_\mu F^{\mu\nu} = 0$ $\quad\Rightarrow\quad$ $E = \text{constant}$

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\mathrm{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x \sqrt{-g} \left[e^{-2\phi} \left(R + \frac{8}{L^2} \right) - \frac{L^2}{4} F^2 \right]$$

- Metric g has signature $-,+$ and Ricci-scalar $R < 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\,\varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field
- Cosmological constant $\Lambda = -\frac{8}{L^2}$ parameterized by AdS radius L
- Coupling constant α usually is positive
- $\delta\phi$ EOM: $R = -\frac{8}{L^2} \quad \Rightarrow \quad \mathrm{AdS}_2$!
- δA EOM: $\nabla_\mu F^{\mu\nu} = 0 \quad \Rightarrow \quad E = \text{constant}$
- δg EOM: complicated for non-constant dilaton...

$$\nabla_\mu \nabla_\nu e^{-2\phi} - g_{\mu\nu} \nabla^2 e^{-2\phi} + \frac{4}{L^2} e^{-2\phi} g_{\mu\nu} + \frac{L^2}{2} F_\mu{}^\lambda F_{\nu\lambda} - \frac{L^2}{8} g_{\mu\nu} F^2 = 0$$

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\text{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x \sqrt{-g}\, \Big[e^{-2\phi}\,\Big(R + \frac{8}{L^2}\Big) - \frac{L^2}{4}\,F^2\Big]$$

- Metric g has signature $-,+$ and Ricci-scalar $R< 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\,\varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field
- Cosmological constant $\Lambda = -\frac{8}{L^2}$ parameterized by AdS radius L
- Coupling constant α usually is positive
- $\delta\phi$ EOM: $R = -\frac{8}{L^2}$ $\quad\Rightarrow\quad$ AdS_2!
- δA EOM: $\nabla_\mu F^{\mu\nu} = 0$ $\quad\Rightarrow\quad$ $E = \text{constant}$
- δg EOM: ...but simple for constant dilaton: $e^{-2\phi} = \frac{L^4}{4}\,E^2$

$$\nabla_\mu\nabla_\nu e^{-2\phi} - g_{\mu\nu}\,\nabla^2 e^{-2\phi} + \frac{4}{L^2}\,e^{-2\phi}\,g_{\mu\nu} + \frac{L^2}{2}\,F_\mu{}^\lambda\,F_{\nu\lambda} - \frac{L^2}{8}\,g_{\mu\nu}\,F^2 = 0$$

Some surprising results

Hartman, Strominger = HS   Castro, DG, Larsen, McNees = CGLM

- Holographic renormalization leads to boundary mass term (CGLM)

$$I \sim \int \mathrm{d}x \sqrt{|\gamma|}\, m A^2$$

Nevertheless, total action gauge invariant

Some surprising results

Hartman, Strominger = HS   Castro, DG, Larsen, McNees = CGLM

- Holographic renormalization leads to boundary mass term (CGLM)

$$I \sim \int \mathrm{d}x \sqrt{|\gamma|}\, m A^2$$

  Nevertheless, total action gauge invariant

- Boundary stress tensor transforms anomalously (HS)

$$(\delta_\xi + \delta_\lambda)\, T_{tt} = 2T_{tt}\partial_t\xi + \xi\partial_t T_{tt} - \frac{c}{24\pi} L\partial_t^3\xi$$

  where $\delta_\xi + \delta_\lambda$ is combination of diffeo- and gauge trafos that preserve the boundary conditions (similarly: $\delta_\lambda J_t = -\frac{k}{4\pi} L\partial_t\lambda$)

Some surprising results

Hartman, Strominger = HS   Castro, DG, Larsen, McNees = CGLM

- Holographic renormalization leads to boundary mass term (CGLM)

$$I \sim \int \mathrm{d}x \sqrt{|\gamma|}\, m A^2$$

  Nevertheless, total action gauge invariant

- Boundary stress tensor transforms anomalously (HS)

$$(\delta_\xi + \delta_\lambda)\, T_{tt} = 2T_{tt}\partial_t\xi + \xi\partial_t T_{tt} - \frac{c}{24\pi} L\partial_t^3\xi$$

  where $\delta_\xi + \delta_\lambda$ is combination of diffeo- and gauge trafos that preserve the boundary conditions (similarly: $\delta_\lambda J_t = -\frac{k}{4\pi} L\partial_t\lambda$)

- Anomalous transformation above leads to central charge (HS, CGLM)

$$c = -24\alpha e^{-2\phi} = \frac{3}{G_2} = \frac{3}{2} k E^2 L^2$$

Some surprising results

Hartman, Strominger = HS   Castro, DG, Larsen, McNees = CGLM

- Holographic renormalization leads to boundary mass term (CGLM)

$$I \sim \int \mathrm{d}x \sqrt{|\gamma|}\, m A^2$$

  Nevertheless, total action gauge invariant
- Boundary stress tensor transforms anomalously (HS)

$$(\delta_\xi + \delta_\lambda)\, T_{tt} = 2T_{tt}\partial_t\xi + \xi\partial_t T_{tt} - \frac{c}{24\pi} L \partial_t^3 \xi$$

  where $\delta_\xi + \delta_\lambda$ is combination of diffeo- and gauge trafos that preserve the boundary conditions (similarly: $\delta_\lambda J_t = -\frac{k}{4\pi} L \partial_t \lambda$)
- Anomalous transformation above leads to central charge (HS, CGLM)

$$c = -24\alpha e^{-2\phi} = \frac{3}{G_2} = \frac{3}{2} k E^2 L^2$$

- Positive central charge only for negative coupling constant α (CGLM)

$$\alpha < 0$$