Semi–Classical Unitarity in 3d Higher Spin Gravity

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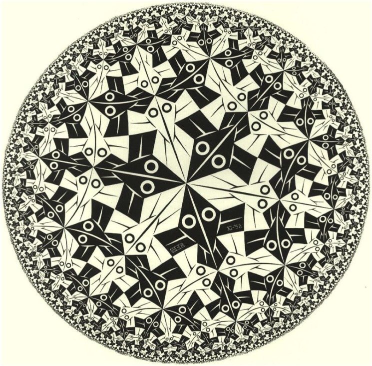
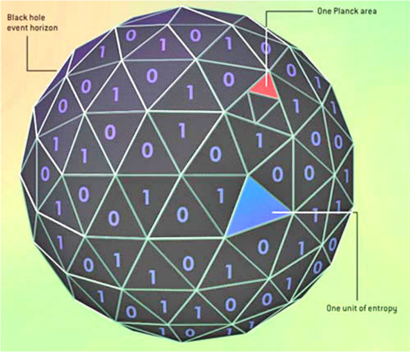
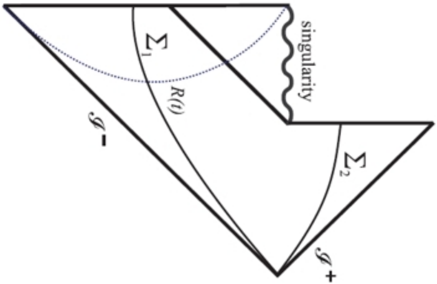
1201.0013, 1209.2860, 1211.4454




Black hole event horizon
One Planck area
One unit of entropy


singularity
Σ_1
$R(t)$
Σ_2
$\mathscr{I}^-$
$\mathscr{I}^+$

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  - Simplicity: topological field theories, AdS_3/CFT_2, no Weyl tensor, ...

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- Classical reformulation as $SL(2)\times SL(2)$ Chern–Simons (CS) [Achucarro, Townsend 1986, Witten 1988]

$$I = I_{\text{CS}}[A] - I_{\text{CS}}[\bar{A}]$$

$$I_{\text{CS}}[A] = \frac{k}{4\pi}\int \mathsf{Tr}(A\wedge \mathrm{d}A + \tfrac{2}{3}A\wedge A\wedge A) + B[A]$$

- Can exploit $\text{AdS}_3/\text{CFT}_2$, e.g. Brown–Henneaux result for central charges

$$c_L = c_R = \frac{3}{2G_N}$$

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> Search for alternative theories with same advantages and no disadvantages!

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$SL(N) \times SL(N)$ CS with suitable boundary conditions called "Higher Spin Gravity in 3 Dimensions"

Example: Spin 3 gravity (Henneaux, Rey 2010, Campoleoni, Fredenhagen, Pfenninger, Theisen 2010), principal embedding of $SL(2)$ into $SL(3)$

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- Particularly, no obvious restriction on k from unitarity

Either work harder or look for alternatives!

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- No-go result! (Castro, Hijano, Lepage-Jutier 2012)

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Back to square one or circumvent no-go result!

Essence of no-go result

- All non-principal embeddings have singlet factor
- Leads to Kac–Moody algebra as part of asymptotic symmetry algebra

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- Unitarity requires non-negative central charge, $c \geq 0$
- CHL proved in semi-classical limit $|c| \to \infty$ inequality

$$\text{sign}(c) = -\text{sign}(\kappa)$$

- The minus sign proves the no-go result!

Circumventing no-go result

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Main results

- At least one such family exists
- Next-to-principal embedding (W_N^2 gravity)
- Asymptotic symmetry algebra is Feigin–Semikhatov algebra
- Unitarity for given family member maintained if

$$c \leq \frac{N}{4} - \frac{1}{8} + \mathcal{O}(1/N)$$

Polyakov–Bershadsky example

Simplest family member is W_3^2 gravity with asymptotic symmetry algebra

$$[J_n,\, L_m] = nJ_{n+m} \quad [J_n,\, G^\pm_m] = \pm G^\pm_{m+n} \quad [L_n,\, G^\pm_m] = \Big(\frac{n}{2} - m\Big)\, G^\pm_{n+m}$$

$$[J_n,\, J_m] = \kappa\, n\, \delta_{n+m,\,0}$$

$$[L_n,\, L_m] = (n-m)L_{m+n} + \frac{c}{12}\, n(n^2-1)\, \delta_{n+m,\,0}$$

$$[G^+_n,\, G^-_m] = \frac{\lambda}{2}\, \Big(n^2 - \frac{1}{4}\Big)\, \delta_{n+m,\,0} + \dots$$

with level

$$\kappa = \frac{2k+3}{3}$$

central charge

$$c = 25 - \frac{24}{k+3} - 6(k+3)$$

and central term in $G^\pm$ commutator

$$\lambda = (k+1)(2k+3)\,.$$

Restrictions from unitarity

1209.2680 with Afshar, Gary, Rashkov and Riegler

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Result for Polyakov–Bershadsky

Central charge must be either $c = 0$ or $c = 1$

W_N^2 gravity (with spins up to $N-1$)

1211.4454 with Afshar, Gary, Rashkov and Riegler

Feigin–Semikhatov algebra similar to Polyakov–Bershadsky

$$[J_n,\, L_m] = nJ_{n+m}\;[J_n,\, G^{\pm}_m] = \pm G^{\pm}_{m+n}\;[L_n,\, G^{\pm}_m] = \big(n(\tfrac{N}{2}-1)-m\big)\,G^{\pm}_{n+m}$$

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$$[W^l_n,\, \text{anything}] = \dots$$

with central term in $G^{\pm}$ commutator

$$\lambda = \prod_{m=1}^{N-1}\big(m(N+k-1)-1\big)$$

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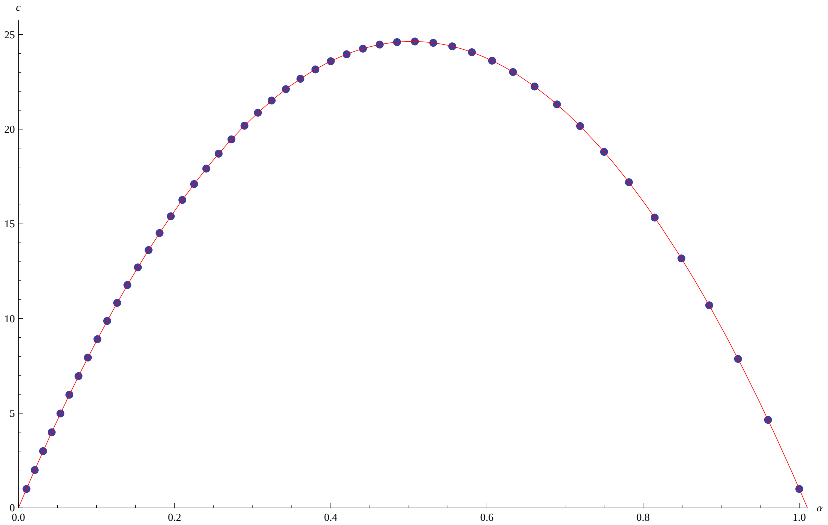
As before unitarity requires $\lambda = 0$

Allowed values of central charge ($N = 100$, $\alpha = \kappa N$)



Solid red curve: allowed by non-negativity of central charge and level

Blue dots: consistent with $\lambda = 0$

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$$c = \alpha(1-\alpha)N + \mathcal{O}(1) \leq \frac{N}{4} + \mathcal{O}(1)$$

- Large $\alpha \sim \mathcal{O}(1)$: dual quantum regime with central charge $c \sim \mathcal{O}(1)$

Open issues

- Limit $N \to \infty$ and relation to minimal model holography (Gaberdiel, Gopakumar, 2011)?

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- Further consistency checks? (e.g. partition function)
- Other non-principal embeddings? (Next-to-next-to-principal, least principal, and the whole zoo in between)

Illustration by uberkraaft (Matt Williams, 2012), used with permission of artist

Thanks for your attention!

Collaborators:

- Hamid Afshar (postdoc at VUT, outgoing to Groningen)
- Michael Gary (postdoc at VUT)
- Radoslav Rashkov (guest professor at VUT)
- Max Riegler (PhD student at VUT)

M. Gary, D. Grumiller and R. Rashkov, "Towards non-AdS holography in 3-dimensional higher spin gravity," *JHEP* **1203** (2012) 022, 1201.0013.

H. Afshar, M. Gary, D. Grumiller, R. Rashkov and M. Riegler, "Non-AdS holography in 3-dimensional higher spin gravity," *JHEP* **1211** (2012) 099, 1209.2860.

H. Afshar, M. Gary, D. Grumiller, R. Rashkov and M. Riegler, "Semi-classical unitarity in 3-dimensional higher-spin gravity for non-principal embeddings," 1211.4454.

Thanks to Bob McNees for providing the LaTeX beamerclass!

Backup slide I

Full expressions for central charge, level and dual level

Level k in terms of parameter α

$$k = -N + 1 + \frac{\alpha + 1}{N - 1}$$

Dual level $\tilde{k}$

$$\tilde{k} = \frac{N+1}{N-2}\,\frac{1}{N+k} - N$$

Duality in terms of α

$$\tilde{\alpha} = \frac{N\big(N(1-\alpha) + 2\alpha - 1\big) + 1}{(N-2)(N+\alpha)} = 1 - \alpha + \mathcal{O}(1/N)$$

Full expression for central charge (duality invariant!)

$$c = \alpha(1-\alpha)\,N + \alpha\,(\alpha^2 + \alpha - 1) - \sum_{m=1}^{\infty} (1+\alpha)^2 (1-\alpha)\Big(-\frac{\alpha}{N}\Big)^m$$

Backup slide II

Holographic algorithm from gravity point of view

Canonical recipe:

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
7. Identify/constrain dual field theory
8. If unhappy with result go back to previous items and modify

Backup slide III
Quantum asymptotic symmetry algebra

Introducing normal ordering in expressions like

$$\sum_{p\in\mathbb{Z}} :J_{n-p}J_p: = \sum_{p\geq 0} J_{n-p}J_p + \sum_{p<0} J_pJ_{n-p}$$

can make semi-classical algebra inconsistent

First example I am aware of: Henneaux–Rey 2010 in spin-3 AdS gravity

Quantum violations of Jacobi-identities possible!

Resolution: deform suitable structure constants/functions and demand validity of Jacobi identities