Holography and phase-transition of flat space

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FWF

based on work with
Afshar, Bagchi, Detournay, Fareghbal, Rosseel, Schöller, Simon

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Address these question in 3D!

"Gravity 3D is a spellbinding experience"

... so let us consider 3D gravity!

(Higher spin) gravity as Chern–Simons gauge theory...

...with weird boundary conditions (Achucarro & Townsend '86; Witten '88; Bañados '96)

CS action:

$$S_{\text{CS}} = \frac{k}{4\pi} \int \text{CS}(A) - \frac{k}{4\pi} \int \text{CS}(\bar{A})$$

Variational principle:

$$\delta S_{\text{CS}}\big|_{\text{EOM}} = \frac{k}{4\pi} \int \text{tr}\left(A \wedge \delta A - \bar{A} \wedge \delta \bar{A}\right)$$

Well-defined for boundary conditions (similarly for $\bar{A}$)

$$A_+ = 0 \qquad \text{or} \qquad A_- = 0 \qquad \text{boundary coordinates } x^\pm$$

Example: asymptotically AdS_3 (Arctan-version of Brown–Henneaux)

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Example: asymptotically AdS_3 (Cartan-version of Brown–Henneaux)

$$A_\rho = L_0 \qquad \bar{A}_\rho = -L_0$$
$$A_+ = e^\rho L_1 + e^{-\rho} L(x^+) L_{-1} \qquad \bar{A}_+ = 0$$
$$A_- = 0 \qquad \bar{A}_- = -e^\rho L_{-1} - e^{-\rho} \bar{L}(x^-) L_1$$

Dreibein: $e/\ell \sim A - \bar{A}$, spin-connection: $\omega \sim A + \bar{A}$

Non-AdS holography

Variational principle for non-vanishing boundary connection (Gary, DG & Rashkov '12)

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- Add boundary term to CS action

$$S_B = \frac{k}{4\pi} \int \mathrm{d}x^+ \, \mathrm{d}x^- \, \mathrm{tr} \left(A_+ A_- + \bar{A}_+ \bar{A}_- \right)$$

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- Simplest examples: Lobachevsky holography for $SO(3,2)$ broken to $SO(2,2) \times U(1)$ (conformal CS gravity, Bertin, Ertl, Ghorbani, DG, Johansson & Vassilevich '12) and null warped holography for principally embedded spin-3 gravity (in prep. with Gary & Perlmutter)

İnönü–Wigner contraction of Virasoro (Barnich & Compère '06)

BMS_3 and GCA_2 (or rather, $URCA_2$)

- Take two copies of Virasoro, generators $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$, central charges c, $\bar{c}$

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- Define superrotations L_n and supertranslations M_n

$$L_n := \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n := \tfrac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

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Example TMG (with gravitational CS coupling μ and Newton constant G):

$$c_L = \frac{3}{\mu G} \qquad c_M = \frac{3}{G}$$

Consequence of ultrarelativistic boost for AdS boundary

AdS-boundary:

Flat space boundary:

Limit $\ell \to \infty$

Null infinity holography!

Contraction on gravity side

AdS metric ($\varphi \sim \varphi + 2\pi$):

$$\mathrm{d}s^2_{\text{AdS}} = \mathrm{d}(\ell\rho)^2 - \cosh^2\big(\tfrac{\ell\rho}{\ell}\big)\ \mathrm{d}t^2 + \ell^2\ \sinh^2\big(\tfrac{\ell\rho}{\ell}\big)\ \mathrm{d}\varphi^2$$

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Limit $\ell \to \infty$ ($r = \ell\rho$):

$$\mathrm{d}s^2_{\text{Flat}} = \mathrm{d}r^2 - \mathrm{d}t^2 + r^2\ \mathrm{d}\varphi^2 = -\,\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\ \mathrm{d}\varphi^2$$

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Limit $\ell \to \infty$ ($\hat{r}_+ = \frac{r_+}{\ell} = \text{finite}$):

$$\mathrm{d}s^2_{\text{FSC}} = \hat{r}_+^2\ \big(1 - \tfrac{r_-^2}{r^2}\big)\ \mathrm{d}t^2 - \frac{1}{1 - \frac{r_-^2}{r^2}}\ \frac{\mathrm{d}r^2}{\hat{r}_+^2} + r^2\ \Big(\mathrm{d}\varphi - \frac{\hat{r}_+\,r_-}{r^2}\ \mathrm{d}t\Big)^2$$

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Shifted-boost orbifold studied by Cornalba & Costa more than decade ago

Describes expanding (contracting) Universe in flat space

Cosmological horizon at $r = r_-$, screening CTCs at $r < 0$

Flat-space boundary conditions

Barnich & Compere '06; Bagchi, DG, Detournay '12; Barnich & González '13

- In metric formulation:

$$\begin{pmatrix} g_{uu} = \mathcal{O}(1) & g_{ur} = -1 + \mathcal{O}(1/r) & g_{u\varphi} = \mathcal{O}(1) \\ & g_{rr} = \mathcal{O}(1/r^2) & g_{r\varphi} = h_0 + \mathcal{O}(1/r) \\ & & g_{\varphi\varphi} = r^2 + (h_1 u + h_2)\, r + \mathcal{O}(1) \end{pmatrix}$$

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- Angular momentum tower:

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Asymptotic symmetry algebra

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- Conformal CS gravity ($G \to \infty$, $8\mu G = 1/k$ finite): $c_L \neq 0$, $c_M = 0$

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- Canonical analysis of TMG yields ASA

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$$[L_n,\, M_m] = (n-m)\, M_{n+m} + c_M\, \tfrac{1}{12}\, \delta_{n+m,\,0}$$
$$[M_n,\, M_m] = 0$$

with central charges

$$c_L = \frac{3}{\mu G} \qquad c_M = \frac{3}{G}$$

- Coincides precisely with results from İnönü–Wigner contraction!
- Einstein gravity ($\mu \to \infty$, G finite): $c_L = 0$, $c_M \neq 0$
- Conformal CS gravity ($G \to \infty$, $8\mu G = 1/k$ finite): $c_L \neq 0$, $c_M = 0$
- Note: generalizations to higher curvature theories like NMG, GMG, PMG, ... should be straightforward

Phase transition (1305.2919)

Hot flat space melts into expanding Universe

General philosophy:

Everything in AdS/CFT could have counterpart in flat space limit

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- "Melting point" of flat space:

$$T_{\text{critical}} = \frac{1}{2\pi\, r_-}$$

Hot flat space (at large T) $\rightarrow$ ~~The Universe Flat~~ The Flat Universe (FSC)

Unitarity in 3D (higher spin) gravity?

General remarks on unitarity (Afshar, Gary, DG, Rashkov, Riegler '12)

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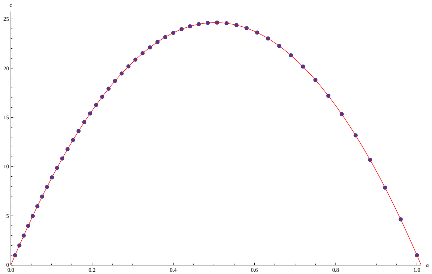
- Circumvented: $\lim \to \infty$ replaced by "finite, but arbitrarily large"
- Example: W_N^2-gravity, with discrete set of unitary values of c:

Plot: spin-100 gravity (W_{101}^2)



Plotted: central charge as function of CS level

Points in plot correspond to unitary points

For large N: $c \leq \frac{N}{4} - \frac{1}{8} - \mathcal{O}(1/N)$

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- Canonical analysis yields $c_L = 24k$ (and $c_M = 0$)
- Entropy of FSC in flat space chiral gravity ($h_L = k\, r_+^2$):

$$S = 4\pi k\, \hat{r}_+ = 2\pi \sqrt{\frac{c_L h_L}{6}}$$

Coincides with chiral version of Cardy formula

Outlook to higher spins

Towards flat space higher spin gravity in 3D

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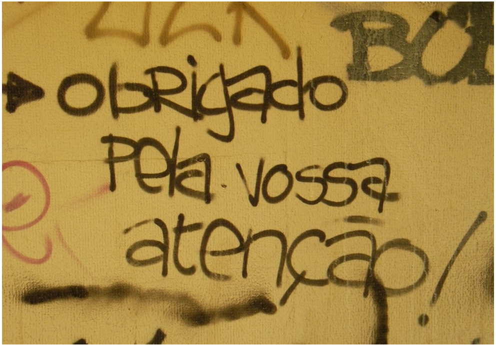
Answers in talk by Jan Rosseel!



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