Holography and phase-transition of flat space

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based on work with
Afshar, Bagchi, Detournay, Fareghbal, Rosseel, Schöller, Simon
Motivation

- Holographic principle, if correct, must work beyond AdS/CFT
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Address these question in 3D!
“Gravity 3D is a spellbinding experience”

... so let us consider 3D gravity!
(Higher spin) gravity as Chern–Simons gauge theory...
...with weird boundary conditions (Achucarro & Townsend ’86; Witten ’88; Bañados ’96)

CS action:

\[ S_{CS} = \frac{k}{4\pi} \int CS(A) - \frac{k}{4\pi} \int CS(\bar{A}) \]

Variational principle:

\[ \delta S_{CS}|_{EOM} = \frac{k}{4\pi} \int \text{tr} \left( A \wedge \delta A - \bar{A} \wedge \delta \bar{A} \right) \]

Well-defined for boundary conditions (similarly for \( \bar{A} \))

\[ A_+ = 0 \quad \text{or} \quad A_- = 0 \quad \text{boundary coordinates} \quad x^\pm \]

Example: asymptotically AdS\(_3\) (Arctan-version of Brown–Henneaux)
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Example: asymptotically AdS_3 (Cartan-version of Brown–Henneaux)
\[ A_\rho = L_0 \quad \quad \bar{A}_\rho = -L_0 \]
\[ A_+ = e^\rho L_1 + e^{-\rho} L(x^+) L_{-1} \quad \bar{A}_+ = 0 \]
\[ A_- = 0 \quad \quad \bar{A}_- = -e^\rho L_{-1} - e^{-\rho} \bar{L}(x^-) L_1 \]

Dreibein: \( e/\ell \sim A - \bar{A} \), spin-connection: \( \omega \sim A + \bar{A} \)
Non-AdS holography
Variational principle for non-vanishing boundary connection (Gary, DG & Rashkov ’12)

- Want to relax $A_- = \bar{A}_+ = 0$, e.g. to $\delta A_- = \delta \bar{A}_+ = 0$
Non-AdS holography

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▶ Want to relax $A_- = \bar{A}_+ = 0$, e.g. to $\delta A_- = \delta \bar{A}_+ = 0$

▶ Add boundary term to CS action

$$S_B = \frac{k}{4\pi} \int dx^+ dx^- \text{tr} \left( A_+ A_- + \bar{A}_+ \bar{A}_- \right)$$
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- Invariant under (anti-)holomorphic boundary diffeos, but not fully boundary diff-invariant!

Gain: can do non-AdS holography in (higher-spin) gravity, including Lobachevsky, warped AdS, Lifshitz and Schrödinger

Simplest examples: Lobachevsky holography for $SO(3,2)$ broken to $SO(2,2) \times U(1)$ (conformal CS gravity, Bertin, Ertl, Ghorbani, DG, Johansson & Vassilevich '12) and null warped holography for principally embedded spin-3 gravity (in prep. with Gary & Perlmutter)
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- Variation of full action:

$$\delta(S_{CS} + S_B) = \frac{k}{2\pi} \int dx^+ dx^- (A_+ \delta A_- + \bar{A}_- \delta \bar{A}_+)$$
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İnönü–Wigner contraction of Virasoro (Barnich & Compère ’06)
BMS$_3$ and GCA$_2$ (or rather, URCA$_2$)

- Take two copies of Virasoro, generators $\mathcal{L}_n, \bar{\mathcal{L}}_n$, central charges $c, \bar{c}$
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- Define superrotations $L_n$ and supertranslations $M_n$

\[
L_n := L_n - \bar{L}_{-n} \quad M_n := \frac{1}{\ell} (L_n + \bar{L}_{-n})
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- Make ultrarelativistic boost, $\ell \to \infty$

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\begin{align*}
\left[ L_n, L_m \right] & = (n - m) \mathcal{L}_{n+m} + c_L \frac{1}{12} \delta_{n+m,0} \\
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- Is precisely the (centrally extended) BMS$_3$ algebra!
- Central charges:
  
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  c_L = c - \bar{c} \quad c_M = (c + \bar{c})/\ell
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Example TMG (with gravitational CS coupling $\mu$ and Newton constant $G$):

\[
\frac{c_L}{\mu G} = \frac{3}{\mu G} \quad \frac{c_M}{G} = \frac{3}{G}
\]
Consequence of ultrarelativistic boost for AdS boundary

AdS-boundary: Flat space boundary:

Limit $\ell \to \infty$

Null infinity holography!
Contraction on gravity side

AdS metric ($\varphi \sim \varphi + 2\pi$):

$$ds^2_{\text{AdS}} = d(\ell \rho)^2 - \cosh^2\left(\frac{\ell \rho}{\ell}\right) dt^2 + \ell^2 \sinh^2\left(\frac{\ell \rho}{\ell}\right) d\varphi^2$$
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Limit $\ell \to \infty$ ($r = \ell \rho$):

$$d s^2_{\text{Flat}} = dr^2 - dt^2 + r^2 d\varphi^2 = -du^2 - 2 du dr + r^2 d\varphi^2$$
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BTZ metric:

$$ds^2_{\text{BTZ}} = -\left( \frac{r^2}{\ell^2} - \frac{r_+^2}{\ell^2} \right) (r^2 - r_-^2) \, dt^2 + \frac{r^2 \, dr^2}{(r^2 - \frac{r_+^2}{\ell^2})(r^2 - r_-^2)} + r^2 \left( d\varphi - \frac{r_+}{r^2} \frac{r_-}{r^2} \, dt \right)^2$$
Contraction on gravity side

**AdS metric** ($\varphi \sim \varphi + 2\pi$):

$$ds_{\text{AdS}}^2 = d(\ell \rho)^2 - \cosh^2 \left( \frac{\ell \rho}{\ell} \right) \, dt^2 + \ell^2 \sinh^2 \left( \frac{\ell \rho}{\ell} \right) \, d\varphi^2$$

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Limit $\ell \to \infty$ ($\hat{r}_+ = \frac{r_+}{\ell} = \text{finite}$):

$$ds_{\text{FSC}}^2 = \hat{r}_+^2 \left( 1 - \frac{r_+^2}{r^2} \right) \, dt^2 - \frac{1}{1 - \frac{r_+^2}{r^2}} \frac{dr^2}{\hat{r}_+^2} + r^2 \left( d\varphi - \frac{\hat{r}_+ + r_-}{r^2} \, dt \right)^2$$
Contraction on gravity side

AdS metric ($\varphi \sim \varphi + 2\pi$):

$$ds_{AdS}^2 = d(l\rho)^2 - \cosh^2\left(\frac{l\rho}{\ell}\right) dt^2 + \ell^2 \sinh^2\left(\frac{l\rho}{\ell}\right) d\varphi^2$$

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Shifted-boost orbifold studied by Cornalba & Costa more than decade ago

Describes expanding (contracting) Universe in flat space

Cosmological horizon at $r = r_-$, screening CTCs at $r < 0$
Flat-space boundary conditions
Barnich & Compere ’06; Bagchi, DG, Detournay ’12; Barnich & González ’13

▶ In metric formulation:
\[
\begin{pmatrix}
g_{uu} = \mathcal{O}(1) & g_{ur} = -1 + \mathcal{O}(1/r) & g_{u\varphi} = \mathcal{O}(1) \\
g_{rr} = \mathcal{O}(1/r^2) & g_{r\varphi} = h_0 + \mathcal{O}(1/r) & g_{\varphi\varphi} = r^2 + (h_1 u + h_2) r + \mathcal{O}(1)
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▶ Functions $h_i$ depend on $\varphi$ only
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- Canonical boundary charges finite, integrable and conserved in TMG
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- Mass tower:
  \[
  M_n \sim \int d\varphi e^{i n \varphi} (g_{uu} + h_1)
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M_n \sim \int d\varphi e^{in\varphi} (g_{uu} + h_1)
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► Angular momentum tower:

\[
L_n \sim \frac{1}{\mu} M_n + \int d\varphi e^{in\varphi} \left( inu g_{uu} + inr(1 + g_{ur}) + 2g_{u\varphi} + r\partial_u g_{r\varphi} - h_0 h_1 - inh_2 \right)
\]
Asymptotic symmetry algebra

- Canonical analysis of TMG yields ASA

\[
\begin{align*}
[L_n, L_m] &= (n - m) L_{n+m} + c_L \frac{1}{12} \delta_{n+m,0} \\
[L_n, M_m] &= (n - m) M_{n+m} + c_M \frac{1}{12} \delta_{n+m,0} \\
[M_n, M_m] &= 0
\end{align*}
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with central charges

\[
c_L = \frac{3}{\mu G} \quad c_M = \frac{3}{G}
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- Coincides precisely with results from İnönü–Wigner contraction!

Note: generalizations to higher curvature theories like NMG, GMG, ... should be straightforward
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- Einstein gravity (\(\mu \to \infty, G \text{ finite}\)): \(c_L = 0, c_M \neq 0\)
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- Einstein gravity ($\mu \to \infty$, $G$ finite): $c_L = 0$, $c_M \neq 0$
- Conformal CS gravity ($G \to \infty$, $8\mu G = 1/k$ finite): $c_L \neq 0$, $c_M = 0$
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with central charges

\[
c_L = \frac{3}{\mu G} \quad c_M = \frac{3}{G}
\]

- Coincides precisely with results from İnönü–Wigner contraction!
- Einstein gravity ($\mu \to \infty$, $G$ finite): $c_L = 0$, $c_M \neq 0$
- Conformal CS gravity ($G \to \infty$, $8\mu G = 1/k$ finite): $c_L \neq 0$, $c_M = 0$
- Note: generalizations to higher curvature theories like NMG, GMG, PMG, ... should be straightforward
Phase transition (1305.2919)
Hot flat space melts into expanding Universe

General philosophy:

Everything in AdS/CFT could have counterpart in flat space limit

▶ Take $\ell \to \infty$ of 9000+ AdS/CFT papers?

"Melting point" of flat space:

$T_{\text{critical}} = \frac{1}{2\pi r}$
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Hot flat space (at large $T$) $\rightarrow$ The Universe Flat The Flat Universe (FSC)
Unitarity in 3D (higher spin) gravity?
General remarks on unitarity (Afshar, Gary, DG, Rashkov, Riegler '12)

- Wanted: unitary family of topological models with $c \gg 1$
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- Example: $W^2_N$-gravity, with discrete set of unitary values of $c$:

Plot: spin-100 gravity ($W^2_{101}$)

Plotted: central charge as function of CS level

Points in plot correspond to unitary points

For large $N$: $c \leq \frac{N}{4} - \frac{1}{8} - \mathcal{O}(1/N)$
Unitarity in flat space?
Flat-space chiral gravity (1208.1658)

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For $c_M \neq 0$: always positive and negative norm state!

Unitarity requires $c_M = 0$ (impossible in Einstein gravity)

Within TMG family: only conformal CS gravity can be unitary

$$S_{CSG}[g] = k4\pi\int (\Gamma \wedge d\Gamma + 2\frac{3}{3}\Gamma \wedge \Gamma \wedge \Gamma)$$

Canonical analysis yields $c_L = 24k$ (and $c_M = 0$)

Entropy of FSC in flat space chiral gravity ($h_L = kr^2$):

$$S = 4\pi k\hat{r} + 2\pi\sqrt{c_L h_L}$$

Coincides with chiral version of Cardy formula
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Towards flat space higher spin gravity in 3D

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Answers in talk by Jan Rosseel!
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References


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