Flat Space Holography

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Some of our papers on flat space holography

A. Bagchi, D. Grumiller and W. Merbis,
“Stress tensor correlators in three-dimensional gravity,”

A. Bagchi, R. Basu, D. Grumiller and M. Riegler,
“Entanglement entropy in Galilean conformal field theories and flat holography,”

H. Afshar, A. Bagchi, R. Fareghbal, D. Grumiller and J. Rosseel,
“Spin-3 Gravity in Three-Dimensional Flat Space,”

A. Bagchi, S. Detournay, D. Grumiller and J. Simon,
“Cosmic Evolution from Phase Transition of Three-Dimensional Flat Space,”

A. Bagchi, S. Detournay and D. Grumiller,
“Flat-Space Chiral Gravity,”
Outline

Motivations

Assumptions

Flat space holography basics

Recent results

Generalizations & open issues
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Holography in our Universe?

This talk focuses on holography (in the quantum gravity sense).
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Main question: how general is holography?
In memoriam Jakob Bekenstein (May 1, 19447—August 16, 2015)
Testing the holographic principle

How general is holography?

- Holographic principle realized in AdS/CFT correspondence
- Special case or generic lesson for quantum gravity?

\[ \text{AdS}_{d+1} \rightarrow \text{CFT}_d \]
- Use (classical) gravity to learn more about CFTs
- Strong coupling large $N$ limit: classical gravity
- Useful tool to calculate correlation functions
- Useful tool to calculate entanglement entropy

\[ \text{CFT}_d \rightarrow \text{AdS}_{d+1} \]
- Use CFTs to learn more about (quantum) gravity
- Gravity in ultra-quantum limit: simple CFT?
- Useful tool to address black hole microstates
- Useful tool for qu-gr puzzles (information paradox)

To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?
- Does holography apply only to unitary theories?
- Can we establish a flat space holographic dictionary?
- Generic non-AdS holography/higher spin holography?
Testing the holographic principle

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see numerous talks at KITP workshop “Bits, Branes, Black Holes” 2012

and at ESI workshop “Higher Spin Gravity” 2012
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- originally holography motivated by unitarity
- plausible AdS/CFT-like correspondence could work non-unitarily
- AdS/log CFT first example of non-unitary holography DG, (Jackiw), Johansson ’08; Skenderis, Taylor, van Rees ’09; Henneaux, Martinez, Troncoso ’09; Maloney, Song, Strominger ’09; DG, Sachs/Hohm ’09; Gaberdiel, DG, Vassilevich ’10; ... DG, Riedler, Rosseel, Zojer ’13
- recent proposal by Vafa ’14
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The answer appears to be yes — see my current talk and recent papers by Bagchi et al., Barnich et al., Strominger et al., ’12–’15
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Non-trivial hints that it might work at least in 2+1 dimensions
Gary, DG Rashkov ’12; Afshar et al ’12; Gutperle et al ’14–’15; Gary, DG, Prohazka, Rey ’14; ...
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Address questions above in simple class of 3D toy models
- Exploit gauge theoretic Chern–Simons formulation
- Restrict to kinematic questions, like (asymptotic) symmetries
Goals of this talk

1. Review general aspects of holography in 3D

Address these issues in 3D!
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Address these issues in 3D!
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Working assumptions:

- 3D
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- Restrict to “pure gravity” theories
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- Restrict to “pure gravity” theories
- Define quantum gravity by its dual field theory

Interesting dichotomy:

- Either dual field theory exists $\rightarrow$ useful toy model for quantum gravity
- Or gravitational theory needs UV completion (within string theory) $\rightarrow$ indication of inevitability of string theory
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This talk:

- Remain agnostic about dichotomy
- Focus on generic features of dual field theories that do not require string theory embedding
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AdS$_3$ gravity

- Lowest dimension with black holes and (off-shell) gravitons
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- Simple microstate counting from AdS$_3$/CFT$_2$
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Caveat: while there are many string compactifications with AdS$_3$ factor, applying holography just to AdS$_3$ factor does not capture everything!
Picturesque analogy: soap films

Both soap films and Chern–Simons theories have

- essentially no bulk dynamics
- highly non-trivial boundary dynamics
- most of the physics determined by boundary conditions
- esthetic appeal (at least for me)
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if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

▶ Works straightforwardly sometimes, otherwise not

▶ Example where it works nicely: asymptotic symmetry algebra

▶ Take linear combinations of Virasoro generators $L_n, \bar{L}_n$

$\ell = L_n - \bar{L}_n = \ell (L_n + \bar{L}_n)$

▶ Make In" on"u–Wigner contraction $\ell \to \infty$ on ASA

$[L_n, L_m] = (n - m) L_n + m + c L_{1/2} (n^3 - n)$

$[M_n, M_m] = (n - m) M_n + m + c M_{1/2} (n^3 - n)$

$[M_n, L_m] = 0$

▶ This is nothing but the BMS

$BMS_3$ algebra (or GCA$_2$, URCA$_2$, CCA$_2$)

▶ Example where it does not work easily: boundary conditions

▶ Example where it does not work: highest weight conditions
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L_n = L_n - \bar{L}_{-n} \quad M_n = \frac{1}{\ell} \left( L_n + \bar{L}_{-n} \right)
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$$[L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} \left( n^3 - n \right) \delta_{n+m,0}$$

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- This is nothing but the BMS\(_3\) algebra (or GCA\(_2\), URCA\(_2\), CCA\(_2\))!
  Ashtekar, Bicak, Schmidt, ’96; Barnich, Compere ’06

\( L_n \): diffeos of circle, \( M_n \): supertranslations, \( c_{L/M} \): central extensions
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If dual field theory exists it must be a 2D Galilean CFT!

Bagchi et al., Barnich et al.
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Flat space Einstein gravity as $\text{isl}(2)$ Chern–Simons theory

For details, references and spin-3 generalization see Gary, DG, Riegler, Rosseel ’14
Flat space Einstein gravity as $\mathfrak{isl}(2)$ Chern–Simons theory

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- AdS gravity in CS formulation: $\mathfrak{sl}(2) \oplus \mathfrak{sl}(2)$ gauge algebra

Achucarro, Townsend '86; Witten '88
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- AdS gravity in CS formulation: sl(2)⊕sl(2) gauge algebra
- Flat space: isl(2) gauge algebra

\[ S_{\text{CS}}^{\text{flat}} = \frac{k}{4\pi} \int \langle A \wedge dA + \frac{2}{3} A \wedge A \wedge A \rangle \]

with isl(2) connection \((a = 0, \pm 1)\)

\[ A = e^a M_a + \omega^a L_a \]

isl(2) algebra (global part of BMS/GCA)

\[ [L_a, L_b] = (a - b) L_{a+b} \]
\[ [L_a, M_b] = (a - b) M_{a+b} \]
\[ [M_a, M_b] = 0 \]

Note: \(e^a\) dreibein, \(\omega^a\) (dualized) spin-connection

Bulk EOM: gauge flatness → Einstein equations

\[ \mathcal{F} = dA + A \wedge A = 0 \]
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- Boundary conditions in CS formulation:

\[ A(r, u, \varphi) = b^{-1}(r) \left( d + a(u, \varphi) + o(1) \right) b(r) \]
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$$a(u, \varphi) = (M_1 - M(\varphi)M_{-1}) \, du + (L_1 - M(\varphi)L_{-1} - N(u, \varphi)M_{-1}) \, d\varphi$$

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► metric

$$g_{\mu\nu} \sim \frac{1}{2} \tilde{\text{tr}} \langle A_\mu A_\nu \rangle \quad \rightarrow \quad ds^2 = M \, du^2 - 2 \, du \, dr + 2N \, du \, d\varphi + r^2 \, d\varphi^2$$
Correlation functions in flat space holography

AdS/CFT good tool for calculating correlators
What about flat space/Galilean CFT correspondence?

\[ \langle T(z_1) T(z_2) \ldots T(z_{42}) \rangle_{\text{CFT}} \sim \delta_{42} \delta_{g_{42}} \Gamma_{\text{EH-AdS}} \mid \mid \mid \text{EOM} \]

▶ Does it work?
▶ What is the left hand side in a Galilean CFT?
▶ Shortcut to right hand side other than varying EH-action 42 times?

Start slowly with 0-point function
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Start slowly with 0-point function
0-point function (on-shell action)
Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action $\Gamma$

\begin{align*}
F_{HFS} &= -\frac{1}{8} G N \\
F_{FSC} &= -r + 8 G N
\end{align*}

Result of this comparison

- $r > 1$: FSC dominant saddle
- $r < 1$: HFS dominant saddle

Critical temperature: $T_c = \frac{1}{2} \pi r_0 = \Omega^2 \pi$

HFS "melts" into FSC at $T > T_c$

Bagchi, Detournay, DG, Simon '13
0-point function (on-shell action)
Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action $\Gamma$
- Variational principle?

$$\Gamma = -\frac{1}{16\pi G_N} \int d^3x \sqrt{g} R - \frac{1}{8\pi G_N} \int d^2x \sqrt{\gamma} K - I_{\text{counter-term}}$$

with $I_{\text{counter-term}}$ chosen such that

$$\delta \Gamma \bigg|_{\text{EOM}} = 0$$

for all $\delta g$ that preserve flat space bc’s
0-point function (on-shell action)
Not check of flat space holography but interesting in its own right

▶ Calculate the full on-shell action $\Gamma$
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with $I_{\text{counter-term}}$ chosen such that

\[ \delta \Gamma \bigg|_{\text{EOM}} = 0 \]

for all $\delta g$ that preserve flat space bc’s

Result (Detournay, DG, Schöller, Simon ’14):

\[ \Gamma = -\frac{1}{16\pi G_N} \int d^3x \sqrt{g} R - \frac{1}{16\pi G_N} \int d^2x \sqrt{\gamma} K \]

\[ \frac{1}{2} \text{GHY!} \]

follows also as limit from AdS using Mora, Olea, Troncoso, Zanelli ’04

independently confirmed by Barnich, Gonzalez, Maloney, Oblak ’15
0-point function (on-shell action)
Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action $\Gamma$
- Variational principle?
- Phase transitions?
Standard procedure (Gibbons, Hawking '77; Hawking, Page '83)

Evaluate Euclidean partition function in semi-classical limit

$$Z(T, \Omega) = \int \mathcal{D}g \, e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T, \Omega)]} \times Z_{\text{fluct.}}.$$ 

path integral bc’s specified by temperature $T$ and angular velocity $\Omega$

Two Euclidean saddle points in same ensemble if
- same temperature $T = 1/\beta$ and angular velocity $\Omega$
- obey flat space boundary conditions
- solutions without conical singularities

Periodicities fixed:

$$(\tau_E, \varphi) \sim (\tau_E + \beta, \varphi + \beta \Omega) \sim (\tau_E, \varphi + 2\pi)$$
0-point function (on-shell action)
Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action $\Gamma$
- Variational principle?
- Phase transitions?

3D Euclidean Einstein gravity: for each $T$, $\Omega$ two saddle points:
  - Hot flat space
    $$ds^2 = d\tau_E^2 + dr^2 + r^2 d\phi^2$$
  - Flat space cosmology
    $$ds^2 = r_+^2 \left( 1 - \frac{r_0^2}{r^2} \right) d\tau_E^2 + \frac{r^2 dr^2}{r_+^2 (r^2 - r_0^2)} + r^2 \left( d\phi - \frac{r + r_0}{r^2} d\tau_E \right)^2$$

shifted-boost orbifold, see Cornalba, Costa ’02
0-point function (on-shell action)
Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action $\Gamma$
- Variational principle?
- Phase transitions?
- Plug two Euclidean saddles in on-shell action and compare free energies

$$F_{\text{HFS}} = -\frac{1}{8G_N} \quad F_{\text{FSC}} = -\frac{r_+}{8G_N}$$
0-point function (on-shell action)
Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action $\Gamma$
- Variational principle?
- Phase transitions?
- Plug two Euclidean saddles in on-shell action and compare free energies

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- Result of this comparison
  - $r_+ > 1$: FSC dominant saddle
  - $r_+ < 1$: HFS dominant saddle

Critical temperature:

$$T_c = \frac{1}{2\pi r_0} = \frac{\Omega}{2\pi}$$

HFS “melts” into FSC at $T > T_c$

Bagchi, Detournay, DG, Simon ’13
1-point functions (conserved charges)

First check of entries in holographic dictionary: identification of sources and vevs

In AdS$_3$:

$$\delta \Gamma \bigg|_{\text{EOM}} \sim \int_{\partial \mathcal{M}} \text{vev} \times \delta \text{source} \sim \int_{\partial \mathcal{M}} T^{\mu \nu}_{\text{BY}} \times \delta g^{\text{NN}}_{\mu \nu}$$

Note that $T^{\mu \nu}_{\text{BY}}$ follows from canonical analysis as well (conserved charges)
1-point functions (conserved charges)
First check of entries in holographic dictionary: identification of sources and vevs

In AdS$_3$:

$$\delta \Gamma \big|_{EOM} \sim \int_{\partial M} \text{vev} \times \delta \text{source} \sim \int_{\partial M} T_{BY}^{\mu\nu} \times \delta g^{NN}_{\mu\nu}$$

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In flat space:

- non-normalizable solutions to linearized EOM?
- analogue of Brown–York stress tensor?
- comparison with canonical results
1-point functions (conserved charges)
First check of entries in holographic dictionary: identification of sources and vevs

In AdS$_3$:

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Note that $T_{\text{BY}}^{\mu \nu}$ follows from canonical analysis as well (conserved charges)

In flat space:

- non-normalizable solutions to linearized EOM?
- analogue of Brown–York stress tensor?
- comparison with canonical results

everything works (Detournay, DG, Schöller, Simon, ’14)

mass and angular momentum:

$$M = \frac{g_{tt}}{8G}, \quad N = \frac{g_{t\phi}}{4G}$$

full tower of canonical charges: see Barnich, Compere ’06
2-point functions (anomalous terms)
First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder \((\varphi \sim \varphi + 2\pi)\):

\[
\langle M(u_1, \varphi_1) M(u_2, \varphi_2) \rangle = 0
\]

\[
\langle M(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_M}{2s_{12}^4}
\]

\[
\langle N(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_L - 2c_M \tau_{12}}{2s_{12}^4}
\]

with \(s_{ij} = 2 \sin[(\varphi_i - \varphi_j)/2]\), \(\tau_{ij} = (u_i - u_j) \cot[(\varphi_i - \varphi_j)/2]\)

Fourier modes of Galilean CFT stress tensor on cylinder:

\[
M := \sum_n M_n e^{-in\varphi} - \frac{c_M}{24}
\]

\[
N := \sum_n (L_n - inuM_n) e^{-in\varphi} - \frac{c_L}{24}
\]

Conservation equations: \(\partial_u M = 0, \partial_u N = \partial_\varphi M\)
2-point functions (anomalous terms)
First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder ($\varphi \sim \varphi + 2\pi$):

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Short-cut on gravity side:

- Do not calculate second variation of action
2-point functions (anomalous terms)
First check sensitive to central charges in symmetry algebra

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Short-cut on gravity side:

- Do not calculate second variation of action
- Calculate first variation of action on non-trivial background
2-point functions (anomalous terms)
First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder ($\varphi \sim \varphi + 2\pi$):

$$\langle M(u_1, \varphi_1) M(u_2, \varphi_2) \rangle = 0$$

$$\langle M(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_M}{2s_{12}^4}$$

$$\langle N(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_L - 2c_M\tau_{12}}{2s_{12}^4}$$

with $s_{ij} = 2\sin[(\varphi_i - \varphi_j)/2]$, $\tau_{ij} = (u_i - u_j) \cot[(\varphi_i - \varphi_j)/2]$

Short-cut on gravity side:

- Do not calculate second variation of action
- Calculate first variation of action on non-trivial background
- Can iterate this procedure to higher $n$-point functions
2-point functions (anomalous terms)
First check sensitive to central charges in symmetry algebra

Galilean CFT on cylinder ($\varphi \sim \varphi + 2\pi$):

\[
\langle M(u_1, \varphi_1) M(u_2, \varphi_2) \rangle = 0
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\[
\langle M(u_1, \varphi_1) N(u_2, \varphi_2) \rangle = \frac{c_M}{2s_{12}^4}
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with $s_{ij} = 2 \sin[(\varphi_i - \varphi_j)/2]$, $\tau_{ij} = (u_i - u_j) \cot[(\varphi_i - \varphi_j)/2]$

Short-cut on gravity side:

- Do not calculate second variation of action
- Calculate first variation of action on non-trivial background
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Summarize first how this works in the AdS case
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

- On CFT side deform free action $S_0$ by source term $\mu$ for stress tensor

$$S_\mu = S_0 + \int d^2 z \mu(z, \bar{z}) T(z)$$
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

- On CFT side deform free action $S_0$ by source term $\mu$ for stress tensor

\[ S_\mu = S_0 + \int \mathrm{d}^2 z \mu(z, \bar{z}) T(z) \]

- Localize source

\[ \mu(z, \bar{z}) = \epsilon \delta^{(2)}(z - z_2, \bar{z} - \bar{z}_2) \]
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

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$$\mu(z, \bar{z}) = \epsilon \delta^{(2)}(z - z_2, \bar{z} - \bar{z}_2)$$

- 1-point function in $\mu$-vacuum $\rightarrow$ 2-point function in 0-vacuum

$$\langle T^1 \rangle_\mu = \langle T^1 \rangle_0 + \epsilon \langle T^1 T^2 \rangle_0 + O(\epsilon^2)$$
2-point functions (anomalous terms)

Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

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  $$\langle T^1 \rangle_\mu = \langle T^1 \rangle_0 + \epsilon \langle T^1 T^2 \rangle_0 + \mathcal{O}(\epsilon^2)$$

- On gravity side exploit sl(2) CS formulation with chemical potentials

  $$A = b^{-1}(d+a)b \quad \quad b = e^{\rho L_0}$$

  $$a_z = L_+ - \frac{\mathcal{L}}{k}L_- \quad \quad a_{\bar{z}} = \mu L_+ + \ldots$$

  Drinfeld, Sokolov ’84, Polyakov ’87, H. Verlinde ’90
  Bañados, Caro ’04
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

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$$S_\mu = S_0 + \int d^2 z \mu(z, \bar{z}) T(z)$$

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- On gravity side exploit sl(2) CS formulation with chemical potentials

$$A = b^{-1} (d + a) b \quad \quad b = e^{\rho L_0}$$
$$a_z = L_+ - \frac{\mathcal{L}}{\kappa} L_- \quad \quad a_{\bar{z}} = \mu L_+ + \ldots$$

- Expand $\mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + O(\epsilon^2)$
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

- On gravity side exploit CS formulation with chemical potentials
  \[ A = b^{-1}(d + a)b \quad b = e^{\rho L_0} \]
  \[ a_z = L_+ - \frac{L}{k}L_- \quad a_{\bar{z}} = \mu L_+ + \ldots \]

- Expand \( \mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2) \)

- Write EOM to first subleading order in \( \epsilon \)
  \[ \bar{\partial} \mathcal{L}^{(1)}(z) = -\frac{k}{2} \bar{\partial}^3 \delta^{(2)}(z - z_2) \]
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

- On gravity side exploit CS formulation with chemical potentials

\[
A = b^{-1}(d + a)b \quad b = e^{\rho L_0}
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- Expand $\mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2)$
- Write EOM to first subleading order in $\epsilon$

\[
\bar{\partial} \mathcal{L}^{(1)}(z) = -\frac{k}{2} \partial^3 \delta^{(2)}(z - z_2)
\]

- Solve them using the Green function on the plane $G = \ln (z_{12} \bar{z}_{12})$

\[
\mathcal{L}^{(1)}(z) = -\frac{k}{2} \partial_{z_1}^4 G(z_{12}) = \frac{3k}{z_{12}^4}
\]
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

- On gravity side exploit CS formulation with chemical potentials

\[
A = b^{-1}(d + a)b \\
\frac{a_z}{b} = L_+ - \frac{L}{k} L_- \\
\frac{a_{\bar{z}}}{b} = \mu L_+ + \ldots
\]

- Expand $\mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2)$
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\[
\mathcal{L}^{(1)}(z) = -\frac{k}{2} \partial^4 \bar{z}_1 G(z_{12}) = \frac{3k}{z_{12}^4}
\]

- This is the correct CFT 2-point function on the plane with $c = 6k$
2-point functions (anomalous terms)
Illustrate shortcut in AdS$_3$/CFT$_2$ (restrict to one holomorphic sector)

- On gravity side exploit CS formulation with chemical potentials
  \[ A = b^{-1}(d + a)b \quad b = e^{\rho L_0} \]
  \[ a_z = L_+ - \frac{\mathcal{L}}{k} L_- \quad a_{\bar{z}} = \mu L_+ + \ldots \]

- Expand \( \mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2) \)

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- This is the correct CFT 2-point function on the plane with \( c = 6k \)

- Generalize to cylinder
2-point functions (anomalous terms)
Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis ’15)

▶ Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel ’14)
2-point functions (anomalous terms)
Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis ’15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel ’14)
- Localize chemical potentials $\mu_{M/L} = \epsilon_{M/L} \delta^{(2)}(u - u_2, \varphi - \varphi_2)$
2-point functions (anomalous terms)
Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis '15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel '14)
- Localize chemical potentials $\mu_{M/L} = \epsilon_{M/L} \delta^{(2)}(u - u_2, \varphi - \varphi_2)$
- Expand around global Minkowski space

\[
M = -k/2 + M^{(1)} \quad N = N^{(1)}
\]
2-point functions (anomalous terms)

Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis ’15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel ’14)
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- Expand around global Minkowski space

\[
M = -k/2 + M^{(1)} \quad N = N^{(1)}
\]

- Write EOM to first subleading order in \( \epsilon_{M/L} \)

\[
\partial_u M^{(1)} = - k \epsilon_L \left( \partial_\varphi^3 \delta + \partial_\varphi \delta \right) \\
\partial_u N^{(1)} = - k \epsilon_M \left( \partial_\varphi^3 \delta + \partial_\varphi \delta \right) + \partial_\varphi M^{(1)}
\]
2-point functions (anomalous terms)
Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis '15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel '14)
- Localize chemical potentials $\mu_{M/L} = \epsilon_{M/L} \delta^{(2)}(u - u_2, \varphi - \varphi_2)$
- Expand around global Minkowski space
  
  $$M = -k/2 + M^{(1)} \quad N = N^{(1)}$$

- Write EOM to first subleading order in $\epsilon_{M/L}$
  
  $$\partial_u M^{(1)} = - k \epsilon_L \left( \partial^3_{\varphi} \delta + \partial_{\varphi} \delta \right)$$
  $$\partial_u N^{(1)} = - k \epsilon_M \left( \partial^3_{\varphi} \delta + \partial_{\varphi} \delta \right) + \partial_{\varphi} M^{(1)}$$

- Solve with Green function on cylinder
  
  $$M^{(1)} = \frac{6k\epsilon_L}{s_{12}^4} \quad N^{(1)} = \frac{6k(\epsilon_M - 2\epsilon_L \tau_{12})}{s_{12}^4}$$
2-point functions (anomalous terms)
Apply shortcut to flat space/Galilean CFT (Bagchi, DG, Merbis ’15)

- Exploit results for flat space gravity in CS formulation in presence of chemical potentials (Gary, DG, Riegler, Rosseel ’14)
- Localize chemical potentials $\mu_{M/L} = \epsilon_{M/L} \delta^{(2)}(u - u_2, \varphi - \varphi_2)$
- Expand around global Minkowski space

$$M = -k/2 + M^{(1)} \quad N = N^{(1)}$$

- Write EOM to first subleading order in $\epsilon_{M/L}$

$$\partial_u M^{(1)} = - k \epsilon L \left( \partial^3 \varphi \delta + \partial \varphi \delta \right)$$

$$\partial_u N^{(1)} = - k \epsilon M \left( \partial^3 \varphi \delta + \partial \varphi \delta \right) + \partial \varphi M^{(1)}$$

- Solve with Green function on cylinder

$$M^{(1)} = \frac{6k\epsilon L}{s_{12}^4} \quad N^{(1)} = \frac{6k(\epsilon M - 2\epsilon L \tau_{12})}{s_{12}^4}$$

- Correct 2-point functions for Einstein gravity with $c_L = 0, c_M = 12k$
3-point functions (check of symmetries)

First non-trivial check of consistency with symmetries of dual Galilean CFT

Check of 2-point functions works nicely with shortcut; 3-point too?

\[ \langle M_{1}N_{2}N_{3} \rangle = c_{M}s_{12}s_{13}s_{23} \]
\[ \langle N_{1}N_{2}N_{3} \rangle = c_{L} - c_{M}\tau_{123}s_{12}s_{13}s_{23} \]

provided we choose again the Einstein values
\[ c_{L} = 0 \] and \[ c_{M} = 12k \]
3-point functions (check of symmetries)
First non-trivial check of consistency with symmetries of dual Galilean CFT

Check of 2-point functions works nicely with shortcut; 3-point too?

► Yes: same procedure, but localize chemical potentials at two points

\[ \mu_{M/L}(u_1, \varphi_1) = \sum_{i=2}^{3} \epsilon_{M/L}^i \delta^{(2)}(u_1 - u_i, \varphi_1 - \varphi_i) \]
3-point functions (check of symmetries)
First non-trivial check of consistency with symmetries of dual Galilean CFT

Check of 2-point functions works nicely with shortcut; 3-point too?

- Yes: same procedure, but localize chemical potentials at two points

\[ \mu_{M/L}(u_1, \varphi_1) = \sum_{i=2}^{3} \epsilon^i_{M/L} \delta^{(2)}(u_1 - u_i, \varphi_1 - \varphi_i) \]

- Iteratively solve EOM

\[
\begin{align*}
\partial_u M &= -k \partial^3 \varphi \mu_L + \mu_L \partial \varphi M + 2M \partial \varphi \mu_L \\
\partial_u N &= -k \partial^3 \varphi \mu_M + (1 + \mu_M) \partial \varphi M + 2M \partial \varphi \mu_M + \mu_L \partial \varphi N + 2N \partial \varphi \mu_L
\end{align*}
\]
3-point functions (check of symmetries)
First non-trivial check of consistency with symmetries of dual Galilean CFT

Check of 2-point functions works nicely with shortcut; 3-point too?

- Yes: same procedure, but localize chemical potentials at two points
  \[
  \mu_{M/L}(u_1, \varphi_1) = \sum_{i=2}^{3} \epsilon_{M/L}^i \delta^{(2)}(u_1 - u_i, \varphi_1 - \varphi_i)
  \]

- Iteratively solve EOM
  \[
  \partial_u M = -k \partial_\varphi^3 \mu_L + \mu_L \partial_\varphi M + 2M \partial_\varphi \mu_L \\
  \partial_u N = -k \partial_\varphi^3 \mu_M + (1 + \mu_M) \partial_\varphi M + 2M \partial_\varphi \mu_M + \mu_L \partial_\varphi N + 2N \partial_\varphi \mu_L
  \]

- Result on gravity side matches precisely Galilean CFT results
  \[
  \langle M^1 N^2 N^3 \rangle = \frac{c_M}{s_{12}^2 s_{23}^2 s_{13}^2} \\
  \langle N^1 N^2 N^3 \rangle = \frac{c_L - c_M \tau_{123}}{s_{12}^2 s_{23}^2 s_{13}^2}
  \]

  provided we choose again the Einstein values \( c_L = 0 \) and \( c_M = 12k \)
4-point functions (enter cross-ratios)
First correlators with non-universal function of cross-ratios

▶ Repeat this algorithm, localizing the sources at three points
4-point functions (enter cross-ratios)
First correlators with non-universal function of cross-ratios

- Repeat this algorithm, localizing the sources at three points
- Derive 4-point functions for Galilean CFTs (Bagchi, DG, Merbis ’15)

\[
\langle M^1 N^2 N^3 N^4 \rangle = \frac{2 c_M g_4(\gamma)}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}
\]

\[
\langle N^1 N^2 N^3 N^4 \rangle = \frac{2 c_L g_4(\gamma) + c_M \Delta_4}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}
\]

with the cross-ratio function

\[
g_4(\gamma) = \frac{\gamma^2 - \gamma + 1}{\gamma} \quad \gamma = \frac{s_{12} s_{34}}{s_{13} s_{24}}
\]

and

\[
\Delta_4 = 4 g'_4(\gamma) \eta_{1234} - (\tau_{1234} + \tau_{14} + \tau_{23}) g_4(\gamma)
\]

\[
\eta_{1234} = \sum (-1)^{1+i-j} (u_i - u_j) \sin(\varphi_k - \varphi_l)/(s_{13}^2 s_{24}^2)
\]
4-point functions (enter cross-ratios)
First correlators with non-universal function of cross-ratios

- Repeat this algorithm, localizing the sources at three points
- Derive 4-point functions for Galilean CFTs (Bagchi, DG, Merbis ’15)

\[
\langle M^1 N^2 N^3 N^4 \rangle = \frac{2c_M g_4(\gamma)}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}
\]

\[
\langle N^1 N^2 N^3 N^4 \rangle = \frac{2c_L g_4(\gamma)}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}} + c_M \Delta_4
\]

with the cross-ratio function

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\]

and

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\Delta_4 = 4g'_4(\gamma)\eta_{1234} - (\tau_{1234} + \tau_{14} + \tau_{23})g_4(\gamma)
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- Gravity side yields precisely the same result!
5-point functions (further check of consistency of flat space holography)
Last new explicit correlators I am showing to you today (I promise)

- Repeat this algorithm, localizing the sources at four points

\[
\langle M_1 N_2 N_3 N_4 N_5 \rangle = 4 c M g_5 (\gamma, \zeta) \prod_{1 \leq i < j \leq 5} s_{ij} \langle N_1 N_2 N_3 N_4 N_5 \rangle = 4 c L g_5 (\gamma, \zeta) + c M \Delta^5 \prod_{1 \leq i < j \leq 5} s_{ij}
\]

with the previous definitions and

\[
\zeta = s_{25} s_{34} s_{35} s_{24}
\]

\[
g_5 (\gamma, \zeta) = \gamma + \zeta 2 (\gamma - \zeta) - (\gamma^2 - \gamma \zeta + \zeta^2) \gamma (\gamma - 1) \zeta (\zeta - 1) (\gamma - \zeta) \times \left[ \gamma (\gamma - 1) + 1 \right] \left[ \zeta (\zeta - 1) + 1 \right] - \gamma \zeta
\]

\[
\Delta^5 = 4 \partial_\gamma g_5 (\gamma, \zeta) \eta_{1234} + 4 \partial_\zeta g_5 (\gamma, \zeta) \eta_{2345} - 2 g_5 (\gamma, \zeta) \tau_{12345}
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\]

\[
\langle N^1 N^2 N^3 N^4 N^5 \rangle = \frac{4c_L g_5(\gamma, \zeta) + c_M \Delta_5}{\prod_{1 \leq i < j \leq 5} s_{ij}}
\]

with the previous definitions and \((\zeta = \frac{s_{25} s_{34}}{s_{35} s_{24}})\)

\[
g_5(\gamma, \zeta) = \frac{\gamma + \zeta}{2(\gamma - \zeta)} - \frac{(\gamma^2 - \gamma \zeta + \zeta^2)}{\gamma(\gamma - 1)\zeta(\zeta - 1)(\gamma - \zeta)} \times \left( [\gamma(\gamma - 1) + 1][\zeta(\zeta - 1) + 1] - \gamma \zeta \right)
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$n$-point functions (holographic Ward identities and recursion relations)
Shortcut to 42 (Bagchi, DG, Merbis ’15)

Smart check of all $n$-point functions?

- Idea: calculate $n$-point function from $(n - 1)$-point function
\( n \)-point functions (holographic Ward identities and recursion relations)
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- Need Galilean CFT analogue of BPZ-recursion relation

\[
\langle T^1 T^2 \ldots T^n \rangle = \sum_{i=2}^{n} \left( \frac{2}{s_{1i}^2} + \frac{c_{1i}}{2} \partial \varphi_i \right) \langle T^2 \ldots T^n \rangle + \text{disconnected}
\]
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- After small derivation we find ($c_{ij} := \cot[(\phi_i - \phi_j)/2]$)

$$
\langle M^1 N^2 \ldots N^n \rangle = \sum_{i=2}^{n} \left( \frac{2}{s_{1i}^2} + \frac{c_{1i}}{2} \partial \phi_i \right) \langle M^2 N^3 \ldots N^n \rangle + \text{disconnected}
$$

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$$
\textit{n}-point functions (holographic Ward identities and recursion relations)
Shortcut to 42 (Bagchi, DG, Merbis ’15)

\begin{tcolorbox}
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\end{tcolorbox}

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\]

› We can also derive same recursion relations on gravity side!
$n$-point functions in flat space holography

Summary

- EH action has variational principle consistent with flat space bc’s (iff we add half the GHY term!)
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- 0-point function shows phase transition exists between hot flat space and flat space cosmologies
- 1-point functions show consistency with canonical charges and lead to first entries in holographic dictionary
- 2-point functions consistent with Galilean CFT for $c_L = 0$, $c_M = 12$, $k = 3/G_N$
- 42nd variation of EH action leads to 42-point Galilean CFT correlators
- all $n$-point correlators of Galilean CFT reproduced precisely on gravity side (recursion relations!)

Fairly non-trivial check that 3D flat space holography can work!
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Other selected recent results

Some further checks that dual field theory is Galilean CFT:

\[ S_{\text{gravity}} = S_{\text{BH}} = \text{Area} \]

as limit from Cardy formula (Riegler '14, Fareghbal, Naseh '14)

\[ S_{\text{GCFT}} = c L^6 \ln \ell x a \]

like CFT + \[ c M^6 \ell y \ell x \]

like grav anomaly

Calculation on gravity side confirms result above (using Wilson lines in CS formulation)
Other selected recent results

Some further checks that dual field theory is Galilean CFT:

▶ Microstate counting?
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  Works! (Bagchi, Detournay, Fareghbal, Simon ’13, Barnich ’13)

\[
S_{\text{gravity}} = S_{\text{BH}} = \frac{\text{Area}}{4G_N} = 2\pi h_L \sqrt{\frac{c_M}{2\hbar_M}} = S_{\text{GCFT}}
\]

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- (Holographic) entanglement entropy?

Works! (Bagchi, Basu, DG, Riegler ’14)

\[ S_{\text{EE}}^{\text{GCFT}} = \frac{c_L}{6} \ln \frac{\ell_x}{a} + \frac{c_M}{6} \frac{\ell_y}{\ell_x} \]

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Outline

Motivations

Assumptions

Flat space holography basics

Recent results

Generalizations & open issues
Generalizations & open issues

Recent generalizations:

- adding chemical potentials

  Works! (Gary, DG, Riegler, Rosseel ’14)

  In CS formulation:

  \[ A_0 \rightarrow A_0 + \mu \]

Some open issues:

- Further checks in 3D (\( n \)-point correlators, partition function, ...)
- Further generalizations in 3D (massive gravity, adding matter, ...)
- Generalization to 4D? (Barnich et al, Strominger et al)
- Flat space limit of usual AdS \( 5/ \) CFT correspondence?
- Holography seems to work in flat space
- Holography more general than AdS/CFT
- (when) does it work even more generally?
Generalizations & open issues

Recent generalizations:
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- 3-derivative theory: flat space chiral gravity (Bagchi, Detournay, DG ’12)

Conformal CS gravity at level $k = 1$ with flat space boundary conditions conjectured to be dual to chiral half of monster CFT.

Action (gravity side):

$$I_{CSG} = \frac{k}{4\pi} \int d^3 x \sqrt{-g} \varepsilon^{\lambda \mu \nu} \Gamma^\rho_{\lambda \sigma} \left( \partial_{\mu} \Gamma^\sigma_{\nu \rho} + \frac{2}{3} \Gamma^\sigma_{\mu \tau} \Gamma^\tau_{\nu \rho} \right)$$

Partition function (field theory side, see Witten ’07):

$$Z(q) = J(q) = \frac{1}{q} + 196884 q + \mathcal{O}(q^2)$$

Note: $\ln 196883 \approx 12.2 = 4\pi + $ quantum corrections
Generalizations & open issues

Recent generalizations:

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- generalization to supergravity

Works! (Barnich, Donnay, Matulich, Troncoso ’14)

Asymptotic symmetry algebra = super-BMS₃

Some open issues:

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Remarkably it exists! (Afshar, Bagchi, Fareghbal, DG, Rosseel ’13; Gonzalez, Matulich, Pino, Troncoso ’13)

New type of algebra: W-like BMS (“BMW”)

\[
[U_n, U_m] = (n - m)(2n^2 + 2m^2 - nm - 8)L_{n+m} + \frac{192}{c_M}(n - m)\Lambda_{n+m} \\
- \frac{96(4L + 44)}{c_M^2}(n - m)\Theta_{n+m} + \frac{c_L}{12} n(n^2 - 1)(n^2 - 4) \delta_{n+m, 0}
\]

\[
[U_n, V_m] = (n - m)(2n^2 + 2m^2 - nm - 8)M_{n+m} + \frac{96}{c_M}(n - m)\Theta_{n+m} \\
+ \frac{c_M}{12} n(n^2 - 1)(n^2 - 4) \delta_{n+m, 0}
\]

\[
[L, L], [L, M], [M, M] \text{ as in BMS}_3 \quad [L, U], [L, V], [M, U], [M, V] \text{ as in isl}(3)
\]
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  Barnich, Gonzalez, Maloney, Oblak ’15: 1-loop partition function matches BMS$_3$ character
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Thanks for your attention!

Vladimir Bulatov, M.C. Escher Circle Limit III in a rectangle