Higher Spin Rindler Holography

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MIAPP programme 'Higher Spin Theory and Duality'
München, May 2016

based on work w. (H. Afshar, S. Detournay, W. Merbis),
{B. Oblak}, A. Perez, S. Prohazka, D. Tempo, R. Troncoso

Simple punchline

Heisenberg algebra

$$[X_n,\, P_m] = i\,\delta_{n,\,m}$$

fundamental not only in quantum mechanics
but also in near horizon physics of (higher spin) gravity theories

Outline

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Bekenstein–Hawking

$$S_{\text{BH}} = \frac{A}{4G_N}$$

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- Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...)

warped CFT: Detournay, Hartman, Hofman '12
Galilean CFT: Bagchi, Detournay, Fareghbal, Simon '13; Barnich '13

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- Main idea: consider near horizon symmetries for non-extremal horizons

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- Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...)
- Main idea: consider near horizon symmetries for non-extremal horizons
- Near horizon line-element with Rindler acceleration a:

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \ldots$$

Meaning of coordinates:

- ρ: radial direction ($\rho = 0$ is horizon)
- $\varphi \sim \varphi + 2\pi$: angular direction
- v: (advanced) time

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of Rindler metric

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  suggestion in `1511.08687`

  We make this choice in this talk!

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$$T_U = \frac{a}{2\pi}$$
- Work in 3d Einstein gravity in Chern–Simons formulation

$$I_{\text{CS}} = \pm \sum_{\pm} \frac{k}{4\pi} \int \langle A^\pm \wedge \mathrm{d}A^\pm + \tfrac{2}{3} A^\pm \wedge A^\pm \wedge A^\pm \rangle$$

with $sl(2)$ connections $A^\pm$ and $k = \ell/(4G_N)$ with AdS radius $\ell = 1$

Outline

Diagonal gauge

Standard trick: partially fix gauge

$$A^{\pm} = b_{\pm}^{-1}(\rho)\left(\mathrm{d}+\mathfrak{a}_{\pm}(x^0,\, x^1)\right)b_{\pm}(\rho)$$

with some group element $b \in SL(2)$ depending on radius ρ with $\delta b = 0$

Drop $\pm$ decorations in most of talk

Manifold topologically a cylinder or torus, with radial coordinate ρ and boundary coordinates $(x^0, x^1) \sim (v, \varphi)$

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- Standard AdS_3 approach: highest weight gauge

$$\mathfrak{a} \sim L_+ + \mathcal{L}(x^0,\, x^1)L_- \qquad b(\rho) = \exp\left(\rho L_0\right)$$

$sl(2)$: $[L_n,\, L_m] = (n-m)L_{n+m}, \quad n, m = -1, 0, 1$

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- Precise boundary conditions (ζ: chemical potential):

  $$\mathfrak{a} = (\mathcal{J}\ \mathrm{d}\varphi + \zeta\ \mathrm{d}v)\ L_0 \qquad \delta\mathfrak{a} = \delta\mathcal{J}\ \mathrm{d}\varphi\, L_0$$

  and $b = \exp\,(\frac{1}{\zeta}\, L_+) \cdot \exp\,(\frac{\rho}{2}\, L_-)$. (assume constant ζ for simplicity)

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yields ($f := 1 + \rho/(2a)$)

$$\mathrm{d}s^2 = -2a\rho f \, \mathrm{d}v^2 + 2 \, \mathrm{d}v \, \mathrm{d}\rho - 2\omega a^{-1} \, \mathrm{d}\varphi \, \mathrm{d}\rho \\ + 4\omega\rho f \, \mathrm{d}v \, \mathrm{d}\varphi + \left[\gamma^2 + \tfrac{2\rho}{a} f(\gamma^2 - \omega^2)\right] \mathrm{d}\varphi^2$$

state-dependent functions $\mathcal{J}^\pm = \gamma \pm \omega$, chemical potentials $\zeta^\pm = -a \pm \Omega$

For simplicity set $\Omega = 0$ and $a = \text{const.}$ in metric above

EOM imply $\partial_v \mathcal{J}^\pm = \pm \partial_\varphi \zeta^\pm$; in this case $\partial_v \mathcal{J}^\pm = 0$

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Comments:

- Recover desired near horizon metric

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- $\gamma = \gamma(\varphi)$: "black flower"

Canonical boundary charges

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Background independent result for Chern–Simons yields

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- Finite
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Meaningful near horizon boundary conditions and non-trivial theory!

Near horizon symmetry algebra

- Near horizon symmetry algebra = all near horizon boundary conditions preserving trafos, modulo trivial gauge trafos

  Most general trafo

  $$\delta_\epsilon \mathfrak{a} = \mathrm{d}\epsilon + [\mathfrak{a},\, \epsilon] = \mathcal{O}(\delta\mathfrak{a})$$

  that preserves our boundary conditions for constant ζ given by

  $$\epsilon = \epsilon^+ L_+ + \eta L_0 + \epsilon^- L_-$$

  with

  $$\partial_v \eta = 0$$

  implying

  $$\delta_\epsilon \mathcal{J} = \partial_\varphi \eta$$

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- Expand charges in Fourier modes

$$J_n^{\pm} = \frac{k}{4\pi} \oint \mathrm{d}\varphi \, e^{in\varphi} \mathcal{J}^{\pm}(\varphi)$$

What should we expect?

  - Virasoro? (spacetime is locally AdS_3)
  - BMS_3? (Rindler boundary similar to scri)
  - warped conformal algebra? (this is what we found for Rindleresque holography and what Donnay, Giribet, Gonzalez, Pino found in their near horizon analysis)

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$$\left[J_n^{\pm},\, J_m^{\pm}\right] = \pm\tfrac{1}{2} k n \delta_{n+m,\,0} \qquad \left[J_n^{+},\, J_m^{-}\right] = 0$$

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- Map

$$P_0 = J_0^+ + J_0^- \qquad P_n = \tfrac{i}{kn}\,(J_{-n}^+ + J_{-n}^-) \text{ if } n \neq 0 \qquad X_n = J_n^+ - J_n^-$$

  yields Heisenberg algebra (with Casimirs X_0, P_0)

$$[X_n,\, X_m] = [P_n,\, P_m] = [X_0,\, P_n] = [P_0,\, X_n] = 0$$
$$[X_n,\, P_m] = i\delta_{n,m} \quad \text{if } n \neq 0$$

Outline

Higher spin near horizon boundary conditions in diagonal gauge

- Inspired by spin-2 take same group element b in $A = b^{-1}(\rho)\big(\mathrm{d}+\mathfrak{a}(v,\,\varphi)\big)b(\rho)$ and choose

$$\mathfrak{a} = \sum_{s=2}^{N} \mathcal{J}_{(s)}\, W_0^{(s)}\ \mathrm{d}\varphi + \sum_{s=2}^{N} \zeta_{(s)}\, W_0^{(s)}\ \mathrm{d}v$$

with $W_n^{(2)} = L_n$ and $\mathcal{J}_{(s)} = \mathcal{J}$

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- Reminder: relevant part of $sl(N)$ algebra:

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  so $W_n^{(s)}$ are generators associated with spin-s

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- (non-trivial) boundary condition preserving trafos generated by

$$\epsilon = \sum_{s=2}^{N} \eta_{(s)} W_0^{(s)}$$

Higher spin near horizon symmetry algebra

- Construct again canonical charges

$$Q[\eta] \sim \oint \mathrm{d}\varphi \, \eta_{(s)}(\varphi) \, \mathcal{J}_{(s)}(\varphi)$$

and introduce again Fourier components $J_{(s)\,n} \sim \oint \mathrm{d}\varphi \, e^{in\varphi} \mathcal{J}_{(s)}\,(\varphi)$

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- Their algebra is again the Heisenberg algebra

$$[(X_s)_n, (X_t)_m] = [(P_s)_n, (P_t)_m] = [(X_s)_0, (P_t)_n] = [(P_s)_0, (X_t)_n] = 0$$
$$[(X_s)_m, (P_t)_n] = i\delta_{s,t}\delta_{m,n} \text{ for } m \neq 0\,.$$

with similar redefinitions as before

$$(P_s)_0 = J^+_{(s)\,0} + J^-_{(s)\,0}$$
$$(P_s)_n \propto \frac{1}{n}(J^+_{(s)\,-n}) + J^-_{(s)\,-n}) \text{ for } n \neq 0$$
$$(X_s)_n = J^+_{(s)\,n} - J^-_{(s)\,n}$$

and $2N-2$ Casimirs $(X_s)_0$, $(P_s)_0$ with $s = 2 \ldots N$

Outline

Soft hair

- Vacuum spin-2 descendants $|\psi(q)\rangle$

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same as energy of vacuum

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Soft hair = zero energy excitations on horizon

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$$S = 2\pi(J_0^+ + J_0^-)$$

calculated directly in Chern–Simons formulation (in spin-2 case: $S = A/(4G_N)$)

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- Suggestive that microstate counting should work

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$$S = 2\pi(J_0^+ + J_0^-)$$

- No contribution from soft hair charges or higher spin charges
- Suggestive that microstate counting should work

Before addressing microstates consider map to aymptotic variables

Map to asymptotic variables in spin-2 case

- Usual asymptotic AdS_3 connection with chemical potential μ:

$$\hat{A} = \hat{b}^{-1}\big(\mathrm{d}+\hat{\mathfrak{a}}\big)\hat{b} \qquad \hat{\mathfrak{a}}_\varphi = L_+ - \tfrac{1}{2}\,\mathcal{L}\,L_-$$

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- Get Virasoro with non-zero central charge $\delta\mathcal{L} = 2\mathcal{L}\varepsilon' + \mathcal{L}'\varepsilon - \varepsilon'''$

Remarks on asymptotic and near horizon variables

- Asymptotic spin-2 currents fulfill Virasoro algebra, but charges obey still Heisenberg algebra

$$\delta Q = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\varepsilon\,\delta\mathcal{L} = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\,\eta\,\delta\mathcal{J}$$

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Near horizon boundary conditions natural for near horizon observer

Cardy counting in spin-2 case

- Idea: use map to asymptotic observables to do standard Cardy counting
- Twisted Sugawara construction expanded in Fourier modes

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Precise numerical factor in twist term crucial for correct results

Warped CFT counting in spin-2 case

- Map near horizon algebra $J_n^{\pm} = \frac{1}{2}(J_n \pm K_n)$

$$Y_n \sim \sum J_{n-p}K_p \qquad T_n \sim J_n$$

to centerless warped conformal algebra

$$[Y_n,\, Y_m] = (n-m)Y_{n+m}$$
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- Assuming $J^{\text{vac}} = 0$ yields

$$S = \beta H = S_{\text{BH}}$$

  Hamiltonian H is product of BH entropy and Unruh temperature

Generalization to higher spins (explicitly for spin-3)

Summary of map to variables in highest weight gauge:

- twisted Sugawara terms for spin-2

  $$\mathcal{L} \sim \mathcal{J}^2 + \mathcal{J}' + \mathcal{J}_3^2$$

  and spin-3 currents

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- holonomy conditions again solved automatically for all states in the theory
- again boundary conditions singled out by demanding that whole spectrum be compatible with regularity

Higher spin entropy in near horizon theory

Similar to spin-2 case, so just quote main results and point out differences:

- Spin-3: $g_{\varphi\varphi} \sim \gamma^2 + \gamma_{(3)}^2 + \dots$ with

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- Result above generalizes to spin-N entropy
- No microscopic attempts yet to calculate higher spin entropy

Outline

Comparison to related approaches

- Brown, Henneaux '86

Our boundary conditions differ from Brown–Henneaux — their chemical potentials depend on our charges and chemical potentials!

Virasoro composite in terms of Heisenberg algebra

Comparison to related approaches

- Brown, Henneaux '86
- Donnay, Giribet, González, Pino 1511.08687
  - Observed already $H = TS_{\mathrm{BH}}$
  - Changing our bc's to

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho - 2\omega a^{-1}\,\mathrm{d}\varphi\,\mathrm{d}\rho + 4\omega\rho\,\mathrm{d}v\,\mathrm{d}\varphi + \left[\gamma^2 + \frac{2\rho}{a}(\gamma^2 - \omega^2)\right]\mathrm{d}\varphi^2 + \mathcal{O}(\rho^2)$$

yields AKVs

$$\xi = T(\varphi)\partial_v + Y(\varphi)\partial_\varphi + \mathcal{O}(\rho^3)$$

  - Up to subleading terms same AKVs as DGGP

But: T and Y state-dependent for our boundary conditions!

Comment: map to Brown–Henneaux variables requires second chemical potential, not just Rindler acceleration!

Warped CFT algebra composite in terms of Heisenberg algebra

Comparison to related approaches

- Brown, Henneaux '86
- Donnay, Giribet, González, Pino 1511.08687
- Afshar, Detournay, DG, Oblak 1512.08233

Rindler acceleration state-dependent in that approach

Twisted warped CFT algebra composite in terms of Heisenberg algebra

Comparison to related approaches

- Brown, Henneaux '86
- Donnay, Giribet, González, Pino 1511.08687
- Afshar, Detournay, DG, Oblak 1512.08233
- Hawking, Perry, Strominger 1601.00921
  - We constructed explicitly gravitational soft hair
  - We find no soft hair contribution to black hole entropy
  - BMS_3 follows from Sugawara-like construction from Heisenberg algebra

BMS algebra (supertranslations + superrotation) composite in terms of near horizon Heisenberg algebra

Comparison to related approaches

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- Hawking, Perry, Strominger 1601.00921
- Comment on complementarity:

> - Asymptotic Virasoro algebra composite from near horizon perspective
> - Same physics described naturally in different variables for asymptotic and near horizon observers
> - In particular, asymptotic chemical potentials depend on near horizon charges and chemical potentials

Elaborations and generalizations

- More on dual field theory — to be done
- Flat space
  - Similar story works!
  - Get centerless BMS_3 as composite algebra from Heisenberg algebra!
  - Soft hairy flat space cosmologies
  - Asymptotic chemical potentials again depend on near horizon charges and chemical potentials
  - Obtain again Bekenstein–Hawking entropy with no soft hair contribution

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- Lower spins — lowest spin gravity! (see Hofman, Rollier 1411.0672)
- 4d — Does it work? Is there soft Heisenberg hair? Is BMS_4 composite? What are near horizon symmetries?

Near horizon symmetries shed new light on soft hair, microstate counting and complementarity

Thanks for your attention!

D. Grumiller, A. Perez, S. Prohazka, D. Tempo and R. Troncoso
in preparation.

H. Afshar, S. Detournay, D. Grumiller, W. Merbis, A. Perez, D. Tempo and R. Troncoso
"Soft Heisenberg hair on black holes in three dimensions,"
Phys.Rev.**D** [R] (2016), in print; 1603.04824.

H. Afshar, S. Detournay, D. Grumiller and B. Oblak
"Near-Horizon Geometry and Warped Conformal Symmetry,"
JHEP **1603** (2016) 187; 1512.08233.

Thanks to Bob McNees for providing the LaTeX beamerclass!

Bonus level: exact metric with generic chemical potentials

Our bc's for the connection $A^{\pm} = b_{\pm}^{-1}(\rho)\left(\mathrm{d}+\mathfrak{a}_{\pm}(x^0,\,x^1)\right)b_{\pm}(\rho)$ with

$$\mathfrak{a}_{\pm} = \left(\mathcal{J}_{\pm}\,\mathrm{d}\varphi + \zeta^{\pm}\,\mathrm{d}v\right)\,L_0$$

and $b_{\pm} = \exp\left(\frac{1}{\zeta^{\pm}}\,L_+\right)\cdot\exp\left(\frac{\rho}{2}\,L_-\right)$ lead to the metric

$\mathrm{d}s^2 = \frac{1}{2}\left\langle\left(A_\mu^+ - A_\mu^-\right)\left(A_\nu^+ - A_\nu^-\right)\right\rangle\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$

$$\begin{aligned}
=&\left(-\frac{(\zeta^{+2}+\partial_v\zeta^+)(\zeta^{-2}+\partial_v\zeta^-)}{\zeta^{+2}\zeta^{-2}}\rho^2+\frac{\zeta^{+3}\zeta^{-2}+\zeta^{+2}\zeta^{-3}+\partial_v\zeta^+\zeta^{-3}+\zeta^{+3}\partial_v\zeta^-}{\zeta^{+2}\zeta^{-2}}\rho+\frac{1}{4}(\zeta^- -\zeta^+)^2\right)\mathrm{d}v^2\\
&+\left(\frac{(-\zeta^{+2}-\partial_v\zeta^+)\partial_\varphi\zeta^-+(-\zeta^{-2}-\partial_v\zeta^-)\partial_\varphi\zeta^+-\mathcal{J}_+\zeta^+\partial_v\zeta^-+\zeta^-(\mathcal{J}_-\zeta^{+2}-\mathcal{J}_+\zeta^+\zeta^-+\mathcal{J}_-\partial_v\zeta^+)}{2\zeta^{+2}\zeta^{-2}}\rho^2\right.\\
&+\frac{\partial_\varphi\zeta^-\zeta^{+3}+\partial_\varphi\zeta^+\zeta^{-3}+\mathcal{J}_+\zeta^{+2}\partial_v\zeta^- -\zeta^-\left(\mathcal{J}_-\partial_v\zeta^+\zeta^-+\zeta^+(\zeta^-+\zeta^+)(\zeta^+\mathcal{J}_- -\zeta^-\mathcal{J}_+)\right)}{2\zeta^{+2}\zeta^{-2}}\rho\\
&\left.-\frac{1}{4}(\zeta^- -\zeta^+)(\mathcal{J}_-+\mathcal{J}_+)\right)\mathrm{d}v\,\mathrm{d}\varphi+\left(1+\frac{\partial_v\zeta^-\zeta^{+2}+\partial_v\zeta^+\zeta^{-2}}{2\zeta^{+2}\zeta^{-2}}\right)\mathrm{d}v\,\mathrm{d}\rho\\
&+\left(\frac{(\mathcal{J}_+\zeta^++\partial_\varphi\zeta_+)(\mathcal{J}_-\zeta^- -\partial_\varphi\zeta^-)}{\zeta^{+2}\zeta^{-2}}\rho^2+\frac{\mathcal{J}_+\partial_\varphi\zeta^-\zeta^{+2}-\zeta^-\mathcal{J}_-(\zeta^-\partial_\varphi\zeta^++\mathcal{J}_+\zeta^+(\zeta^-+\zeta^+))}{\zeta^{+2}\zeta^{-2}}\rho\right.\\
&\left.+\frac{1}{4}(\zeta^-+\zeta^+)^2\right)\mathrm{d}\varphi^2+\left(\frac{\mathcal{J}_+\zeta^+\zeta^{-2}-\mathcal{J}_-\zeta^{+2}\zeta^-+\partial_\varphi\zeta^+\zeta^{-2}+\partial_\varphi\zeta^-\zeta^{+2}}{2\zeta^{+2}\zeta^{-2}}\right)\mathrm{d}\varphi\,\mathrm{d}r
\end{aligned}$$