Quantum Dilaton Gravity with Fermions

René Meyer

Institute for Theoretical Physics
University of Leipzig, Germany

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Outline

1. First Order Gravity with Matter
   - First Order Gravity
   - Fermions
   - Non-linear gauge theory

2. Quantizing Gravity
   - Three steps to Quantized Gravity
   - The effective action

3. Matter perturbation theory
   - 1-Loop effects
   - 4-Point Vertices & Virtual Black Holes

4. Conclusions & Outlook
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First Order Gravity

\[ S_{\text{FOG}} = \int X^a (De)_a + X \, d\omega + \epsilon (X^+ X^- U(X) + V(X)) \]

- Classically equivalent to 2D Dilaton Gravity
- Exactly solvable (P\(\sigma\)M)
- Absolute conserved quantity: \(dC(g) = 0\)

\[ C(g) = e^{Q(X)} X^+ X^- + w(X) \]

\[ Q(X) = \int X U(y) \, dy, \quad w(X) = \int e^{Q(y)} V(y) \, dy \]


\[ \Gamma[\langle e^a, \omega, X^a, X \rangle] = S_{\text{FOG}}[\langle e^a, \omega, X^a, X \rangle] \]

<table>
<thead>
<tr>
<th>Model</th>
<th>$U(X)$</th>
<th>$V(X)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schwarzschild</td>
<td>$-\frac{1}{2X}$</td>
<td>$-\frac{\lambda^2}{2}$</td>
</tr>
<tr>
<td>SRG (generic $D &gt; 3$)</td>
<td>$-\frac{D-3}{(D-2)X}$</td>
<td>$-\lambda^2 X^{(D-4)/(D-2)}$</td>
</tr>
<tr>
<td>Jackiw-Teitelboim</td>
<td>$0$</td>
<td>$-\Lambda X$</td>
</tr>
<tr>
<td>Witten BH/CGHS</td>
<td>$-\frac{1}{X}$</td>
<td>$-2\lambda^2 X$</td>
</tr>
<tr>
<td>(A)dS ground state</td>
<td>$-\frac{a}{X}$</td>
<td>$-\frac{B}{2}X$</td>
</tr>
<tr>
<td>Rindler ground state</td>
<td>$-\frac{a}{X}$</td>
<td>$-\frac{B}{2}Xa$</td>
</tr>
<tr>
<td>BH attractor</td>
<td>$0$</td>
<td>$-\frac{B}{2}X^{-1}$</td>
</tr>
<tr>
<td>All above: $ab$-family</td>
<td>$-\frac{a}{X}$</td>
<td>$-\frac{B}{2}X^{a+b}$</td>
</tr>
<tr>
<td>Reissner-Nordström</td>
<td>$-\frac{1}{2X}$</td>
<td>$-\lambda^2 + \frac{Q^2}{X}$</td>
</tr>
<tr>
<td>Schwarzschild-(A)dS</td>
<td>$-\frac{1}{2X}$</td>
<td>$-\lambda^2 - \ell X$</td>
</tr>
<tr>
<td>Katanaev-Volovich</td>
<td>$\alpha$</td>
<td>$\beta X^2 - \Lambda$</td>
</tr>
<tr>
<td>KK reduced CS</td>
<td>$0$</td>
<td>$\frac{1}{2}X(c - X^2)$</td>
</tr>
<tr>
<td>Symmetric kink</td>
<td>$\text{generic}$</td>
<td>$-X\prod_{i=1}^{n}(X^2 - X_i^2)$</td>
</tr>
<tr>
<td>2D type 0A/0B</td>
<td>$-\frac{1}{X}$</td>
<td>$-2\lambda^2 X + \frac{\lambda^2 q^2}{8\pi}$</td>
</tr>
<tr>
<td>ESBH</td>
<td>$\Rightarrow$</td>
<td>solved</td>
</tr>
</tbody>
</table>

$\Rightarrow$ René Meyer (University of Leipzig)
Fermions

\[ S_\chi = \frac{i}{2} \int F(X) (*e^a) \wedge (\bar{\chi} \gamma^a \d x) + \int \epsilon H(X) \left( m\bar{\chi}\chi + \lambda (\bar{\chi}\chi)^2 \right) \]

- Spin connection \( \omega \) drops out in D=2.
- Exact solutions for chiral fermions exist, but integrability is lost in general.
- Absolute conservation law: \( 0 = d(C^{(g)} + C^{(m)}) \)

\[ dC^{(m)} = e^Q(X) \left( X^+ \frac{\delta S_\chi}{\delta e^+} + X^- \frac{\delta S_\chi}{\delta e^-} \right) \]

- 4D minimally coupled Dirac fermions \( \rightarrow \) 2 Dirac Fermions in 2D + intertwiner term
FOG as Non-linear gauge theory

- The system is invariant under: Local SO(1,1) $\gamma$, diffeos $\xi^\mu$
- Generated by three 1st class constraints $G_i \approx 0$, $i = 1, 2, 3$
- Hamiltonian Constraint: $\mathcal{H} = - \sum_{i=1}^{3} \bar{q}^i G_i$, $\bar{q}^i = (\omega_0, e^0_-, e^0_+)$
- Symmetry algebra:

  \[
  \{ G_i, G'_i \}^* = 0 \quad \{ G_1, G'_{2/3} \}^* = \mp G_{2/3} \delta
  \]

\[
\{ G_2, G'_3 \}^* = \left[ - \sum_{i=1}^{3} \frac{d\gamma}{dp_i} G_i + \left( gH' - \frac{H}{F} F' g' \cdot (\bar{\chi}\chi) \right) G_1 \right] \delta
\]

with $g(\bar{\chi}\chi) = m\bar{\chi}\chi + \lambda(\bar{\chi}\chi)^2$, $g'(\bar{\chi}\chi) = \frac{\partial g}{\partial (\bar{\chi}\chi)}$, $p_i = (\omega_1, e^-_1, e^+_1)$.

- 2nd class constraints: Relate $\chi$ to canonical conjugate momentum
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Quantizing Gravity

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Three steps to Quantized Gravity

1. Ghosts \((c^i, p^c_j)\) for the \(G_i\), BRST charge \(\Omega = c^i G_i + \frac{1}{2} c^i c^j C^k_{ij} p^c_k\) terminates at Yang-Mills level

2. Fix EF gauge \((\omega_0, e^-_0, e^+_0) = (0, 1, 0)\): \(g_{\mu\nu} = e^+_1 \begin{pmatrix} 0 & 1 \\ 1 & 2e^-_1 \end{pmatrix}\)

3. Path integration:

\[
\mathcal{D}c^i \mathcal{D}p^c_i \rightarrow \mathcal{D}P_\chi \delta(\Phi_\alpha) \rightarrow \mathcal{D}(\omega_1, e^-_1, e^+_1) \rightarrow \mathcal{D}(X, X^+, X^-) \rightarrow \mathcal{D}(\hat{X}, \hat{X}^+, \hat{X}^-)
\]

\[
\text{Det}M \quad \text{trivial} \quad \delta(\text{EOM}(X, X^+X^-)) \quad (\text{Det}M)^{-1}
\]

\[
\partial_0 X = j_1 + X^+
\]

\[
\partial_0 X^+ = j_2 - \frac{i}{\sqrt{2}} F(X) (\chi^*_1 \partial_0 \chi_1)
\]

\[
(\partial_0 + U(X) X^+) X^- = j_3 - V(X) + \frac{i}{\sqrt{2}} F(X) (\chi^*_0 \partial_0 \chi_0) + H(X) g(\chi\chi)
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Three steps to Quantized Gravity

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   terminates at Yang-Mills level

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The effective action

\[ \mathcal{L}_{\text{eff}} = J^i \hat{X}_i + \frac{i}{\sqrt{2}} F(\hat{X})(\chi^*_1 \partial_1 \chi_1) \]

\[ + e^{Q(\hat{X})} \left( j_3 - V(\hat{X}) + H(\hat{X})g(\chi \chi) + \frac{i}{\sqrt{2}} F(\hat{X})(\chi^*_0 \partial_0 \chi_0) \right) \]

- **Nonlocal:** $\partial^{-1}_0$
  - Includes all backreactions of matter fields on geometry
  - Procedure is background independent
  - Integration over $(\omega_0, e^-_1, e^+_1)$ is not restricted
  - So far: No quantum corrections from matter included, and matter is still off-shell.
The effective action

\[ L_{\text{eff}} = J^i \dot{X}_i + \frac{i}{\sqrt{2}} F(\dot{X}) (\chi_1^* \overrightarrow{\partial_1} \chi_1) + e^{Q(\dot{X})} \left( j_3 - V(\dot{X}) + H(\dot{X}) g(\bar{\chi} \chi) + \frac{i}{\sqrt{2}} F(\dot{X}) (\chi_0^* \overrightarrow{\partial_0} \chi_0) \right) \]

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1-Loop effects

- Expansion: \( \mathcal{L}_{\text{eff}} = \mathcal{L}^{(0)} + \mathcal{L}^{(2)} + \mathcal{L}_{\text{int}} \)

- Kinetic term (m=0,F(X)=1):

\[
\mathcal{L}^{(2)} = \frac{i}{2} E_{a}[j, J](\bar{\chi} \gamma^a \partial_\mu \chi)
\]

→ effective Background

- Conformal anomaly:

\[
T_\mu^\mu = \frac{R}{24\pi}
\]

- 1-loop in the matter fields:

\[
W_{\text{Poly.}} = - \log \det \mathcal{D} = \frac{1}{96\pi} \int d^2 x \sqrt{-g} R \frac{1}{\Delta} R
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For free massless fermions:

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<thead>
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<tr>
<td>1)</td>
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- Nonlocal of form $\int d^2 y \int d^2 x \Theta(y^0 - x^0) \delta(x^1 - y^1) V(x, y)$.
- Vanish for $x^0 = y^0$.
- (1) & (2) same as for a real scalar
- (1) & (2) conformally invariant; (3) not conformally invariant, but external legs $\chi_0$ also have conformal weight $-2$
- No gravitational interaction of $\chi_0$ with itself in this gauge.
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- (1) & (2) conformally invariant; (3) not conformally invariant, but external legs $\chi_0$ also have conformal weight $-2$
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VBH: Intermediary effective geometry in scattering processes encoding the vertices and the classical background
e.g. for Spherical Reduced Gravity:

\[(ds)^2 = 2 \, dr \, du + \left( 1 - \theta(r_y - r) \delta(u - u_y) \left( \frac{2m}{r} + ar \right) \right) (du)^2\]

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   - First Order Gravity
   - Fermions
   - Non-linear gauge theory

2. Quantizing Gravity
   - Three steps to Quantized Gravity
   - The effective action

3. Matter perturbation theory
   - 1-Loop effects
   - 4-Point Vertices & Virtual Black Holes

4. Conclusions & Outlook
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- Tree-level S-Matrix, Unitarity, CPT invariance
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