How general is holography?
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Fundamental forces (xkcd 1489)

There are four fundamental forces between particles:

1. Gravity, which obeys this inverse-square law:
   \[ F_{\text{gravity}} = G \frac{m_1 m_2}{d^2} \]

2. Electromagnetism, which obeys this inverse-square law:
   \[ F_{\text{static}} = k_e \frac{q_1 q_2}{d^2} \]

   And also Maxwell’s equations

   Also what?

3. The strong nuclear force, which obeys, uh...
   ... well, umm...
   ... it holds protons and neutrons together.

   I see.

   It’s strong.

4. And (4) the weak force: it [mumble mumble] radioactive decay [mumble mumble]
   That’s not a sentence.
   You just said ‘radio–

   --and those are the four fundamental forces!
“Of these four forces, there’s one we don’t really understand.” “Is it the weak force or the strong—” “It’s gravity.”
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- **Newton–Einstein world:** Gravity best understood force
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- **Newton–Einstein world**: Gravity best understood force
- **Bohr–Schrödinger world**: Gravity least understood force
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Main goal: understand quantum gravity
Outline

Motivation

Holography in 3d
  Chern–Simons formulation
  Asymptotic symmetries
  Example: flat space holography

Outlook - how general is holography?
General motivations

▶ Quantum gravity
  ▶ Address conceptual issues of quantum gravity
General motivations

- Quantum gravity
  - Address conceptual issues of quantum gravity
  - Black holes (thermodynamics, evaporation, information loss, microstate counting, entanglement entropy, firewalls, ...)

- Holography
  - Holographic principle realized in Nature? (yes/no)
  - Quantum gravity via AdS/CFT? (define quantum gravity in AdS by constructing/postulating dual CFT)
  - How general is holography? (non-unitary holography, higher spin holography, flat space holography, non-AdS holography, ...)

- Applications (will not address them in my talk)
  - Gauge gravity correspondence (non-abelian plasmas, condensed matter)
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  - Holographic principle realized in Nature? (yes/no)
  - Quantum gravity via AdS/CFT? (define quantum gravity in AdS by constructing/postulating dual CFT)
  - How general is holography? (non-unitary holography, higher spin holography, flat space holography, non-AdS holography, ...)

- **Applications (will not address them in my talk)**
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Testing the holographic principle

How general is holography?

▶ To what extent do (previous) lessons rely on the particular constructions used to date?
▶ Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?
▶ Does holography apply only to unitary theories?
▶ Can we establish a flat space holographic dictionary?
▶ Generic non-AdS holography/higher spin holography?
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see numerous talks at KITP workshop “Bits, Branes, Black Holes” 2012

and at ESI workshop “Higher Spin Gravity” 2012
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- plausible AdS/CFT-like correspondence could work non-unitarily
- AdS/log CFT first example of non-unitary holography DG, (Jackiw), Johansson ’08; Skenderis, Taylor, van Rees ’09; Henneaux, Martinez, Troncoso ’09; Maloney, Song, Strominger ’09; DG, Sachs/Hohm ’09; Gaberdiel, DG, Vassilevich ’10; ... DG, Riedler, Rosseel, Zojer ’13
- recent proposal by Vafa ’14
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The answer appears to be yes — see my current talk and recent papers by Bagchi et al., Barnich et al., Strominger et al., ’12-’15
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*non-trivial hints that it might work at least in 2+1 dimensions*

Gary, DG Rashkov ’12; Afshar et al ’12; Gutperle et al ’14-’15; Gary, DG, Prohazka, Rey ’14; …
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- Address questions above in simple class of 3d toy models
- Exploit gauge theoretic Chern–Simons formulation
- Restrict to kinematic questions, like (asymptotic) symmetries
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Outlook - how general is holography?
Clarification of nomenclature

- Conformal CS gravity (Deser, Jackiw, Templeton '82)

\[ I_{CSG} = \frac{k}{4\pi} \int d^3 x \sqrt{-g} \varepsilon^{\lambda \mu \nu} \Gamma^\rho_{\lambda \sigma} \left( \partial_\mu \Gamma^\sigma_{\nu \rho} + \frac{2}{3} \Gamma^\sigma_{\mu \tau} \Gamma^\tau_{\nu \rho} \right) \]

0 local physical degrees of freedom
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▶ Einstein gravity in CS formulation ([Achucarro, Townsend '86; Witten '88])

\[
I_{EH} = \frac{1}{16\pi G_N} \int d^3 x \sqrt{-g} \left( R + \frac{2}{\ell^2} \right) \sim I_{CS}(A) - I_{CS}(\bar{A})
\]

\(A, \bar{A}: \text{sl}(2)\) connections (sum/diff of Dreibein and spin-connection)

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  0 local physical degrees of freedom

- **Gravitational CS term in topologically massive gravity (Deser, Jackiw, Templeton ’82)**
  \[ I_{TMG} = I_{EH} + I_{CSG} \]

  0 + 0 = 1 local physical degree of freedom (massive graviton)
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- This talk: gravity-like CS theories
CS bulk theory

Action:

\[ I_{CS} = \frac{k}{4\pi} \int_{\mathcal{M}} \langle A \wedge dA + \frac{2}{3} A \wedge A \wedge A \rangle \]

- \( k \): CS-level
- \( \mathcal{M} \): 3d or (2+1)d manifold (this talk: filled cylinder or filled torus)
- \( A = A^a_{\mu} T^a dx^\mu \): (non-abelian) connection 1-form
- \( \langle,\rangle \): bilinear form
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EOM:

\[ F = dA + [A, A] = 0 \]

Solutions: (locally) gauge-flat connections, \( A = g^{-1} \, dg \)
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Solutions: (locally) gauge-flat connections, \( A = g^{-1} \, dg \)

- Chern–Simons theory locally trivial
- Boundary conditions/fall-off behavior crucial
Overview of gravity-like CS theories

(spin-2) gravity
- with negative cosmological constant: $sl(2) \oplus sl(2)$ with suitable bc’s
- in flat space: $isl(2)$ with suitable bc’s

Vasiliev type higher spin gravity
- with negative cosmological constant: $hs(\lambda) \oplus hs(\lambda)$ with suitable bc’s
- in flat space: probably exists?
Overview of gravity-like CS theories

**Spin-2 gravity**
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**Spin-3 gravity**
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**generic higher spin/lower spin gravity**
- higher spin with negative cosmological constant: some gauge algebra containing $sl(2) \oplus sl(2)$ with suitable bc’s (e.g. $sl(N) \oplus sl(N)$)
- higher spin in flat space: some gauge algebra containing $isl(2)$ with suitable bc’s (e.g. $isl(N)$)
- higher spin in Lobachevsky/warped AdS/Schrödinger/Lifshitz: some gauge algebra containing $sl(2) \oplus sl(2)$ with suitable bc’s
- lower spin: $sl(2) \oplus u(1)$ with suitable bc’s
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- with negative cosmological constant: $hs(\lambda) \oplus hs(\lambda)$ with suitable bc’s
- in flat space: probably exists?
Gravity-like CS with asymptotic boundary

Spin-2: Bañados '94; Spin-3: Henneaux, Rey '10; Campoleoni et al '10

Assume cylinder topology:

radius: $\rho$
boundary: $\rho \to \infty$
boundary coord’s: $x^i$

Impose fall-off conditions on connection

$$\lim_{\rho \to \infty} A(\rho, x^i) = \hat{A}_\mu^a(\rho, x^i) T^a d\rho^\mu + \delta A(\rho, x^i) + \ldots$$

asympt. bg.
state dep.

Lot of guesswork!

Ansatz that works in all cases so far:

$$A(\rho, x^i) = b^{-1}(\rho) \left( d + a(x^i) \right) + o(1) b(\rho)$$

Radial dependence captured by $b$.
Gravity-like CS with asymptotic boundary

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- Lot of guesswork!
- Ansatz that works in all cases so far:
  \[ A(\rho, x^i) = b^{-1}(\rho) (d + a(x^i) + o(1)) b(\rho) \]
- Radial dependence captured by \( b \)
- Connection \( a = \hat{a}_i \, dx^i + \delta a_i \, dx^i \) subject to asymptotic on-shell conditions
  \[ F_{ij} = 0 \iff da + [a, a] = 0 \]
Gravity-like CS with asymptotic boundary

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- \( F_{\rho i} = 0 \) automatically
Gravity-like CS with asymptotic boundary

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- \( F_{\rho i} = 0 \) automatically

- Subleading fluctuation terms \( o(1) \) irrelevant

radius: \( \rho \)

boundary: \( \rho \to \infty \)

boundary coord’s: \( x^i \)
(Non-)equivalence of gravity-like CS to gravity

Gravity-like CS:

- Need suitable gauge algebra
- Need appropriate boundary conditions on connection
(Non-)equivalence of gravity-like CS to gravity

Gravity-like CS:
- Need suitable gauge algebra
- Need appropriate boundary conditions on connection
- Need map to metric variables

Spin-2 field: $g_{\mu\nu} = \frac{1}{2} \tilde{\text{tr}}(A^\mu A^\nu)$

Spin-3 field: $\Phi_{\mu\nu\lambda} = \frac{1}{6} \tilde{\text{tr}}(A^\mu A^\nu A^\lambda)$

Classically equivalent to gravity(-like) theory

Probably quantum inequivalent

Debatable which version is correct at quantum level
(Non-)equivalence of gravity-like CS to gravity

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- Need appropriate boundary conditions on connection
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Spin-2 field:
\[ g_{\mu\nu} = \frac{1}{2} \tilde{\text{tr}} \left( A_{\mu} A_{\nu} \right) \]

Spin-3 field:
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etc.
(Non-)equivalence of gravity-like CS to gravity

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etc.

- Classically equivalent to gravity(-like) theory
- Probably quantum inequivalent
- Debatable which version is correct at quantum level
Best known example: AdS$_3$ spin-2 gravity (Bañados ’94)

- Gauge algebra: $sl(2) \oplus sl(2)$

- Split full connection into left ($A$) and right ($\bar{A}$) moving part

- Choose $b = e^\rho L_0$

- Choose $a$

- Full connection (remember, $A = b - 1 (d + a)$): $A = \ldots$

- Analogous choices in bar-sector $\bar{A} = b (d + (L - 1 + \bar{L} (x - L) + \ldots) d x - 1)$

- Metric:

$$g_{\mu\nu} = \frac{1}{2} \text{tr} \left( (A_\mu - \bar{A}_\mu)(A_\nu - \bar{A}_\nu) \right)$$

- Boundary conditions above = partly gauge-fixed Brown–Henneaux:

$$g_{\mu\nu} d x^\mu d x^\nu = d \rho^2 + 2 e^2 \rho d x + d x - \ldots + L (x + \ldots) (d x + \ldots)^2 + \bar{L} (x - \ldots) (d x - \ldots)^2 + \ldots$$
Best known example: AdS$_3$ spin-2 gravity (Bañados ’94)

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Best known example: \( \text{AdS}_3 \) spin-2 gravity (Bañados ’94)

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\[
b = e^\rho L_0
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Best known example: $\text{AdS}_3$ spin-2 gravity (Bañados ’94)

- Gauge algebra: $sl(2) \oplus sl(2)$
- Split full connection into left ($A$) and right ($\bar{A}$) moving part
- Choose
  \[ b = e^{\rho L_0} \]
- Choose
  \[ a = (\hat{L}_+ + \mathcal{L}(x^+)\hat{L}_-) \, dx^+ + \mathcal{O}(e^{-2\rho}) \]
  \[ \hat{a}_+ + \delta a_+ \]

\[ \mathcal{L}(x^+) \hat{L}_- = \delta a_+ \]
Best known example: AdS$_3$ spin-2 gravity (Bañados ‘94)

- **Gauge algebra:** $sl(2) \oplus sl(2)$
- **Split full connection into left ($A$) and right ($\bar{A}$) moving part**
- **Choose**

$$b = e^\rho L_0$$

- **Choose**

$$a = (L_+ + \mathcal{L}(x^+)\mathcal{L}_-)\, dx^+ + \mathcal{O}(e^{-2\rho})$$

- **Full connection** (remember, $A = b^{-1}(d + a)b$):

$$A = L_0\, d\rho + e^\rho L_+\, dx^+ + e^{-\rho}\mathcal{L}(x^+)\mathcal{L}_-\, dx^+ + \ldots$$
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- Analogous choices in bar-sector
  \[ \bar{A} = b \left( d+(L_- + \bar{\mathcal{L}}(x^-)L_+ + \ldots) \, dx^- \right) b^{-1} \]
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  \[ A = L_0 \, d\rho + e^{\rho} L_+ \, dx^+ + e^{-\rho} \mathcal{L}(x^+)L_- \, dx^+ + \ldots \]
- Analogous choices in bar-sector
  \[ \bar{A} = b \left( d+(L_- + \bar{\mathcal{L}}(x^-)L_+ + \ldots) \, dx^- \right) b^{-1} \]
- Metric:
  \[ g_{\mu\nu} = \frac{1}{2} \text{tr} \left((A_\mu - \bar{A}_\mu)(A_\nu - \bar{A}_\nu)\right) \]
Best known example: AdS$_3$ spin-2 gravity (Bañados ’94)

- Gauge algebra: $sl(2) \oplus sl(2)$
- Split full connection into left ($A$) and right ($\bar{A}$) moving part
- Choose
  \[ b = e^{\rho L_0} \]
- Choose
  \[ a = (L_+ + \mathcal{L}(x^+) L_-) \, dx^+ + \mathcal{O}(e^{-2\rho}) \]
- Full connection (remember, $A = b^{-1}(d+a)b$):
  \[ A = L_0 \, d\rho + e^\rho L_+ \, dx^+ + e^{-\rho} \mathcal{L}(x^+) L_- \, dx^+ + \ldots \]
- Analogous choices in bar-sector
  \[ \bar{A} = b \left( d + (L_- + \bar{\mathcal{L}}(x^-) L_+ + \ldots) \, dx^- \right) b^{-1} \]
- Metric:
  \[ g_{\mu\nu} = \frac{1}{2} \text{tr} \left( (A_\mu - \bar{A}_\mu)(A_\nu - \bar{A}_\nu) \right) \]
- Boundary conditions above = partly gauge-fixed Brown–Henneaux:
  \[ g_{\mu\nu} \, dx^\mu \, dx^\nu = d\rho^2 + 2e^{2\rho} \, dx^+ \, dx^- + \mathcal{L}(x^+) \, (dx^+)^2 + \bar{\mathcal{L}}(x^-) \, (dx^-)^2 + \ldots \]
Outline

Motivation

Holography in 3d
   Chern–Simons formulation
   Asymptotic symmetries
   Example: flat space holography

Outlook - how general is holography?
Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle

   Essentially did this: CS theory with suitable gauge algebra (will not talk about variational principle)
Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions

Essentially did this: \( A = b^{-1}(d + \hat{a} + \delta a + o(1))b \)

Still need to choose asymptotic background \( \hat{a} \) and state dependent fluctuations \( \delta a \)
Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
   - Find and classify all constraints
   - Construct canonical gauge generators
   - Add boundary terms and get (variation of) canonical charges
   - Check integrability of canonical charges
   - Check finiteness of canonical charges
   - Check conservation (in time) of canonical charges
   - Calculate Dirac bracket algebra of canonical charges
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
7. Identify/constrain dual field theory
8. If unhappy with result go back to previous items and modify

Previous Ansatz for connection simplifies above algorithm considerably!
Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
4. Derive (classical) asymptotic symmetry algebra and central charges

Reminder: ASA = quotient algebra of asymptotic symmetries by ‘trivial’ asymptotic symmetries with zero canonical charges
Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
   Example: semi-classical ASA in spin-3 gravity (Henneaux, Rey ’10; Campoleoni, Pfenninger, Fredenhagen, Theisen ’10)

\[
[W_n, W_m] = \frac{16}{5c} \sum_p L_p L_{n+m-p} + \ldots
\]

quantum ASA

\[
[W_n, W_m] = \frac{16}{5c + 22} \sum_p : L_p L_{n+m-p} : + \ldots
\]
Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
6. Study unitary representations of quantum ASA

Example:

Afshar et al ’12
Discrete set of Newton constant values compatible with unitarity
(3D spin-N gravity in next-to-principal embedding)
see my talk at MIT March ’13
Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
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Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
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Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
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4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
7. Identify/constrain dual field theory
8. If unhappy with result go back to previous items and modify

Previous Ansatz for connection simplifies above algorithm considerably!
Boundary condition preserving transformations and canonical charges

Generic (non-)AdS holography in higher spin gravity: see Afshar et al ’12

- Boundary-condition preserving transformations generated by $\epsilon$:

$$\delta_\epsilon A = dA + [\epsilon, A] = O(\delta A)$$
Boundary condition preserving transformations generated by $\epsilon$:

$$\delta_\epsilon A = dA + [\epsilon, A] = \mathcal{O}(\delta A)$$

Exploit Ansatz:

$$\epsilon = b^{-1}(\rho) \varepsilon(x^i) b(\rho) + \ldots$$
Boundary condition preserving transformations generated by $\epsilon$:

$$\delta_{\epsilon} A = dA + [\epsilon, A] = O(\delta A)$$

Exploit Ansatz:

$$\epsilon = b^{-1}(\rho) \varepsilon(x^i) b(\rho) + \ldots$$

Background independent canonical analysis yields canonical currents:

$$\delta Q[\epsilon] = \frac{k}{2\pi} \lim_{\rho \to \infty} \oint \langle \epsilon \delta A \rangle = \frac{k}{2\pi} \oint \langle \epsilon \delta a \rangle$$
Boundary condition preserving transformations generated by $\epsilon$:

$$\delta_\epsilon A = dA + [\epsilon, A] = O(\delta A)$$

**Exploit Ansatz:**

$$\epsilon = b^{-1}(\rho) \varepsilon(x^i) b(\rho) + \ldots$$

**Background independent canonical analysis yields canonical currents:**

$$\delta Q[\epsilon] = \frac{k}{2\pi} \lim_{\rho \to \infty} \oint \langle \epsilon \delta A \rangle = \frac{k}{2\pi} \oint \langle \varepsilon \delta a \rangle$$

- Manifestly finite! (all $b(\rho)$ cancel)
- Non-trivial?
- Integrable to canonical charges $Q[\epsilon]$?
- Conserved?
Boundary condition preserving trafos and canonical charges

Generic (non-)AdS holography in higher spin gravity: see Afshar et al ’12

- Boundary-condition preserving transformations generated by $\epsilon$:
  \[ \delta_\epsilon A = dA + [\epsilon, A] = O(\delta A) \]

- Exploit Ansatz:
  \[ \epsilon = b^{-1}(\rho) \varepsilon(x^i) b(\rho) + \ldots \]

- Background independent canonical analysis yields canonical currents:
  \[ \delta Q[\epsilon] = \frac{k}{2\pi} \lim_{\rho \to \infty} \oint \langle \epsilon \delta A \rangle = \frac{k}{2\pi} \oint \langle \varepsilon \delta a \rangle \]
  - Manifestly finite! (all $b(\rho)$ cancel)
  - Non-trivial?
  - Integrable to canonical charges $Q[\epsilon]$?
  - Conserved?

If any of these is answered with ‘no’ then back to square one in algorithm
Otherwise: may have new holographic correspondence!
Outline

Motivation

Holography in 3d
  Chern–Simons formulation
  Asymptotic symmetries
  Example: flat space holography

Outlook - how general is holography?
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...) 

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true \( \Rightarrow \) must work in flat space

Just take large AdS radius limit of \( 10^4 \) AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...) 

if holography is true $\Rightarrow$ must work in flat space

- Just take large AdS radius limit of $10^4$ AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra

- Example where it does not work easily: boundary conditions!
- Example where it does not work at all: highest weight conditions!
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, …)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $L_n, \bar{L}_n$

$$L_n = L_n - \bar{L}_{-n} \quad M_n = \frac{1}{\ell} (L_n + \bar{L}_{-n})$$

- Example where it does not work easily: boundary conditions!
- Example where it does not work at all: highest weight conditions!
Flat space holography \((\text{Barnich et al, Bagchi et al, Strominger et al, ...})\)

if holography is true \(\Rightarrow\) must work in flat space

Just take large AdS radius limit of \(10^4\) AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators \(L_n, \bar{L}_n\)
  \[
  L_n = L_n - \bar{L}_{-n} \quad M_n = \frac{1}{\ell} \left( L_n + \bar{L}_{-n} \right)
  \]
- Make Inönü–Wigner contraction \(\ell \rightarrow \infty\) on ASA
  \[
  [L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0} \\
  [L_n, M_m] = (n - m) M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m,0} \\
  [M_n, M_m] = 0
  \]
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true $\Rightarrow$ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $L_n, \tilde{L}_n$
  \[
  L_n = L_n - \tilde{L}_{-n}, \quad M_n = \frac{1}{\ell} \left( L_n + \tilde{L}_{-n} \right)
  \]
- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA
  \[
  \begin{align*}
  [L_n, L_m] &= (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0} \\
  [L_n, M_m] &= (n - m) M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m,0} \\
  [M_n, M_m] &= 0
  \end{align*}
  \]
- This is nothing but the BMS$_3$ algebra (or GCA$_2$, URCA$_2$, CCA$_2$)!
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $L_n, \tilde{L}_n$

$$L_n = L_n - \tilde{L}_{-n} \quad M_n = \frac{1}{\ell} \left( L_n + \tilde{L}_{-n} \right)$$

- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0}$$

$$[L_n, M_m] = (n - m) M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m,0}$$

$$[M_n, M_m] = 0$$

- This is nothing but the BMS$_3$ algebra (or GCA$_2$, URCA$_2$, CCA$_2$)!
- Example where it does not work easily: boundary conditions!
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

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- Works straightforwardly sometimes, otherwise not
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- Take linear combinations of Virasoro generators $L_n, \bar{L}_n$

\[
L_n = L_n - \bar{L}_{-n} \quad M_n = \frac{1}{\ell} \left( L_n + \bar{L}_{-n} \right)
\]

- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

\[
\begin{align*}
[L_n, L_m] &= (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0} \\
[L_n, M_m] &= (n - m) M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m,0} \\
[M_n, M_m] &= 0
\end{align*}
\]

- This is nothing but the BMS$_3$ algebra (or GCA$_2$, URCA$_2$, CCA$_2$)!
- Example where it does not work easily: boundary conditions!
- Example where it does not work at all: highest weight conditions!
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

Not in general! Must (also) work intrinsically in flat space!
Interesting example:
- unitarity of flat space quantum gravity
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

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- unitarity of flat space quantum gravity
- extrapolate from AdS: should be unitary (?)
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

Not in general! Must (also) work intrinsically in flat space!

Interesting example:

- unitarity of flat space quantum gravity
- extrapolate from AdS: should be unitary (?)
- extrapolate from dS: should be non-unitary (?)
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of $10^4$ AdS/CFT papers?

Not in general! Must (also) work intrinsically in flat space!

Interesting example:

- unitarity of flat space quantum gravity
- extrapolate from AdS: should be unitary (?)
- extrapolate from dS: should be non-unitary (?)
- directly in flat space: both options realized, depending on details of model
Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

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Interesting example:

▶ unitarity of flat space quantum gravity
▶ extrapolate from AdS: should be unitary (?)
▶ extrapolate from dS: should be non-unitary (?)
▶ directly in flat space: both options realized, depending on details of model

Many open issues in flat space holography!

Next few slides: mention a couple of recent results
Overview of selected recent results

- Applying algorithm just described to flat space theories

  Barnich, Gonzalez '13; Afshar '13
Overview of selected recent results

- Applying algorithm just described to flat space theories
- Flat space chiral gravity

Bagchi, Detournay, DG ’13
Overview of selected recent results

- Applying algorithm just described to flat space theories
- Flat space chiral gravity
- Cosmic evolution from phase transition

Bagchi, Detournay, DG, Simon ’13
Overview of selected recent results

- Applying algorithm just described to flat space theories
- Flat space chiral gravity
- Cosmic evolution from phase transition
- (Holographic) entanglement entropy

Bagchi, Basu, DG, Riegler ’14
Overview of selected recent results

- Applying algorithm just described to flat space theories
- Flat space chiral gravity
- Cosmic evolution from phase transition
- (Holographic) entanglement entropy
- Flat space higher spin gravity

Afshar, Bagchi, Fareghbal, DG, Rosseel ’13
Gonzalez, Matulich, Pino, Troncoso ’13
Overview of selected recent results

- Applying algorithm just described to flat space theories
- Flat space chiral gravity
- Cosmic evolution from phase transition
- (Holographic) entanglement entropy
- Flat space higher spin gravity
- Unitarity of dual field theory

DG, Riegler, Rosseel '14
Overview of selected recent results

- Applying algorithm just described to flat space theories
- Flat space chiral gravity
- Cosmic evolution from phase transition
- (Holographic) entanglement entropy
- Flat space higher spin gravity
- Unitarity of dual field theory
- Adding chemical potentials

Gary, DG, Riegler, Rosseel '14
Overview of selected recent results

- Applying algorithm just described to flat space theories
- Flat space chiral gravity
- Cosmic evolution from phase transition
- (Holographic) entanglement entropy
- Flat space higher spin gravity
- Unitarity of dual field theory
- Adding chemical potentials

See backup slides or discuss with me privately!
Focus here on flat space higher spin gravity
Flat space higher spin gravity
Afshar, Bagchi, Fareghbal, DG, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13

Interacting theories of massless higher spin fields heavily constrained by no-go results!

- Coleman, Mandula '67
- Haag, Lopuszanski, Sohnius '75
- Aragone, Deser '79
- Weinberg, Witten '80
- ...
- review: Bekaert, Boulanger, Sundell '10

Vasiliev '90: circumvents no-go's by going to (A)dS

we circumvent them by going to 3d (no local physical degrees of freedom)
Flat space higher spin gravity
Afshar, Bagchi, Fareghbal, DG, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13

▶ AdS gravity in CS formulation: spin 2 $\rightarrow$ spin 3 $\sim$ \text{sl}(2) $\rightarrow$ \text{sl}(3)
Flat space higher spin gravity
Afshar, Bagchi, Fareghbal, DG, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13

- AdS gravity in CS formulation: spin 2 → spin 3 \sim \mathfrak{sl}(2) \to \mathfrak{sl}(3)
- Flat space: similar!

\[ S_{\text{CS}}^{\text{flat}} = \frac{k}{4\pi} \int \text{CS}(A) \]

with \( \mathfrak{is}(3) \) connection (\( e^a = \text{"zuvielbein"} \))

\[ A = e^a T_a + \omega^a J_a \quad T_a = (M_n, V_m) \quad J_a = (L_n, U_m) \]

\( \mathfrak{is}(3) \) algebra (spin 3 extension of global part of BMS/GCA algebra)

\[
\begin{align*}
[L_n, L_m] &= (n - m)L_{n+m} \\
[L_n, M_m] &= (n - m)M_{n+m} \\
[L_n, U_m] &= (2n - m)U_{n+m} \\
[M_n, U_m] &= [L_n, V_m] = (2n - m)V_{n+m} \\
[U_n, U_m] &= (n - m)(2n^2 + 2m^2 - nm - 8)L_{n+m} \\
[U_n, V_m] &= (n - m)(2n^2 + 2m^2 - nm - 8)M_{n+m}
\end{align*}
\]
AdS gravity in CS formulation: spin 2 → spin 3 \sim \mathfrak{sl}(2) \rightarrow \mathfrak{sl}(3)

Flat space: similar!

$$S_{\text{CS}}^\text{flat} = \frac{k}{4\pi} \int \text{CS}(\mathcal{A})$$

with is\ell(3) connection ($e^a = \text{“zuvielbein”}$)

$$\mathcal{A} = e^a T_a + \omega^a J_a \quad T_a = (M_n, V_m) \quad J_a = (L_n, U_m)$$

Same type of boundary conditions as for spin 2:

$$\mathcal{A}(r, t, \varphi) = b^{-1}(r) \left( d+a(t, \varphi) + o(1) \right) b(r)$$
Flat space higher spin gravity
Afshar, Bagchi, Fareghbal, DG, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13

- AdS gravity in CS formulation: spin 2 $\rightarrow$ spin 3 $\sim sl(2) \rightarrow sl(3)$
- Flat space: similar!
  \[
  S_{CS}^{\text{flat}} = \frac{k}{4\pi} \int \text{CS}(\mathcal{A})
  \]
  with isl(3) connection ($e^a = \text{“zuvielbein”}$)
  \[
  \mathcal{A} = e^a T_a + \omega^a J_a \quad T_a = (M_n, V_m) \quad J_a = (L_n, U_m)
  \]
- Same type of boundary conditions as for spin 2:
  \[
  \mathcal{A}(r, t, \varphi) = b^{-1}(r) \left( d + a(t, \varphi) + o(1) \right) b(r)
  \]
- Flat space boundary conditions: $b(r) = \exp \left( \frac{1}{2} r M_{-1} \right)$ and
  \[
  a(t, \varphi) = \left( M_1 - M(\varphi) M_{-1} - V(\varphi) V_{-2} \right) dt
  + \left( L_1 - M(\varphi) L_{-1} - V(\varphi) U_{-2} - L(\varphi) M_{-1} - Z(\varphi) V_{-2} \right) d\varphi
  \]
Flat space higher spin gravity
Afshar, Bagchi, Fareghbal, DG, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13

- AdS gravity in CS formulation: spin 2 → spin 3 ∼ sl(2) → sl(3)
- Flat space: similar!

\[ S_{\text{CS}}^{\text{flat}} = \frac{k}{4\pi} \int \text{CS}(\mathcal{A}) \]

with isl(3) connection \((e^a = \text{"zuvielbein"})\)

\[ \mathcal{A} = e^a T_a + \omega^a J_a \quad T_a = (M_n, V_m) \quad J_a = (L_n, U_m) \]

- Same type of boundary conditions as for spin 2:

\[ \mathcal{A}(r, t, \varphi) = b^{-1}(r) \left( d + a(t, \varphi) + o(1) \right) b(r) \]

- Flat space boundary conditions: \(b(r) = \exp \left( \frac{1}{2} r M_{-1} \right)\) and

\[ a(t, \varphi) = \left( M_1 - M(\varphi) M_{-1} - V(\varphi) V_{-2} \right) dt \\
+ \left( L_1 - M(\varphi) L_{-1} - V(\varphi) U_{-2} - L(\varphi) M_{-1} - Z(\varphi) V_{-2} \right) \, d\varphi \]

- Spin-2 and spin-3 charges:

\[ Q[\varepsilon_M, \varepsilon_L, \varepsilon_V, \varepsilon_U] \sim \oint \left( \varepsilon_M(\varphi) M(\varphi) + \varepsilon_L(\varphi) L(\varphi) + \varepsilon_V(\varphi) V(\varphi) + \varepsilon_U(\varphi) U(\varphi) \right) \]
Defining $\langle , \rangle$ and $\widetilde{\text{tr}}$ using Grassmann trick by Krishnan, Raju, Roy ’13

- $\text{isl}(n)$ and $\text{BMW}_n$ have $\mathbb{Z}_2$ grading
- even generators $L_n, U_n, \ldots$: $\text{ad}[sl(n)] \otimes 1_{2 \times 2}$
- odd generators $M_n, V_n, \ldots$: $\epsilon \cdot \text{ad}[sl(n)] \otimes \sigma_3$ with $\epsilon^2 = 0$
- reproduces $\text{isl}(n)$ algebra from $\text{sl}(n)$ algebra
- bilinear form between two generators $G_{n_1}, G_{n_2}$:

$$\langle G_{n_1}, G_{n_2} \rangle = \frac{d}{d\epsilon} \text{tr}\left( \frac{1}{2} G_{n_1} \frac{1}{2} G_{n_2} \gamma^* \right)$$

where $\gamma^* = 1 \otimes \sigma_3$

- tilde-trace of product of $m$ generators $G_{n_1}, \ldots, G_{n_m}$:

$$\widetilde{\text{tr}}\left( \prod_{i=1}^{m} G_{n_i} \right) = \frac{1}{2} \text{tr}\left( \prod_{i=1}^{m} \left( \frac{d}{d\epsilon} G_{n_i} \gamma^* \right) \right)$$

- for further details see Gary, DG, Riegler, Rosseel ’14
Flat space higher spin gravity
Asymptotic symmetry algebra at finite level $k$: Afshar, Bagchi, Fareghbal, DG, Rosseel ’13

- Do either Brown–Henneaux type of analysis or İnönü–Wigner contraction of two copies of quantum $W_3$-algebra
Do either Brown–Henneaux type of analysis or İnönü–Wigner contraction of two copies of quantum $W_3$-algebra

Obtain new type of $W$-algebra as extension of BMS ("BMW")

\[
[L_n, L_m] = (n - m)L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m, 0}
\]

\[
[L_n, M_m] = (n - m)M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m, 0}
\]

\[
[U_n, U_m] = (n - m)(2n^2 + 2m^2 - nm - 8)L_{n+m} + \frac{192}{c_M} (n - m)\Lambda_{n+m}
\]

\[- \frac{96(c_L + \frac{44}{5})}{c_M^2} (n - m)\Theta_{n+m} + \frac{c_L}{12} n(n^2 - 1)(n^2 - 4) \delta_{n+m, 0}
\]

\[
[U_n, V_m] = (n - m)(2n^2 + 2m^2 - nm - 8)M_{n+m} + \frac{96}{c_M} (n - m)\Theta_{n+m}
\]

\[+ \frac{c_M}{12} n(n^2 - 1)(n^2 - 4) \delta_{n+m, 0}
\]

\[
\Lambda_n = \sum_p: L_p M_{n-p}: - \frac{3}{10} (n + 2)(n + 3)M_n
\]

\[
\Theta_n = \sum_p M_p M_{n-p}
\]

other commutators as in $isl(3)$ with $n \in \mathbb{Z}$
Flat space higher spin gravity
Asymptotic symmetry algebra at finite level $k$: Afshar, Bagchi, Fareghbal, DG, Rosseel '13

- Do either Brown–Henneaux type of analysis or İnönü–Wigner contraction of two copies of quantum $W_3$-algebra
- Obtain new type of $W$-algebra as extension of BMS ("BMW")

\[
[L_n, L_m] = (n - m)L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0}
\]

\[
[L_n, M_m] = (n - m)M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m,0}
\]

\[
[U_n, U_m] = (n - m)(2n^2 + 2m^2 - nm - 8)L_{n+m} + \frac{192}{c_M} (n - m)\Lambda_{n+m}
\]

\[
- \frac{96(c_L + \frac{44}{5})}{c_M^2} (n - m)\Theta_{n+m} + \frac{c_L}{12} n(n^2 - 1)(n^2 - 4) \delta_{n+m,0}
\]

\[
[U_n, V_m] = (n - m)(2n^2 + 2m^2 - nm - 8)M_{n+m} + \frac{96}{c_M} (n - m)\Theta_{n+m}
\]

\[
+ \frac{c_M}{12} n(n^2 - 1)(n^2 - 4) \delta_{n+m,0}
\]

- Note quantum shift and poles in central terms!
Do either Brown–Henneaux type of analysis or İnönü–Wigner contraction of two copies of quantum $W_3$-algebra
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$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0}$$

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$$[U_n, U_m] = (n - m)(2n^2 + 2m^2 - nm - 8)L_{n+m} + \frac{192}{c_M} (n - m)\Lambda_{n+m}$$

$$- \frac{96(c_L + 44)}{c_M^2} (n - m)\Theta_{n+m} + \frac{c_L}{12} n(n^2 - 1)(n^2 - 4) \delta_{n+m,0}$$

$$[U_n, V_m] = (n - m)(2n^2 + 2m^2 - nm - 8)M_{n+m} + \frac{96}{c_M} (n - m)\Theta_{n+m}$$

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Note quantum shift and poles in central terms!
Analysis generalizes to flat space contractions of other $W$-algebras
Outline

Motivation

Holography in 3d
- Chern–Simons formulation
- Asymptotic symmetries
- Example: flat space holography

Outlook - how general is holography?
Selected open issues

We have answered an $\epsilon$ of the open questions.

- Checks of flat space chiral gravity (Bagchi et al '12-'15)
- $\exists$ flat space chiral higher spin gravity? (DG, Riegler, Rosseel '14)
- Flat space local quantum quench? (Nozaki, Numasawa, Takayanagi '13)
- (holographic) entanglement entropy in other non-CFT contexts?
- Other non-AdS holography examples? (Gary et al '12-'15)
- Existence of UV-complete 3d theory/no-go result?
- Dimensions $> 3$? (Barnich et al '10-'15; Strominger et al '14-'15)
- Flat limit of $\text{AdS}_5 \times S^5$/CFT$_4$? (Polchinski; Susskind; Giddings '99)

Many open issues that can and should be addressed!
Selected open issues

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Here are a few more $\epsilon$s for 3d models:
- checks of flat space chiral gravity (Bagchi et al ’12-'15)
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- Many open issues that can and should be addressed!
Thanks for your attention!

Vladimir Bulatov, M.C. Escher Circle Limit III in a rectangle
Selected references to own work


Apply algorithm just described to flat space theories
Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
   Topologically massive gravity with mixed boundary conditions

\[ I = I_{EH} + \frac{1}{32\pi G} \int \! d^3 x \sqrt{-g} \varepsilon^{\lambda\mu\nu} \Gamma^\rho_{\lambda\sigma} \left( \partial_\mu \Gamma^\sigma_{\nu\rho} + \frac{2}{3} \Gamma^\sigma_{\mu\tau} \Gamma^\tau_{\nu\rho} \right) \]

with \( \delta g = \text{fixed} \) and \( \delta K_L = \text{fixed} \) at the boundary

Deser, Jackiw & Templeton ’82
Apply algorithm just described to flat space theories
Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions 
asymptotically flat adapted to lightlike infinity \((\varphi \sim \varphi + 2\pi)\)

\[d\tilde{s}^2 = -du^2 - 2du dr + r^2 d\varphi^2\]

\[g_{uu} = h_{uu} + O\left(\frac{1}{r}\right)\]
\[g_{ur} = -1 + h_{ur}/r + O\left(\frac{1}{r^2}\right)\]
\[g_{u\varphi} = h_{u\varphi} + O\left(\frac{1}{r}\right)\]
\[g_{rr} = h_{rr}/r^2 + O\left(\frac{1}{r^3}\right)\]
\[g_{r\varphi} = h_1(\varphi) + h_{r\varphi}/r + O\left(\frac{1}{r^2}\right)\]
\[g_{\varphi\varphi} = r^2 + (h_2(\varphi) + uh_3(\varphi))r + O(1)\]

Barnich & Compere '06
Bagchi, Detournay & DG '12
Apply algorithm just described to flat space theories
Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s

Obtain canonical boundary charges

\[ Q_{Mn} = \frac{1}{16\pi G} \int d\varphi e^{in\varphi} (h_{uu} + h_3) \]

\[ Q_{Ln} = \frac{1}{16\pi G\mu} \int d\varphi e^{in\varphi} (h_{uu} + \partial_u h_{ur} + \frac{1}{2} \partial^2_u h_{rr} + h_3) \]

\[ + \frac{1}{16\pi G} \int d\varphi e^{in\varphi} (inuh_{uu} + inh_{ur} + 2h_{w\varphi} + \partial_u h_{r\varphi} \]

\[ - (n^2 + h_3)h_1 - inh_2 - in\partial_\varphi h_1) \]

Bagchi, Detournay & DG '12
Apply algorithm just described to flat space theories

Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
4. Derive (classical) asymptotic symmetry algebra and central charges

\[
[L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0}
\]

\[
[L_n, M_m] = (n - m) M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m,0}
\]

\[
[M_n, M_m] = 0
\]

with central charges

\[
c_L = \frac{3}{\mu G} \quad c_M = \frac{3}{G}
\]

Note:

- \( c_L = 0 \) in Einstein gravity
- \( c_M = 0 \) in conformal Chern–Simons gravity (\( \mu \to 0, \mu G = \frac{1}{8k} \))

Flat space chiral gravity!
Apply algorithm just described to flat space theories
Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
   Trivial here
Apply algorithm just described to flat space theories
Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
   ▶ Straightforward in flat space chiral gravity
   ▶ Difficult/impossible otherwise
Apply algorithm just described to flat space theories
Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
7. Identify/constrain dual field theory
   Monster CFT in flat space chiral gravity
   Witten '07
   Li, Song & Strominger '08
   Bagchi, Detournay & DG '12

\[ Z(q) = J(q) = \frac{1}{q} + (1 + 196883) q + \mathcal{O}(q^2) \]

Note: \( \ln 196883 \approx 12.2 \approx 4\pi + \text{quantum corrections} \)
Apply algorithm just described to flat space theories
Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
7. Identify/constrain dual field theory
8. If unhappy with result go back to previous items and modify

We are happy!
Conjecture:

Conformal Chern–Simons gravity at level $k = 1 \simeq$ chiral extremal CFT with central charge $c = 24$

$$I_{CSG} = \frac{k}{4\pi} \int \left( \Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma \right) + \text{flat space bc's}$$
Conjecture:

Conformal Chern–Simons gravity at level $k = 1 \simeq$ chiral extremal CFT with central charge $c = 24$

\[ I_{\text{CSG}} = \frac{k}{4\pi} \int (\Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma) + \text{flat space bc}'s \]

- Symmetries match (Brown–Henneaux type of analysis)
Conjecture:

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- Symmetries match (Brown–Henneaux type of analysis)
- Trace and gravitational anomalies match
Conjecture:

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- Symmetries match (Brown–Henneaux type of analysis)
- Trace and gravitational anomalies match
- Perturbative states match (Virasoro descendants of vacuum)
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- Symmetries match (Brown–Henneaux type of analysis)
- Trace and gravitational anomalies match
- Perturbative states match (Virasoro descendants of vacuum)
- Gaps in spectra match

$Z(q) = J(q) = 1 + 196884q + O(q^2)$
Conjecture:

Conformal Chern–Simons gravity at level $k = 1 \simeq$ chiral extremal CFT with central charge $c = 24$

$$I_{\text{CSG}} = \frac{k}{4\pi} \int (\Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma) + \text{flat space bc’s}$$

- Symmetries match (Brown–Henneaux type of analysis)
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- Microscopic counting of $S_{\text{FSC}}$ reproduced by chiral Cardy formula
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- Perturbative states match (Virasoro descendants of vacuum)
- Gaps in spectra match
- Microscopic counting of $S_{FSC}$ reproduced by chiral Cardy formula
- No issues with logarithmic modes/log CFTs
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- No issues with logarithmic modes/log CFTs

Missing: full partition function on gravity side

$$Z(q) = J(q) = \frac{1}{q} + 196884 q + \mathcal{O}(q^2)$$
Cosmic evolution from phase transition
Flat space version of Hawking–Page phase transition

Hot flat space

\( d s^2 = \pm d t^2 + d r^2 + r^2 \, d \varphi^2 \)
Cosmic evolution from phase transition
Flat space version of Hawking–Page phase transition

Hot flat space

\[ ds^2 = \pm dt^2 + dr^2 + r^2 \, d\varphi^2 \]

Flat space cosmology

\[ ds^2 = \pm d\tau^2 + \frac{(E\tau)^2}{1 + (E\tau)^2} \, dx^2 + (1 + (E\tau)^2) \left( dy + \frac{(E\tau)^2}{1 + (E\tau)^2} \, dx \right)^2 \]

\[ (y \sim y + 2\pi r_0) \]

Bagchi, Detournay, DG & Simon ’13
Flat space cosmologies (Cornalba & Costa ’02)

▶ Start with BTZ in AdS:

\[
ds^2 = -\frac{(r^2 - R_+^2)(r^2 - r_-^2)}{r^2 \ell^2} \, dt^2 + \frac{r^2 \ell^2 \, dr^2}{(r^2 - R_+^2)(r^2 - r_-^2)} + r^2 \left( d\varphi - \frac{R_+ r_-}{\ell r^2} \, dt \right)^2
\]
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▶ Consider region between the two horizons \( r_- < r < R_+ \)
Flat space cosmologies (Cornalba & Costa ’02)

- Start with BTZ in AdS:
  \[ ds^2 = -\frac{(r^2 - R_+^2)(r^2 - r_-^2)}{r^2 \ell^2} \, dt^2 + \frac{r^2 \ell^2 \, dr^2}{(r^2 - R_+^2)(r^2 - r_-^2)} + r^2 \left( d\varphi - \frac{R_+ + r_-}{\ell r^2} \, dt \right)^2 \]

- Consider region between the two horizons \( r_- < r < R_+ \)

- Take the \( \ell \to \infty \) limit (with \( R_+ = \ell \hat{r}_+ \) and \( r_- = r_0 \))
  \[ ds^2 = \hat{r}_+^2 \left( 1 - \frac{r_0^2}{r^2} \right) \, dt^2 - \frac{r^2 \, dr^2}{\hat{r}_+^2 (r^2 - r_0^2)} + r^2 \left( d\varphi - \frac{\hat{r}_+ r_0}{r^2} \, dt \right)^2 \]
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\]

- Go to Euclidean signature \( (t = i \tau_E, \, \hat{r}_+ = -ir_+) \)

\[
\text{ds}^2 = r_+^2 \left( 1 - \frac{r_0^2}{r^2} \right) \, d\tau_E^2 + \frac{r^2 \, dr^2}{r_+^2 (r^2 - r_0^2)} + r^2 \left( \, d\varphi - \frac{r_+ + r_0}{r^2} \, d\tau_E \right)^2
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- Note peculiarity: no conical singularity, but asymptotic conical defect!
Flat space cosmologies (Cornalba & Costa ’02)

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\]

- Note peculiarity: no conical singularity, but asymptotic conical defect!

Question we want to address:

Is FSC or HFS the preferred Euclidean saddle?
Euclidean path integral

Evaluate Euclidean partition function in semi-classical limit

\[ Z(T, \Omega) = \int \mathcal{D}g \, e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T, \Omega)]} \times Z_{\text{fluct}}. \]

Boundary conditions specified by temperature \( T \) and angular velocity \( \Omega \).
Euclidean path integral

Evaluate Euclidean partition function in semi-classical limit

\[ Z(T, \Omega) = \int \mathcal{D}g \, e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T, \Omega)]} \times Z_{\text{fluct}}. \]

boundary conditions specified by temperature \( T \) and angular velocity \( \Omega \)

Two Euclidean saddle points in same ensemble if

- same temperature \( T = 1/\beta \) and angular velocity \( \Omega \)
- obey flat space boundary conditions
- solutions without conical singularities
Euclidean path integral

Evaluate Euclidean partition function in semi-classical limit

\[ Z(T, \Omega) = \int \mathcal{D}g \, e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T,\Omega)]} \times Z_{\text{fluct.}} \]

boundary conditions specified by temperature \( T \) and angular velocity \( \Omega \)

Two Euclidean saddle points in same ensemble if

- same temperature \( T = 1/\beta \) and angular velocity \( \Omega \)
- obey flat space boundary conditions
- solutions without conical singularities

Periodicities fixed:

\[ (\tau_E, \varphi) \sim (\tau_E + \beta, \varphi + \beta \Omega) \sim (\tau_E, \varphi + 2\pi) \]
Results

On-shell action:

\[ \Gamma = -\frac{1}{16\pi G_N} \int d^3x \sqrt{g} R - \frac{1}{16\pi G_N} \int d^2x \sqrt{\gamma} K \]

Free energy:

\[ F_{HFS} = -\frac{1}{8} G_N F_{FSC} = -r + \frac{8}{G_N} \]

\[ F_{HFS} \quad \text{dominant saddle} \]

\[ F_{FSC} \quad \text{dominant saddle} \]

Critical temperature:

\[ T_c = \frac{1}{2\pi r_0} = \frac{\Omega^2}{\pi^2} \]

HFS "melts" into FSC at \( T > T_c \)
Results

On-shell action:

\[ \Gamma = -\frac{1}{16\pi G_N} \int d^3x \sqrt{g} R - \frac{1}{16\pi G_N} \int d^2x \sqrt{\gamma} K \]

Free energy:

\[ F_{\text{HFS}} = -\frac{1}{8G_N} \quad F_{\text{FSC}} = -\frac{r_+}{8G_N} \]
Results

On-shell action:

$$\Gamma = -\frac{1}{16\pi G_N} \int d^3x \sqrt{g} R - \frac{1}{16\pi G_N} \int d^2x \sqrt{\gamma} K$$

Free energy:

$$F_{\text{HFS}} = -\frac{1}{8G_N} \quad F_{\text{FSC}} = -\frac{r_+}{8G_N}$$

- $r_+ > 1$: FSC dominant saddle
- $r_+ < 1$: HFS dominant saddle

Critical temperature:

$$T_c = \frac{1}{2\pi r_0} = \frac{\Omega}{2\pi}$$

HFS “melts” into FSC at $T > T_c$
Entanglement entropy of Galilean CFTs and flat space holography
Bagchi, Basu, DG, Riegler ’14

Using methods similar to CFT:

$$S_{EE}^{GCFT} = \frac{c_L}{6} \ln \frac{\ell_x}{a} + \frac{c_M}{6} \frac{\ell_y}{\ell_x}$$

like CFT

like grav anomaly
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\[ S_{EE}^{GCFT} = \frac{c_L}{6} \ln \frac{\ell_x}{a} \] 

like CFT

\[ + \frac{c_M}{6} \frac{\ell_y}{\ell_x} \]

like grav anomaly

with

\[ [L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0} \]

\[ [L_n, M_m] = (n - m) M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m,0} \]

and

▶ \( \ell_x \): spatial distance
▶ \( \ell_y \): temporal distance
▶ \( a \): UV cutoff (lattice size)
Using methods similar to CFT:

\[ S_{EE}^{GCFT} = \frac{c_L}{6} \ln \frac{\ell_x}{a} \]

▶ flat space chiral gravity: \( c_L \neq 0, \ c_M = 0 \)
Using methods similar to CFT:

\[
S_{EE}^{GCFT} = \frac{c_M}{6} \frac{\ell_y}{\ell_x} \langle x \rangle
\]

like grav anomaly

- flat space chiral gravity: \( c_L \neq 0, c_M = 0 \)
- flat space Einstein gravity: \( c_L = 0, c_M \neq 0 \)
Entanglement entropy of Galilean CFTs and flat space holography

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- flat space chiral gravity: \( c_L \neq 0, c_M = 0 \)
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Same results obtained holographically!

- Using methods similar to Ammon, Castro Iqbal ’13, de Boer, Jottar ’13, Castro, Detournay, Iqbal, Perlmutter ’14
- geodesics \( \Rightarrow \) Wilson lines
Unitarity in flat space
Unitarity leads to further contraction DG, Riegler, Rosseel ’14

Facts:
- Unitarity in GCA requires $c_M = 0$ (see paper for caveats!)
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- Generalization to contracted higher spin algebras straightforward
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- All of them contain GCA as subalgebra

Higher spin states decouple and become null states!
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- $c_M = 0$ is necessary for unitarity
Unitarity in flat space
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Limit $c_M \to 0$ requires further contraction: $U_n \to c_M U_n$
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Limit $c_M \to 0$ requires further contraction: $U_n \to c_M U_n$

Doubly contracted algebra has unitary representations:

\[
[L_n, L_m] = (n - m)L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0}
\]

\[
[L_n, M_m] = (n - m)M_{n+m}
\]

\[
[L_n, U_m] = (2n - m)U_{n+m}
\]

\[
[M_n, U_m] = [L_n, V_m] = (2n - m)V_{n+m}
\]

\[
[U_n, U_m] \propto [U_n, V_m] = 96(n - m) \sum_p M_p M_{n-p}
\]

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Higher spin states decouple and become null states!
1. NO–GO:
Generically (see paper) you can have only two out of three:
  ▶ Unitarity
  ▶ Flat space
  ▶ Non-trivial higher spin states
Unitarity in flat space
Generic flat space $W$-algebras DG, Riegler, Rosseel ’14

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Example:
Flat space chiral gravity
Bagchi, Detournay, DG, 1208.1658
Unitarity in flat space
Generic flat space $W$-algebras DG, Riegler, Rosseel ’14

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Example:
Minimal model holography
Gaberdiel, Gopakumar, 1011.2986, 1207.6697
Unitarity in flat space
Generic flat space $\mathcal{W}$-algebras DG, Riegler, Rosseel ’14

1. NO–GO:
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  ▶ Unitarity
  ▶ Flat space
  ▶ Non-trivial higher spin states

Example:
Flat space higher spin gravity (Galilean $\mathcal{W}_3$ algebra)
Afshar, Bagchi, Fareghbal, DG and Rosseel, 1307.4768
Gonzalez, Matulich, Pino and Troncoso, 1307.5651
Unitarity in flat space
Generic flat space $W$-algebras DG, Riegler, Rosseel ’14

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Generically (see paper) you can have only two out of three:

- Unitarity
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Compatible with “spirit” of various no-go results in higher dimensions!

2. YES–GO:
There is (at least) one counter-example, namely a Vasiliev-type of theory,
where you can have all three properties!

Unitary, non-trivial flat space higher spin algebra exists!

Vasiliev-type flat space chiral higher spin gravity?
Unitarity in flat space
Generic flat space $W$-algebras DG, Riegler, Rosseel ’14

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Unitarity in flat space
Flat space $W_\infty$-algebra compatible with unitarity DG, Riegler, Rosseel ’14

- We do not know if flat space chiral higher spin gravity exists...

\[
\begin{align*}
\left[ V_i^m, V_j^n \right] &= \lfloor \frac{i+j}{2} \rfloor \sum_{r=0} g_{ij} 2^r (m,n) V_i^m + j - 2r m + n + c_i V(m) \delta_{ij} \delta_{m+n,0} \\
\left[ W_i^m, W_j^n \right] &= 0 \\
\left[ W_i^m, V_j^n \right] &= 0
\end{align*}
\]

where $c_i V(m) = \#(i,m) \times c$ and $c = -\bar{c}$

Vacuum descendants $W_i^m |0 \rangle$ are null states for all $i$ and $m$!
Unitarity in flat space
Flat space $W_\infty$-algebra compatible with unitarity DG, Riegler, Rosseel ’14

- We do not know if flat space chiral higher spin gravity exists...
- ...but its existence is at least not ruled out by the no-go result!

\[ [V_{im}, V_{jn}] = \left\lfloor \frac{i+j}{2} \right\rfloor \sum_{r=0}^{\infty} g_{ij}^{(m,n)} V_{i+j-2r}^{m+n+c} \delta_{ij} \delta_{m+n,0} \]
\[ [W_{im}, W_{jn}] = 0 \]

where \( c_i V_{(m)} = \#(i, m) \times c \) and \( c = -\bar{c} \)

Vacuum descendants \( W_{im} |0\rangle \) are null states for all \( i \) and \( m \)!
Unitarity in flat space
Flat space $W_\infty$-algebra compatible with unitarity DG, Riegler, Rosseel ’14

- We do not know if flat space chiral higher spin gravity exists...
- ...but its existence is at least not ruled out by the no-go result!
- If it exists, this must be its asymptotic symmetry algebra:

$$\left[ \mathcal{V}_m^i, \mathcal{V}_n^j \right] = \sum_{r=0}^{\left\lfloor \frac{i+j}{2} \right\rfloor} g_{2r}^{ij}(m, n) \mathcal{V}_{m+n}^{i+j-2r} + c^i_Y(m) \delta^{ij} \delta_{m+n,0}$$

$$\left[ \mathcal{V}_m^i, \mathcal{W}_n^j \right] = \sum_{r=0}^{\left\lfloor \frac{i+j}{2} \right\rfloor} g_{2r}^{ij}(m, n) \mathcal{W}_{m+n}^{i+j-2r}$$

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\]

\[
[W^i_m, W^j_n] = \frac{i+j}{2} \sum_{r=0}^{\infty} g_{2r}^{ij}(m, n) W^{i+j-2r}_{m+n}
\]

where

\[
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[V^i_m, V^j_n] &= \sum_{r=0}^{\lfloor\frac{i+j}{2}\rfloor} g_{2r}^{ij}(m, n) V^{i+j-2r}_{m+n} + c_V^i(m) \delta^{ij} \delta_{m+n,0} \\
[W^i_m, W^j_n] &= \sum_{r=0}^{\lfloor\frac{i+j}{2}\rfloor} g_{2r}^{ij}(m, n) W^{i+j-2r}_{m+n} \\
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where

\[c_V^i(m) = \#(i, m) \times c \quad \text{and} \quad c = -\bar{c}\]

- Vacuum descendants $\mathcal{W}_m^i |0\rangle$ are null states for all $i$ and $m$!
- AdS parent theory: no trace anomaly, but gravitational anomaly
  (Like in conformal Chern–Simons gravity $\rightarrow$ Vasiliev type analogue?)
Adding chemical potentials Gary, DG, Riegler, Rosseel ’14

Long story short:
Adding chemical potentials Gary, DG, Riegler, Rosseel ’14

Long story short:

\[ A_u \rightarrow A_u + \mu \]

Works nicely in Chern–Simons formulation!
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Works nicely in Chern–Simons formulation!
Line-element with spin-2 and spin-3 chemical potentials:

\[
g_{\mu \nu} \, dx^\mu \, dx^\nu = \left( r^2 \left( \mu_L^2 - 4 \mu'' U \mu_U + 3 \mu_U^2 + 4 \mathcal{M} \mu_U^2 \right) + r g_{uu}^{(r)} + g_{uu}^{(0)} + g_{uu}^{(0')} \right) \, du^2 + \\
\left( r^2 \mu_L - r \mu'_M + N (1 + \mu_M) + 8 \mathcal{Z} \mu_V \right) \, 2 \, du \, d\varphi - (1 + \mu_M) \, 2 \, dr \, du + r^2 \, d\varphi^2
\]

\[
g_{uu}^{(0)} = \mathcal{M} (1 + \mu_M)^2 + 2 (1 + \mu_M) (N \mu_L + 12 \mathcal{V} \mu_V + 16 \mathcal{Z} \mu_U) + 16 \mathcal{Z} \mu_L \mu_V + \frac{4}{3} (\mathcal{M}^2 \mu_V^2 + 4 \mathcal{M} N \mu_U \mu_V + N^2 \mu_U^2)
\]

Spin-3 field with same chemical potentials:

\[
\Phi_{\mu \nu \lambda} \, dx^\mu \, dx^\nu \, dx^\lambda = \Phi_{uuu} \, du^3 + \Phi_{ruu} \, dr \, du^2 + \Phi_{uuu} \varphi \, du^2 \, d\varphi - (2 \mu_U r^2 - r \mu'_V + 2 N \mu_V) \, dr \, du \, d\varphi
\]

\[
+ \mu_V \, dr^2 \, du - (\mu'_U r^3 - \frac{1}{3} r^2 (\mu''_V - \mathcal{M} \mu_V + 4 N \mu_U) + r N \mu'_V - N^2 \mu_V) \, du \, d\varphi^2
\]

\[
\Phi_{uuu} = r^2 \left[ 2 (1 + \mu_M) \mu_U (\mathcal{M} \mu_L - 4 \mathcal{V} \mu_U) - \frac{1}{3} \mu_L^2 (\mathcal{M} \mu_V - 4 \mathcal{N} \mu_U) + 16 \mu_L \mu_U (\mathcal{V} \mu_V + \mathcal{Z} \mu_U) - \frac{4}{3} \mathcal{M} \mu^2_U (\mathcal{M} \mu_V + 2 \mathcal{N} \mu_U) \right] + 2 \mathcal{V} (1 + \mu_M)^3 + \frac{2}{3} (1 + \mu_M)^2 (6 \mathcal{Z} \mu_L + \mathcal{M}^2 \mu_V + 2 \mathcal{M} N \mu_U) + 16 \mu_L \mu_V (\mathcal{N} \mu_V - \frac{1}{3} \mathcal{M} \mathcal{Z}) + \frac{2}{3} (1 + \mu_M) (N \mu_L + 16 \mathcal{Z} \mu_U) (2 \mathcal{M} \mu_V + N \mu_U) + 12 \mathcal{M} \mu_V^2 + \frac{64}{3} \mathcal{Z} \mu_U \mu_V (N \mu_L + 12 \mathcal{V} \mu_V + 12 \mathcal{Z} \mu_U) + N^2 \mu_L \mu_V + 64 \mathcal{V}^2 \mu_V^3 - \frac{8}{27} (\mathcal{M}^3 \mu_V^3 - N^3 \mu_U^3) - \frac{4}{9} \mathcal{M} N \mu_U \mu_V (4 \mathcal{M} \mu_V + 5 N \mu_U) + \sum_{n=0}^{3} r^n \Phi^{(n)}_{uuu}
\]
Adding chemical potentials Gary, DG, Riegler, Rosseel ’14

Long story short:

\[ A_u \rightarrow A_u + \mu \]

Works nicely in Chern–Simons formulation!

Interesting novel phase transitions of zeroth/first order:

Free energy of four branches of regular solutions as function of temperature for different values of higher spin chemical potential ratio (in AdS: see David, Ferlaino, Kumar ’12)
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