How general is holography?

April 2015

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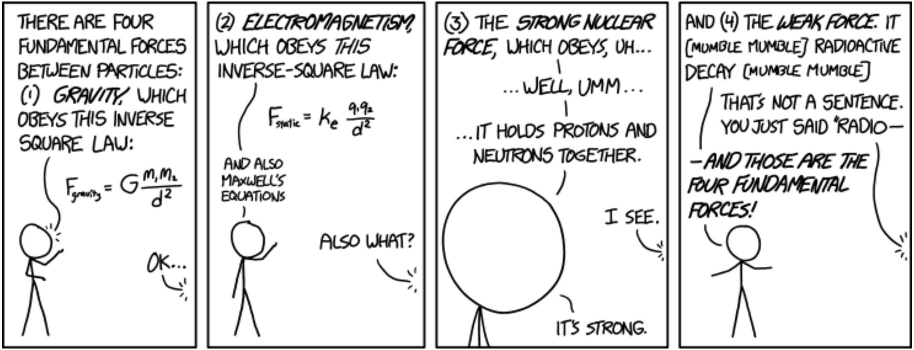
TU Wien

Seminar talk at CTP, MIT

Fundamental forces (xkcd 1489)



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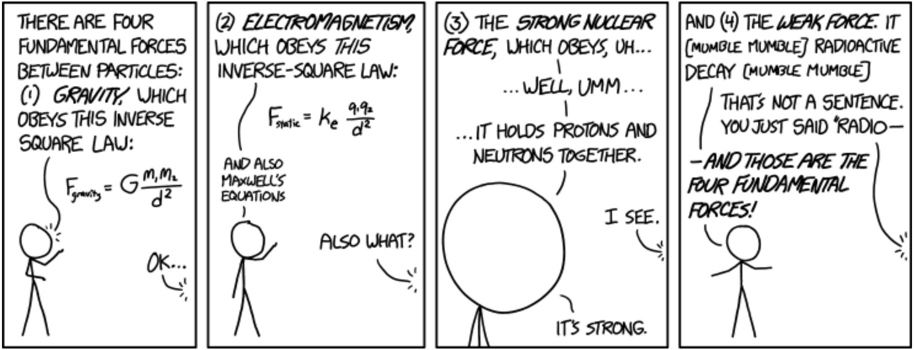

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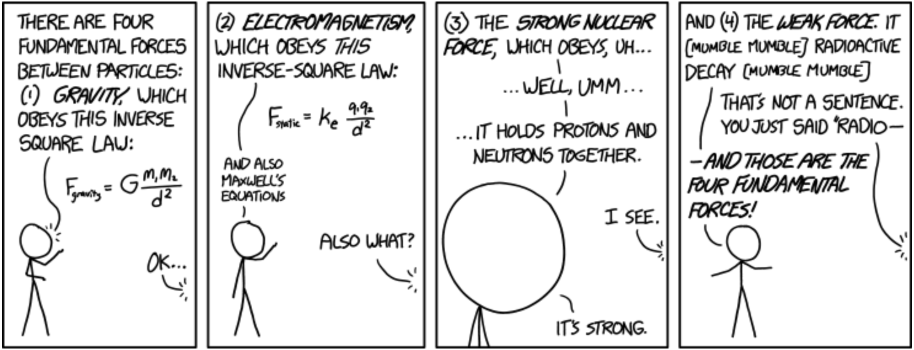

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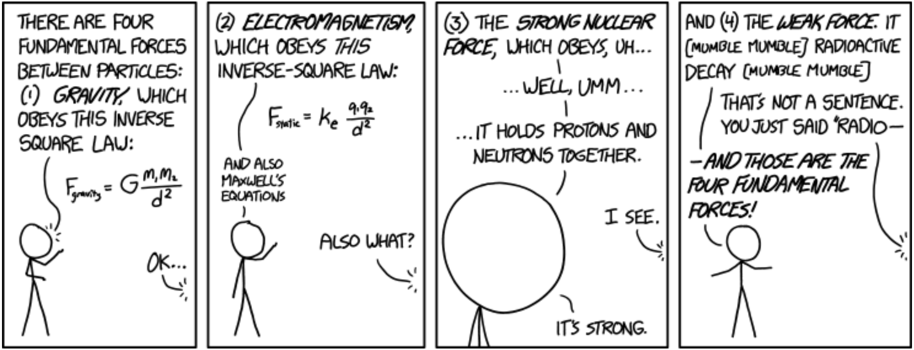

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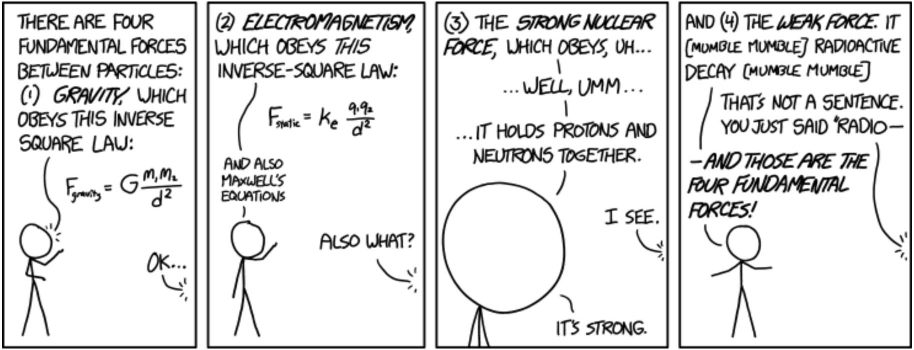

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Main goal: understand quantum gravity

Outline

Motivation

Holography in 3d

Chern–Simons formulation

Asymptotic symmetries

Example: flat space holography

Outlook - how general is holography?

General motivations

- Quantum gravity
  - Address conceptual issues of quantum gravity

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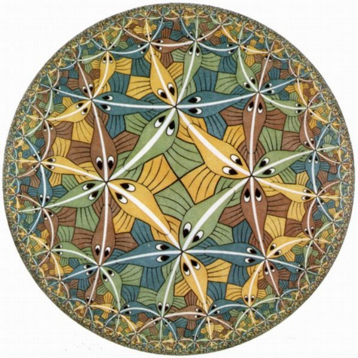

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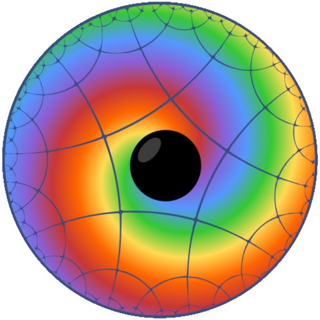

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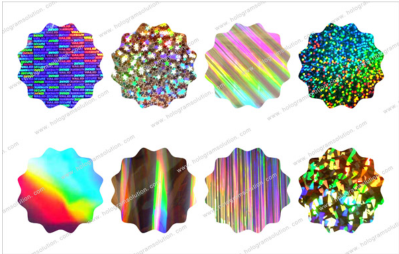
  - How general is holography? (non-unitary holography, higher spin holography, flat space holography, non-AdS holography, ...)



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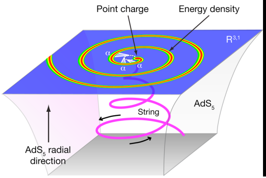
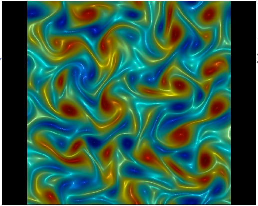
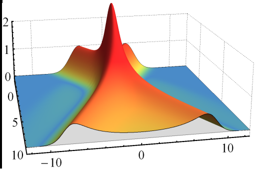
  - How general is holography? (non-unitary holography, higher spin holography, flat space holography, non-AdS holography, ...)
- Applications (will not address them in my talk)
  - Gauge gravity correspondence (non-abelian plasmas, condensed matter)







Testing the holographic principle

How general is holography?

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- To what extent do (previous) lessons rely on the particular constructions used to date?
- Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?

see numerous talks at KITP workshop “Bits, Branes, Black Holes” 2012

and at ESI workshop “Higher Spin Gravity” 2012

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- plausible AdS/CFT-like correspondence could work non-unitarily
- AdS/log CFT first example of non-unitary holography DG, (Jackiw), Johansson '08; Skenderis, Taylor, van Rees '09; Henneaux, Martinez, Troncoso '09; Maloney, Song, Strominger '09; DG, Sachs/Hohm '09; Gaberdiel, DG, Vassilevich '10; ... DG, Riedler, Rosseel, Zojer '13
- recent proposal by Vafa '14

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- Can we establish a flat space holographic dictionary?

the answer appears to be yes — see my current talk and recent papers by Bagchi et al., Barnich et al., Strominger et al., '12-'15

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- Generic non-AdS holography/higher spin holography?

non-trivial hints that it might work at least in 2+1 dimensions
Gary, DG Rashkov '12; Afshar et al '12; Gutperle et al '14-'15; Gary, DG, Prohazka, Rey '14; ...

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- Address questions above in simple class of 3d toy models
- Exploit gauge theoretic Chern–Simons formulation
- Restrict to kinematic questions, like (asymptotic) symmetries

Outline

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Holography in 3d

- Chern–Simons formulation
- Asymptotic symmetries
- Example: flat space holography

Outlook - how general is holography?

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Clarification of nomenclature

- Conformal CS gravity (Deser, Jackiw, Templeton '82)

$$I_{\text{CSG}} = \frac{k}{4\pi} \int \mathrm{d}^3x\sqrt{-g}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)$$

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$$I_{\rm EH} = \frac{1}{16\pi G_N} \int \mathrm{d}^3x\sqrt{-g}\big(R + \frac{2}{\ell^2}\big) \sim I_{\rm CS}(A) - I_{\rm CS}(\bar{A})$$

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- Gravitational CS term in topologically massive gravity (Deser, Jackiw, Templeton '82)

$$I_{\rm TMG}=I_{\rm EH}+I_{\rm CSG}$$

  $0+0=1$ local physical degree of freedom (massive graviton)

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$$I_{\text{TMG}} = I_{\text{EH}} + I_{\text{CSG}}$$

  $0 + 0 = 1$ local physical degree of freedom (massive graviton)
- This talk: gravity-like CS theories

CS bulk theory

Action:

$$I_{\rm CS} = \frac{k}{4\pi} \int_{\mathcal{M}} \langle A \wedge \mathrm{d}A + \tfrac{2}{3}\, A \wedge A \wedge A \rangle$$

- k: CS-level
- $\mathcal{M}$: 3d or (2+1)d manifold (this talk: filled cylinder or filled torus)
- $A = A_\mu^a T^a \, \mathrm{d}x^\mu$: (non-abelian) connection 1-form
- $\langle , \rangle$: bilinear form

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EOM:

$$F = \mathrm{d}A + [A,\, A] = 0$$

Solutions: (locally) gauge-flat connections, $A = g^{-1}\, \mathrm{d}g$

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- Chern–Simons theory locally trivial
- Boundary conditions/fall-off behavior crucial

Overview of gravity-like CS theories

(spin-2) gravity

- with negative cosmological constant: $sl(2) \oplus sl(2)$ with suitable bc's
- in flat space: $isl(2)$ with suitable bc's

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generic higher spin/lower spin gravity

- higher spin with negative cosmological constant: some gauge algebra containing $sl(2) \oplus sl(2)$ with suitable bc's (e.g. $sl(N) \oplus sl(N)$)
- higher spin in flat space: some gauge algebra containing $isl(2)$ with suitable bc's (e.g. $isl(N)$)
- higher spin in Lobachevsky/warped AdS/Schrödinger/Lifshitz: some gauge algebra containing $sl(2) \oplus sl(2)$ with suitable bc's
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Vasiliev type higher spin gravity

- with negative cosmological constant: $hs(\lambda) \oplus hs(\lambda)$ with suitable bc's
- in flat space: probably exists?

Gravity-like CS with asymptotic boundary

Spin-2: Bañados '94; Spin-3: Henneaux, Rey '10; Campoleoni et al '10

Assume cylinder topology:

radius: ρ

boundary: $\rho \to \infty$

boundary coord's: x^i

- Impose fall-off conditions on connection

$$\lim_{\rho\to\infty} A(\rho, x^i) = \underbrace{\widehat{A}^a_\mu(\rho,\, x^i)}_{\text{asympt. bg.}} T^a \, \mathrm{d}x^\mu + \underbrace{\delta A(\rho,\, x^i)}_{\text{state dep.}} + \dots$$

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- Connection $a = \widehat{a}_i\, \mathrm{d}x^i + \delta a_i\, \mathrm{d}x^i$ subject to asymptotic on-shell conditions

$$F_{ij} = 0 \quad \leftrightarrow \quad \mathrm{d}a + [a,\, a] = 0$$

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- Subleading fluctuation terms $o(1)$ irrelevant

(Non-)equivalence of gravity-like CS to gravity

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- Need suitable gauge algebra
- Need appropriate boundary conditions on connection

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Spin-2 field:

$$g_{\mu\nu} = \frac{1}{2}\,\widetilde{\mathrm{tr}}\,\big(A_\mu A_\nu\big)$$

Spin-3 field:

$$\Phi_{\mu\nu\lambda} = \frac{1}{6}\,\widetilde{\mathrm{tr}}\,\big(A_\mu A_\nu A_\lambda\big)$$

etc.

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- Classically equivalent to gravity(-like) theory
- Probably quantum inequivalent
- Debatable which version is correct at quantum level

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- Analogous choices in bar-sector

$$\bar{A} = b\left(\mathrm{d} + (L_- + \bar{\mathcal{L}}(x^-) L_+ + \dots)\, \mathrm{d}x^-\right) b^{-1}$$

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$$b = e^{\rho L_0}$$

- Choose

$$a = (L_+ + \mathcal{L}(x^+) L_-)\, \mathrm{d}x^+ + \mathcal{O}(e^{-2\rho})$$

- Full connection (remember, $A = b^{-1}(d + a)b$):

$$A = L_0 \, \mathrm{d}\rho + e^{\rho} L_+ \, \mathrm{d}x^+ + e^{-\rho} \mathcal{L}(x^+) L_- \, \mathrm{d}x^+ + \dots$$

- Analogous choices in bar-sector

$$\bar{A} = b \left(\mathrm{d} + (L_- + \bar{\mathcal{L}}(x^-) L_+ + \dots)\, \mathrm{d}x^- \right) b^{-1}$$

- Metric:

$$g_{\mu\nu} = \frac{1}{2} \operatorname{tr} \left((A_\mu - \bar{A}_\mu)(A_\nu - \bar{A}_\nu) \right)$$

Best known example: AdS_3 spin-2 gravity (Bañados '94)

- Gauge algebra: $sl(2) \oplus sl(2)$
- Split full connection into left (A) and right ($\bar{A}$) moving part
- Choose

$$b = e^{\rho L_0}$$

- Choose

$$a = (L_+ + \mathcal{L}(x^+) L_-)\, \mathrm{d}x^+ + \mathcal{O}(e^{-2\rho})$$

- Full connection (remember, $A = b^{-1}(d+a)b$):

$$A = L_0 \,\mathrm{d}\rho + e^{\rho} L_+ \,\mathrm{d}x^+ + e^{-\rho} \mathcal{L}(x^+) L_- \,\mathrm{d}x^+ + \dots$$

- Analogous choices in bar-sector

$$\bar{A} = b \left(\mathrm{d} + (L_- + \bar{\mathcal{L}}(x^-) L_+ + \dots)\, \mathrm{d}x^- \right) b^{-1}$$

- Metric:

$$g_{\mu\nu} = \frac{1}{2} \operatorname{tr} \left((A_\mu - \bar{A}_\mu)(A_\nu - \bar{A}_\nu) \right)$$

- Boundary conditions above = partly gauge-fixed Brown–Henneaux:

$$g_{\mu\nu}\, \mathrm{d}x^\mu \,\mathrm{d}x^\nu = \mathrm{d}\rho^2 + 2e^{2\rho}\, \mathrm{d}x^+ \,\mathrm{d}x^- + \mathcal{L}(x^+)\, (\mathrm{d}x^+)^2 + \bar{\mathcal{L}}(x^-)\, (\mathrm{d}x^-)^2 + \dots$$

Outline

Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle

   Essentially did this: CS theory with suitable gauge algebra (will not talk about variational principle)

Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions

   Essentially did this: $A = b^{-1}\big(\mathrm{d} + \widehat{a} + \delta a + o(1)\big)b$

   Still need to choose asymptotic background $\widehat{a}$ and state dependent fluctuations δa

Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
    - Find and classify all constraints
    - Construct canonical gauge generators
    - Add boundary terms and get (variation of) canonical charges
    - Check integrability of canonical charges
    - Check finiteness of canonical charges
    - Check conservation (in time) of canonical charges
    - Calculate Dirac bracket algebra of canonical charges

Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
4. Derive (classical) asymptotic symmetry algebra and central charges

   Reminder: ASA = quotient algebra of asymptotic symmetries by 'trivial' asymptotic symmetries with zero canonical charges

Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
   Example: semi-classical ASA in spin-3 gravity (Henneaux, Rey '10; Campoleoni, Pfenninger, Fredenhagen, Theisen '10)

$$[W_n,\,W_m]=\frac{16}{5c}\sum_p L_p L_{n+m-p}+\dots$$

   quantum ASA

$$[W_n,\,W_m]=\frac{16}{5c+22}\sum_p :L_p L_{n+m-p}:+\dots$$

Holographic algorithm from bulk point of view

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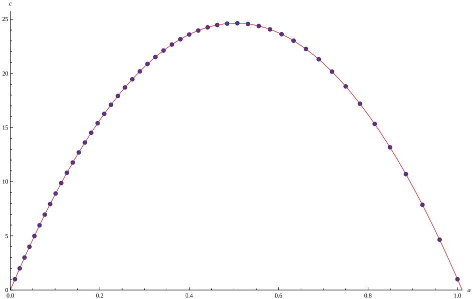
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
6. Study unitary representations of quantum ASA

Example:



Afshar et al '12

Discrete set of Newton constant values compatible with unitarity
(3D spin-N gravity in next-to-principal embedding)
see my talk at MIT March '13

Holographic algorithm from bulk point of view

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
4. Derive (classical) asymptotic symmetry algebra and central charges
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6. Study unitary representations of quantum ASA
7. Identify/constrain dual field theory

Holographic algorithm from bulk point of view

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8. If unhappy with result go back to previous items and modify

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8. If unhappy with result go back to previous items and modify

Previous Ansatz for connection simplifies above algorithm considerably!

Boundary condition preserving trafos and canonical charges

Generic (non-)AdS holography in higher spin gravity: see Afshar et al '12

- Boundary-condition preserving transformations generated by ϵ:

$$\delta_\epsilon A = \mathrm{d}A + [\epsilon,\, A] = \mathcal{O}(\delta A)$$

Boundary condition preserving trafos and canonical charges

Generic (non-)AdS holography in higher spin gravity: see Afshar et al '12

- Boundary-condition preserving transformations generated by ϵ:

$$\delta_\epsilon A = \mathrm{d}A + [\epsilon,\, A] = \mathcal{O}(\delta A)$$

- Exploit Ansatz:

$$\epsilon = b^{-1}(\rho)\,\varepsilon(x^i)\,b(\rho) + \dots$$

Boundary condition preserving trafos and canonical charges

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- Background independent canonical analysis yields canonical currents:

$$\delta Q[\epsilon] = \frac{k}{2\pi}\,\lim_{\rho\to\infty}\oint\langle\epsilon\,\delta A\rangle = \frac{k}{2\pi}\,\oint\langle\varepsilon\,\delta a\rangle$$

Boundary condition preserving trafos and canonical charges

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  - Manifestly finite! (all $b(\rho)$ cancel)
  - Non-trivial?
  - Integrable to canonical charges $Q[\epsilon]$?
  - Conserved?

Boundary condition preserving trafos and canonical charges

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  - Non-trivial?
  - Integrable to canonical charges $Q[\epsilon]$?
  - Conserved?

If any of these is answered with 'no' then back to square one in algorithm

Otherwise: may have new holographic correspondence!

Outline

Motivation

Holography in 3d

Chern–Simons formulation

Asymptotic symmetries

Example: flat space holography

Outlook - how general is holography?

Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true ⇒ must work in flat space

Just take large AdS radius limit of 10^4 AdS/CFT papers?

Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

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- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

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- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

$$[M_n,\, M_m] = 0$$

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- This is nothing but the BMS_3 algebra (or GCA_2, URCA_2, CCA_2)!

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- Example where it does not work easily: boundary conditions!

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$$[M_n,\, M_m] = 0$$

- This is nothing but the BMS_3 algebra (or GCA_2, URCA_2, CCA_2)!
- Example where it does not work easily: boundary conditions!
- Example where it does not work at all: highest weight conditions!

Flat space holography (Barnich et al, Bagchi et al, Strominger et al, ...)

if holography is true $\Rightarrow$ must work in flat space

Just take large AdS radius limit of 10^4 AdS/CFT papers?

Not in general! Must (also) work intrinsically in flat space!
Interesting example:

- unitarity of flat space quantum gravity

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- extrapolate from AdS: should be unitary (?)
- extrapolate from dS: should be non-unitary (?)

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- directly in flat space: both options realized, depending on details of model

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Many open issues in flat space holography!

Next few slides: mention a couple of recent results

Overview of selected recent results

- Applying algorithm just described to flat space theories

  Barnich, Gonzalez '13; Afshar '13

Overview of selected recent results

- Applying algorithm just described to flat space theories
- Flat space chiral gravity

  Bagchi, Detournay, DG '13

Overview of selected recent results

- Applying algorithm just described to flat space theories
- Flat space chiral gravity
- Cosmic evolution from phase transition

Bagchi, Detournay, DG, Simon '13

Overview of selected recent results

- Applying algorithm just described to flat space theories
- Flat space chiral gravity
- Cosmic evolution from phase transition
- (Holographic) entanglement entropy

Bagchi, Basu, DG, Riegler '14

Overview of selected recent results

- Applying algorithm just described to flat space theories
- Flat space chiral gravity
- Cosmic evolution from phase transition
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- Flat space higher spin gravity

Afshar, Bagchi, Fareghbal, DG, Rosseel '13
Gonzalez, Matulich, Pino, Troncoso '13

Overview of selected recent results

- Applying algorithm just described to flat space theories
- Flat space chiral gravity
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- Flat space higher spin gravity
- Unitarity of dual field theory

DG, Riegler, Rosseel '14

Overview of selected recent results

- Applying algorithm just described to flat space theories
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- Unitarity of dual field theory
- Adding chemical potentials

Gary, DG, Riegler, Rosseel '14

Overview of selected recent results

- Applying algorithm just described to flat space theories
- Flat space chiral gravity
- Cosmic evolution from phase transition
- (Holographic) entanglement entropy
- Flat space higher spin gravity
- Unitarity of dual field theory
- Adding chemical potentials

> - See backup slides or discuss with me privately!
> - Focus here on flat space higher spin gravity

Flat space higher spin gravity

Afshar, Bagchi, Fareghbal, DG, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13

> Interacting theories of massless higher spin fields heavily constrained by no-go results!

- Coleman, Mandula '67
- Haag, Lopuszanski, Sohnius '75
- Aragone, Deser '79
- Weinberg, Witten '80
- ...
- review: Bekaert, Boulanger, Sundell '10

Vasiliev '90: circumvents no-go's by going to (A)dS

we circumvent them by going to 3d (no local physical degrees of freedom)

Flat space higher spin gravity

Afshar, Bagchi, Fareghbal, DG, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13

- AdS gravity in CS formulation: spin 2 $\to$ spin 3 $\sim$ sl$(2) \to$ sl(3)

Flat space higher spin gravity

Afshar, Bagchi, Fareghbal, DG, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13

- AdS gravity in CS formulation: spin 2 $\to$ spin 3 $\sim$ sl$(2) \to$ sl(3)
- Flat space: similar!

$$S_{\text{CS}}^{\text{flat}} = \frac{k}{4\pi} \int \text{CS}(\mathcal{A})$$

with isl(3) connection ($e^a =$ "zuvielbein")

$$\mathcal{A} = e^a T_a + \omega^a J_a \qquad T_a = (M_n,\, V_m) \quad J_a = (L_n,\, U_m)$$

isl(3) algebra (spin 3 extension of global part of BMS/GCA algebra)

$$[L_n,\, L_m] = (n-m) L_{n+m}$$
$$[L_n,\, M_m] = (n-m) M_{n+m}$$
$$[L_n,\, U_m] = (2n-m) U_{n+m}$$
$$[M_n,\, U_m] = [L_n,\, V_m] = (2n-m) V_{n+m}$$
$$[U_n,\, U_m] = (n-m)(2n^2+2m^2-nm-8) L_{n+m}$$
$$[U_n,\, V_m] = (n-m)(2n^2+2m^2-nm-8) M_{n+m}$$

Flat space higher spin gravity

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- Same type of boundary conditions as for spin 2:

$$\mathcal{A}(r,\, t,\, \varphi) = b^{-1}(r)\,\big(\,\text{d}+a(t,\, \varphi) + o(1)\big)\, b(r)$$

Flat space higher spin gravity

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$$\mathcal{A}(r,\,t,\,\varphi) = b^{-1}(r)\,\big(\,{\rm d}{+}a(t,\,\varphi) + o(1)\big)\,b(r)$$

- Flat space boundary conditions: $b(r) = \exp\big(\frac{1}{2}\,rM_{-1}\big)$ and

$$a(t,\,\varphi) = \big(M_1 - M(\varphi)M_{-1} - V(\varphi)V_{-2}\big)\,{\rm d}t$$
$$+ \big(L_1 - M(\varphi)L_{-1} - V(\varphi)U_{-2} - L(\varphi)M_{-1} - Z(\varphi)V_{-2}\big)\,{\rm d}\varphi$$

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- spin-2 and spin-3 charges:

$$Q[\varepsilon_M,\, \varepsilon_L,\, \varepsilon_V,\, \varepsilon_U] \sim \oint \big(\varepsilon_M(\varphi) M(\varphi) {+} \varepsilon_L(\varphi) L(\varphi) {+} \varepsilon_V(\varphi) V(\varphi) {+} \varepsilon_U(\varphi) U(\varphi)\big)$$

I will skip this slide

Defining $\langle , \rangle$ and $\widetilde{\mathrm{tr}}$ using Grassmann trick by Krishnan, Raju, Roy '13

- isl(n) and BMW$_n$ have $\mathbb{Z}_2$ grading
- even generators L_n, U_n, ...: $\mathrm{ad}[sl(n)] \otimes 1\!\!1_{2\times 2}$
- odd generators M_n, V_n, ...: $\epsilon \cdot \mathrm{ad}[sl(n)] \otimes \sigma_3$ with $\epsilon^2 = 0$
- reproduces isl(n) algebra from sl(n) algebra
- bilinear form between two generators G_{n_1}, G_{n_2}:

$$\langle G_{n_1}, G_{n_2} \rangle = \frac{\mathrm{d}}{\mathrm{d}\epsilon} \mathrm{tr}\big(\tfrac{1}{2} G_{n_1} \tfrac{1}{2} G_{n_2} \gamma^*\big)$$

  where $\gamma^* = 1\!\!1 \otimes \sigma_3$

- tilde-trace of product of m generators G_{n_1}, ..., G_{n_m}:

$$\widetilde{\mathrm{tr}}\Big(\prod_{i=1}^{m} G_{n_i}\Big) = \frac{1}{2} \mathrm{tr}\Big(\prod_{i=1}^{m} \big(\frac{\mathrm{d}}{\mathrm{d}\epsilon} G_{n_i} \gamma^*\big)\Big)$$

- for further details see Gary, DG, Riegler, Rosseel '14

Flat space higher spin gravity

Asymptotic symmetry algebra at finite level k Afshar, Bagchi, Fareghbal, DG, Rosseel '13

- Do either Brown–Henneaux type of analysis or İnönü–Wigner contraction of two copies of quantum W_3-algebra

Flat space higher spin gravity

Asymptotic symmetry algebra at finite level k Afshar, Bagchi, Fareghbal, DG, Rosseel '13

- Do either Brown–Henneaux type of analysis or İnönü–Wigner contraction of two copies of quantum W_3-algebra
- Obtain new type of W-algebra as extension of BMS ("BMW")

$$[L_n,\, L_m] = (n-m)L_{n+m} + \frac{c_L}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

$$[L_n,\, M_m] = (n-m)M_{n+m} + \frac{c_M}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

$$[U_n,\, U_m] = (n-m)(2n^2+2m^2-nm-8)L_{n+m} + \frac{192}{c_M}(n-m)\Lambda_{n+m} - \frac{96\big(c_L+\frac{44}{5}\big)}{c_M^2}(n-m)\Theta_{n+m} + \frac{c_L}{12}\,n(n^2-1)(n^2-4)\,\delta_{n+m,\,0}$$

$$[U_n,\, V_m] = (n-m)(2n^2+2m^2-nm-8)M_{n+m} + \frac{96}{c_M}(n-m)\Theta_{n+m} + \frac{c_M}{12}\,n(n^2-1)(n^2-4)\,\delta_{n+m,\,0}$$

$$\Lambda_n = \sum_p :L_p\,M_{n-p}: \; - \tfrac{3}{10}\,(n+2)(n+3)M_n \qquad \Theta_n = \sum_p M_p\,M_{n-p}$$

other commutators as in isl(3) with $n \in \mathbb{Z}$

Flat space higher spin gravity

Asymptotic symmetry algebra at finite level k Afshar, Bagchi, Fareghbal, DG, Rosseel '13

- Do either Brown–Henneaux type of analysis or İnönü–Wigner contraction of two copies of quantum W_3-algebra
- Obtain new type of W-algebra as extension of BMS ("BMW")

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- Note quantum shift and poles in central terms!

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$$

- Note quantum shift and poles in central terms!
- Analysis generalizes to flat space contractions of other W-algebras

Outline

Motivation

Holography in 3d

Chern–Simons formulation

Asymptotic symmetries

Example: flat space holography

Outlook - how general is holography?

Selected open issues

We have answered an ϵ of the open questions.

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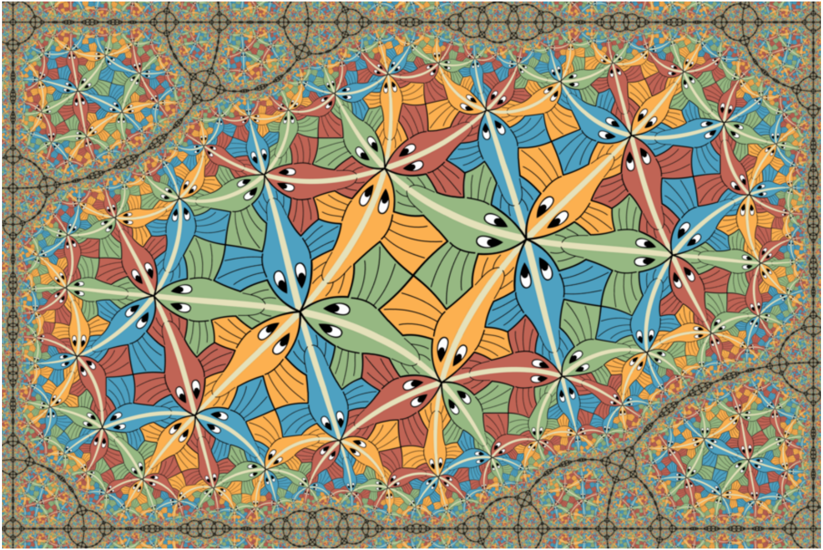
- ...

- Dimensions > 3? (Barnich et al '10-'15; Strominger et al '14-'15)
- Flat limit of $\text{AdS}_5 \times S^5/\text{CFT}_4$? (Polchinski; Susskind; Giddings '99)
- Many open issues that can and should be addressed!

Thanks for your attention!



Vladimir Bulatov, M.C.Escher Circle Limit III in a rectangle

Selected references to own work

- M. Gary, D. Grumiller, M. Riegler and J. Rosseel, "Flat space (higher spin) gravity with chemical potentials," JHEP **1501** (2015) 152, `arXiv:1411.3728`.
- A. Bagchi, R. Basu, D. Grumiller and M. Riegler, "Entanglement entropy in Galilean conformal field theories and flat holography," Phys. Rev. Lett. **114** (2015) 111602, `arXiv:1410.4089`.
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- A. Bagchi, S. Detournay, D. Grumiller and J. Simon, "Cosmic Evolution from Phase Transition of Three-Dimensional Flat Space," Phys. Rev. Lett. **111** (2013) 181301, `arXiv:1305.2919`.
- A. Bagchi, S. Detournay and D. Grumiller, "Flat-Space Chiral Gravity," Phys. Rev. Lett. **109** (2012) 151301, `arXiv:1208.1658`.

Apply algorithm just described to flat space theories

Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
   Topologically massive gravity with mixed boundary conditions

$$I = I_{\text{EH}} + \frac{1}{32\pi\, G\mu} \int \mathrm{d}^3x\sqrt{-g}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)$$

   with $\delta g = \text{fixed}$ and $\delta K_L = \text{fixed}$ at the boundary
   Deser, Jackiw & Templeton '82

Apply algorithm just described to flat space theories
Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
   asymptotically flat adapted to lightlike infinity $(\varphi \sim \varphi + 2\pi)$

$$\mathrm{d}\bar{s}^2 = -\,\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\;\mathrm{d}\varphi^2$$

$$\begin{aligned}
g_{uu} &= h_{uu} + O(\tfrac{1}{r}) \\
g_{ur} &= -1 + h_{ur}/r + O(\tfrac{1}{r^2}) \\
g_{u\varphi} &= h_{u\varphi} + O(\tfrac{1}{r}) \\
g_{rr} &= h_{rr}/r^2 + O(\tfrac{1}{r^3}) \\
g_{r\varphi} &= h_1(\varphi) + h_{r\varphi}/r + O(\tfrac{1}{r^2}) \\
g_{\varphi\varphi} &= r^2 + (h_2(\varphi) + u h_3(\varphi))r + O(1)
\end{aligned}$$

Barnich & Compere '06
Bagchi, Detournay & DG '12

Apply algorithm just described to flat space theories

Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
   Obtain canonical boundary charges

$$Q_{M_n} = \frac{1}{16\pi G} \int \mathrm{d}\varphi \, e^{in\varphi} \, \big(h_{uu} + h_3\big)$$

$$Q_{L_n} = \frac{1}{16\pi G\mu} \int \mathrm{d}\varphi \, e^{in\varphi} \, \big(h_{uu} + \partial_u h_{ur} + \tfrac{1}{2}\partial_u^2 h_{rr} + h_3\big)$$
$$+ \frac{1}{16\pi G} \int \mathrm{d}\varphi \, e^{in\varphi} \, \big(inuh_{uu} + inh_{ur} + 2h_{u\varphi} + \partial_u h_{r\varphi}$$
$$- (n^2 + h_3)h_1 - inh_2 - in\partial_\varphi h_1\big)$$

Bagchi, Detournay & DG '12

Apply algorithm just described to flat space theories
Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
4. Derive (classical) asymptotic symmetry algebra and central charges

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$
$$[L_n,\, M_m] = (n-m)\, M_{n+m} + \frac{c_M}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$
$$[M_n,\, M_m] = 0$$

with central charges

$$c_L = \frac{3}{\mu G} \qquad c_M = \frac{3}{G}$$

Note:

- $c_L = 0$ in Einstein gravity
- $c_M = 0$ in conformal Chern–Simons gravity ($\mu \to 0$, $\mu G = \frac{1}{8k}$)
  Flat space chiral gravity!

Apply algorithm just described to flat space theories
Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
   Trivial here

Apply algorithm just described to flat space theories

Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
    - Straightforward in flat space chiral gravity
    - Difficult/impossible otherwise

Apply algorithm just described to flat space theories

Flat space Einstein and chiral gravity

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2. Fix background and impose suitable boundary conditions
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5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
7. Identify/constrain dual field theory
   Monster CFT in flat space chiral gravity
   Witten '07
   Li, Song & Strominger '08
   Bagchi, Detournay & DG '12

$$Z(q) = J(q) = \frac{1}{q} + (1 + 196883)\, q + \mathcal{O}(q^2)$$

Note: $\ln 196883 \approx 12.2 = 4\pi + \text{quantum corrections}$

Apply algorithm just described to flat space theories

Flat space Einstein and chiral gravity

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
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7. Identify/constrain dual field theory
8. If unhappy with result go back to previous items and modify

   We are happy!

Flat space chiral gravity

Bagchi, Detournay, DG '12

Conjecture:

> Conformal Chern–Simons gravity at level $k = 1 \simeq$ chiral extremal CFT with central charge $c = 24$

$$I_{\text{CSG}} = \frac{k}{4\pi} \int \left(\Gamma \wedge \mathrm{d}\Gamma + \tfrac{2}{3}\Gamma \wedge \Gamma \wedge \Gamma\right) + \text{flat space bc's}$$

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Missing: full partition function on gravity side

$$Z(q) = J(q) = \frac{1}{q} + 196884\, q + \mathcal{O}(q^2)$$

Cosmic evolution from phase transition

Flat space version of Hawking–Page phase transition

Hot flat space $(\varphi \sim \varphi + 2\pi)$

$$\mathrm{d}s^2 = \pm\,\mathrm{d}t^2 + \mathrm{d}r^2 + r^2\;\mathrm{d}\varphi^2$$

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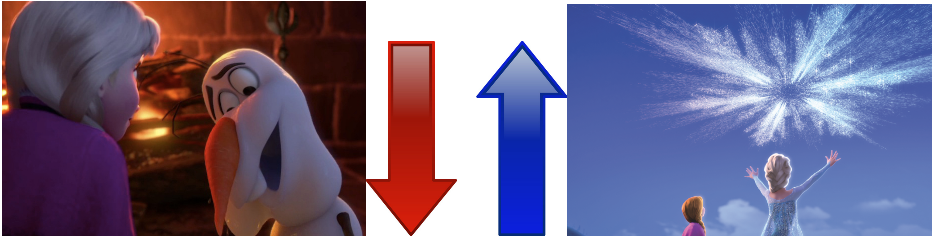

$$\mathrm{d}s^2 = \pm\,\mathrm{d}\tau^2 + \frac{(E\tau)^2\;\mathrm{d}x^2}{1+(E\tau)^2} + \big(1+(E\tau)^2\big)\Big(\,\mathrm{d}y + \frac{(E\tau)^2}{1+(E\tau)^2}\;\mathrm{d}x\Big)^2$$

Flat space cosmology $(y \sim y + 2\pi r_0)$

Bagchi, Detournay, DG & Simon '13

Flat space cosmologies (Cornalba & Costa '02)

- Start with BTZ in AdS:

$$\mathrm{d}s^2 = -\frac{(r^2 - R_+^2)(r^2 - r_-^2)}{r^2\ell^2}\,\mathrm{d}t^2 + \frac{r^2\ell^2\,\mathrm{d}r^2}{(r^2 - R_+^2)(r^2 - r_-^2)} + r^2\left(\mathrm{d}\varphi - \frac{R_+ r_-}{\ell r^2}\,\mathrm{d}t\right)^2$$

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- Take the $\ell \to \infty$ limit (with $R_+ = \ell\hat{r}_+$ and $r_- = r_0$)

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- Go to Euclidean signature ($t = i\tau_E$, $\hat{r}_+ = -ir_+$)

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Question we want to address:

Is FSC or HFS the preferred Euclidean saddle?

Euclidean path integral

Evaluate Euclidean partition function in semi-classical limit

$$Z(T,\,\Omega) = \int \mathcal{D}g\, e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T,\Omega)]} \times Z_{\text{fluct.}}$$

boundary conditions specified by temperature T and angular velocity Ω

Euclidean path integral

Evaluate Euclidean partition function in semi-classical limit

$$Z(T,\,\Omega) = \int \mathcal{D}g\, e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T,\Omega)]} \times Z_{\text{fluct.}}$$

boundary conditions specified by temperature T and angular velocity Ω

Two Euclidean saddle points in same ensemble if

- same temperature $T = 1/\beta$ and angular velocity Ω
- obey flat space boundary conditions
- solutions without conical singularities

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Periodicities fixed:

$$(\tau_E,\,\varphi)\sim(\tau_E+\beta,\,\varphi+\beta\Omega)\sim(\tau_E,\,\varphi+2\pi)$$

Results

On-shell action:

$$\Gamma = -\frac{1}{16\pi G_N}\int \mathrm{d}^3x\sqrt{g}\,R - \underbrace{\frac{1}{16\pi G_N}}_{\frac{1}{2}\text{GHY!}}\int \mathrm{d}^2x\sqrt{\gamma}\,K$$

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- $r_+ > 1$: FSC dominant saddle
- $r_+ < 1$: HFS dominant saddle

Critical temperature:

$$T_c = \frac{1}{2\pi r_0} = \frac{\Omega}{2\pi}$$

HFS "melts" into FSC at $T > T_c$

Entanglement entropy of Galilean CFTs and flat space holography

Bagchi, Basu, DG, Riegler '14

Using methods similar to CFT:

$$S_{\text{EE}}^{\text{GCFT}} = \underbrace{\frac{c_L}{6}\,\ln\frac{\ell_x}{a}}_{\text{like CFT}} + \underbrace{\frac{c_M}{6}\,\frac{\ell_y}{\ell_x}}_{\text{like grav anomaly}}$$

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with

$$[L_n,\,L_m] = (n-m)\,L_{n+m} + \frac{c_L}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

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and

- ℓ_x: spatial distance
- ℓ_y: temporal distance
- a: UV cutoff (lattice size)

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- flat space Einstein gravity: $c_L = 0$, $c_M \neq 0$

Same results obtained holographically!

- Using methods similar to Ammon, Castro Iqbal '13, de Boer, Jottar '13, Castro, Detournay, Iqbal, Perlmutter '14
- geodesics ⇒ Wilson lines

Unitarity in flat space

Unitarity leads to further contraction DG, Riegler, Rosseel '14

Facts:

- Unitarity in GCA requires $c_M = 0$ (see paper for caveats!)

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$$[L_n,\, U_m] = (2n-m)U_{n+m}$$

$$[M_n,\, U_m] = [L_n,\, V_m] = (2n-m)V_{n+m}$$

$$[U_n,\, U_m] \propto [U_n, V_m] = 96(n-m)\sum_p M_p\, M_{n-p}$$

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\end{aligned}
$$

Higher spin states decouple and become null states!

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Generic flat space W-algebras DG, Riegler, Rosseel '14

1. NO–GO:

Generically (see paper) you can have only two out of three:

- Unitarity
- Flat space
- Non-trivial higher spin states

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Example:

Flat space chiral gravity

Bagchi, Detournay, DG, 1208.1658

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Example:

Minimal model holography

Gaberdiel, Gopakumar, 1011.2986, 1207.6697

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Example:

Flat space higher spin gravity (Galilean W_3 algebra)

Afshar, Bagchi, Fareghbal, DG and Rosseel, 1307.4768

Gonzalez, Matulich, Pino and Troncoso, 1307.5651

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> Compatible with "spirit" of various no-go results in higher dimensions!

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Compatible with "spirit" of various no-go results in higher dimensions!

2. YES–GO:

There is (at least) one counter-example, namely a Vasiliev-type of theory, where you can have all three properties!

Unitary, non-trivial flat space higher spin algebra exists!
Vasiliev-type flat space chiral higher spin gravity?

Unitarity in flat space

Flat space W_∞-algebra compatible with unitarity DG, Riegler, Rosseel '14

- We do not know if flat space chiral higher spin gravity exists...

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- ...but its existence is at least not ruled out by the no-go result!
- If it exists, this must be its asymptotic symmetry algebra:

$$[\mathcal{V}^i_m, \mathcal{V}^j_n] = \sum_{r=0}^{\lfloor \frac{i+j}{2} \rfloor} g^{ij}_{2r}(m,n)\, \mathcal{V}^{i+j-2r}_{m+n} + c^i_{\mathcal{V}}(m)\, \delta^{ij}\, \delta_{m+n,0}$$

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where

$$c^i_{\mathcal{V}}(m) = \#(i,\, m) \times c \qquad \text{and} \qquad c = -\bar{c}$$

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- Vacuum descendants $\mathcal{W}^i_m|0\rangle$ are null states for all i and m!
- AdS parent theory: no trace anomaly, but gravitational anomaly (Like in conformal Chern–Simons gravity $\to$ Vasiliev type analogue?)

Adding chemical potentials Gary, DG, Riegler, Rosseel '14

Long story short:

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Line-element with spin-2 and spin-3 chemical potentials:

$$g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \Big(r^2\left(\mu_{\mathrm{L}}^2 - 4\mu_{\mathrm{U}}''\mu_{\mathrm{U}} + 3\mu_{\mathrm{U}}'^{\,2} + 4\mathcal{M}\mu_{\mathrm{U}}^2\right) + r\,g_{uu}^{(r)} + g_{uu}^{(0)} + g_{uu}^{(0')}\Big)\,\mathrm{d}u^2 +$$
$$\Big(r^2\mu_{\mathrm{L}} - r\mu_{\mathrm{M}}' + \mathcal{N}(1+\mu_{\mathrm{M}}) + 8\mathcal{Z}\mu_{\mathrm{V}}\Big)\,2\,\mathrm{d}u\,\mathrm{d}\varphi - (1+\mu_{\mathrm{M}})\,2\,\mathrm{d}r\,\mathrm{d}u + r^2\,\mathrm{d}\varphi^2$$

$$g_{uu}^{(0)} = \mathcal{M}(1+\mu_{\mathrm{M}})^2 + 2(1+\mu_{\mathrm{M}})(\mathcal{N}\mu_{\mathrm{L}} + 12\mathcal{V}\mu_{\mathrm{V}} + 16\mathcal{Z}\mu_{\mathrm{U}})$$
$$+ 16\mathcal{Z}\mu_{\mathrm{L}}\mu_{\mathrm{V}} + \tfrac{4}{3}(\mathcal{M}^2\mu_{\mathrm{V}}^2 + 4\mathcal{M}\mathcal{N}\mu_{\mathrm{U}}\mu_{\mathrm{V}} + \mathcal{N}^2\mu_{\mathrm{U}}^2)$$

Spin-3 field with same chemical potentials:

$$\Phi_{\mu\nu\lambda}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu\,\mathrm{d}x^\lambda = \Phi_{uuu}\,\mathrm{d}u^3 + \Phi_{ruu}\,\mathrm{d}r\,\mathrm{d}u^2 + \Phi_{uu\varphi}\,\mathrm{d}u^2\,\mathrm{d}\varphi - (2\mu_{\mathrm{U}}r^2 - r\mu_{\mathrm{V}}' + 2\mathcal{N}\mu_{\mathrm{V}})\mathrm{d}r\,\mathrm{d}u\,\mathrm{d}\varphi$$
$$+ \mu_{\mathrm{V}}\,\mathrm{d}r^2\,\mathrm{d}u - (\mu_{\mathrm{U}}'r^3 - \tfrac{1}{3}r^2(\mu_{\mathrm{V}}'' - \mathcal{M}\mu_{\mathrm{V}} + 4\mathcal{N}\mu_{\mathrm{U}}) + r\mathcal{N}\mu_{\mathrm{V}}' - \mathcal{N}^2\mu_{\mathrm{V}})\,\mathrm{d}u\,\mathrm{d}\varphi^2$$

$$\Phi_{uuu} = r^2\left[2(1+\mu_{\mathrm{M}})\mu_{\mathrm{U}}(\mathcal{M}\mu_{\mathrm{L}} - 4\mathcal{V}\mu_{\mathrm{U}}) - \tfrac{1}{3}\mu_{\mathrm{L}}^2(\mathcal{M}\mu_{\mathrm{V}} - 4\mathcal{N}\mu_{\mathrm{U}}) + 16\mu_{\mathrm{L}}\mu_{\mathrm{U}}(\mathcal{V}\mu_{\mathrm{V}} + \mathcal{Z}\mu_{\mathrm{U}}) - \tfrac{4}{3}\mathcal{M}\mu_{\mathrm{U}}^2(\mathcal{M}\mu_{\mathrm{V}}\right.$$
$$\left.+ 2\mathcal{N}\mu_{\mathrm{U}})\right] + 2\mathcal{V}(1+\mu_{\mathrm{M}})^3 + \tfrac{2}{3}(1+\mu_{\mathrm{M}})^2(6\mathcal{Z}\mu_{\mathrm{L}} + \mathcal{M}^2\mu_{\mathrm{V}} + 2\mathcal{M}\mathcal{N}\mu_{\mathrm{U}}) + 16\mu_{\mathrm{L}}\mu_{\mathrm{V}}^2(\mathcal{N}\mathcal{V} - \tfrac{1}{3}\mathcal{M}\mathcal{Z})$$
$$+ \tfrac{2}{3}(1+\mu_{\mathrm{M}})\big((\mathcal{N}\mu_{\mathrm{L}} + 16\mathcal{Z}\mu_{\mathrm{U}})(2\mathcal{M}\mu_{\mathrm{V}} + \mathcal{N}\mu_{\mathrm{U}}) + 12\mathcal{M}\mathcal{V}\mu_{\mathrm{V}}^2\big) + \tfrac{64}{3}\mathcal{Z}\mu_{\mathrm{U}}\mu_{\mathrm{V}}(\mathcal{N}\mu_{\mathrm{L}} + 12\mathcal{V}\mu_{\mathrm{V}} + 12\mathcal{Z}\mu_{\mathrm{U}})$$
$$+ \mathcal{N}^2\mu_{\mathrm{L}}^2\mu_{\mathrm{V}} + 64\mathcal{V}^2\mu_{\mathrm{V}}^3 - \tfrac{8}{27}(\mathcal{M}^3\mu_{\mathrm{V}}^3 - \mathcal{N}^3\mu_{\mathrm{U}}^3) - \tfrac{4}{9}\mathcal{M}\mathcal{N}\mu_{\mathrm{U}}\mu_{\mathrm{V}}(4\mathcal{M}\mu_{\mathrm{V}} + 5\mathcal{N}\mu_{\mathrm{U}}) + \textstyle\sum_{n=0}^{3} r^n\,\Phi_{uuu}^{(r^n)}$$

Adding chemical potentials Gary, DG, Riegler, Rosseel '14

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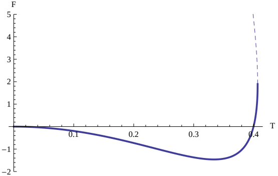
Works nicely in Chern–Simons formulation!

Interesting novel phase transitions of zeroth/first order:



Free energy of four branches of regular solutions as function of temperature for different values of higher spin chemical potential ratio (in AdS: see David, Ferlaino, Kumar '12)

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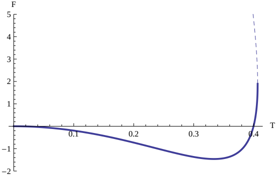

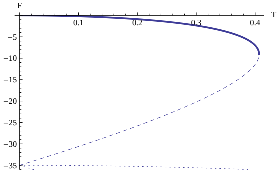

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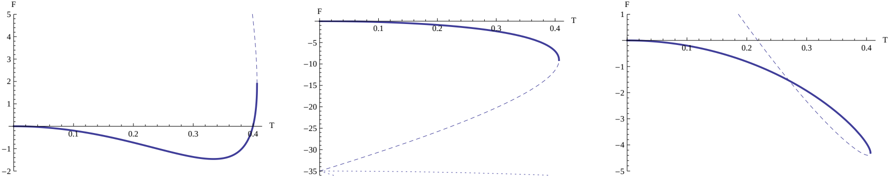

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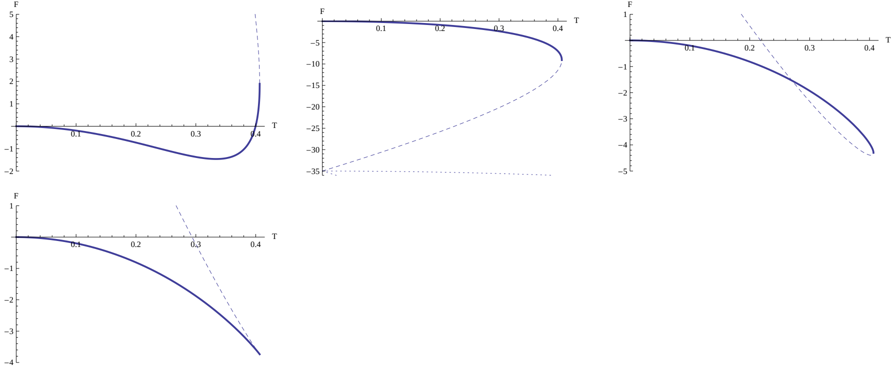

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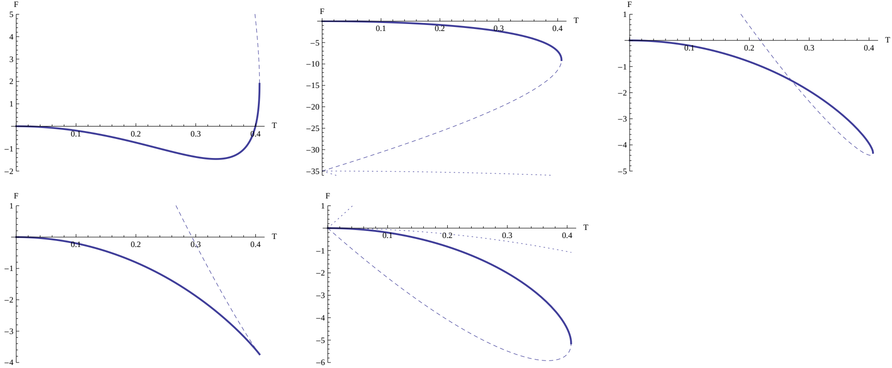

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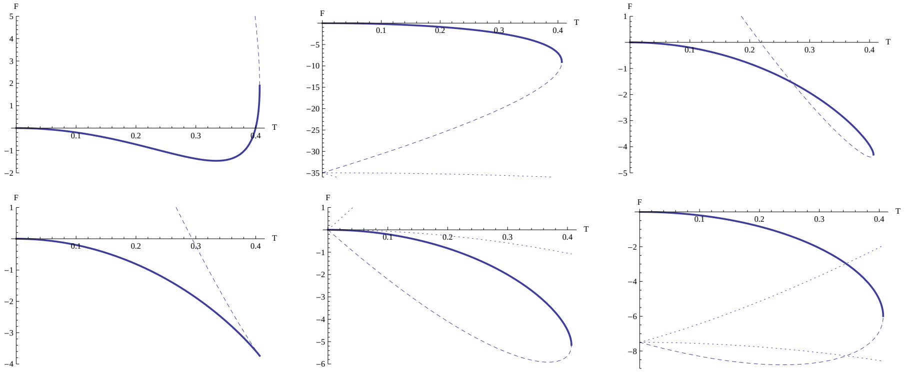

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