Seminar Talk (MIT)

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Fantastic Realism: The "Vienna School" of 2D Dilaton Gravity

speaker: *Daniel Grumiller (DG)*

affiliation: Institute for Theoretical Physics, University of Leipzig, Augustusplatz 10-11, D-04109 Leipzig, Germany

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Review: *DG, W. Kummer, D. Vassilevich,* hep-th/0204253

Fantastic Realism in Art

“Vienna School” emerged in the 1950ies

Brauer

Fuchs

Hausner

Lehmden

Hutter

Guetersloh

- Much later than comparable 20th century Schools (e.g. Surrealism)
- Inspired by these Schools and “The Old Masters” (e.g. P. Breughel, H. Bosch)
- Recognized internationally, many students

Outline

1. Motivation
2. Dilaton gravity in 2D, 2^{nd} order
3. Selected list of models
4. Gravity as gauge theory: $2^{nd} \rightarrow 1^{st}$ order
5. First order action, relation to PSM
6. All classical solutions (locally)
7. Global structure (Penrose diagrams)
8. Hawking temperature, entropy
9. Path integral quantization (with matter)
10. Summary

1. Motivation

- dimensionally reduced models (spherical symmetry)
- strings (2D target space)
- integrable models (mathematical meth.)
- noncommutative geometry (Poisson σ)
- models for BH physics (information loss)

study important conceptual problems without encountering insurmountable technical ones $\rightarrow$ 2D gravity: useful toy model(s) for classical and quantum gravity

most prominent member not just a toy model: Schwarzschild Black Hole ("Hydrogen atom of General Relativity")

2. Dilaton gravity in 2D, 2nd order

Gravity in 2D: EH not useful:

$$S^{(EH)} = \int_{\mathcal{M}_2} \mathrm{d}^2x \sqrt{g} R = \text{Euler}$$

No equations of motion (EOM)!

Generalization of EH:

2D scalar tensor theories (Minkowskian):

[*J. Russo, A. Tseytlin,* `hep-th/9201021`]

$$S^{(SOG)} = \int_{\mathcal{M}_2} \mathrm{d}^2x \sqrt{-g} \left(XR - U(X)(\nabla X)^2 + 2V(X) \right)$$

X: "dilaton field" (scalar)

$g_{\mu\nu}$: 2D metric (tensor)

$U(X), V(X)$: potentials defining the model

Note 1: sometimes first term: $Z(X)R$; if Z invertible: form above! if not: singularities at $Z' = 0$! in each regular patch: again form above!

Note 2: conformal transformation to a different model with $U = 0$ possible – but conformal factor singular in general, thus change of global structure!

3. Selected list of models

Model	$U(X)$	$V(X)$
Schwarzschild	$-\frac{1}{2X}$	$-\lambda^2$
Jackiw-Teitelboim	0	ΛX
Witten BH/CGHS	$-\frac{1}{X}$	$-2\lambda^2 X$
CT Witten BH	0	$-2\lambda^2$
SRG (generic $D > 3$)	$-\frac{D-3}{(D-2)X}$	$-\lambda^2 X^{(D-4)/(D-2)}$
All above: ab-family	$-\frac{a}{X}$	$-\frac{B}{2}X^{a+b}$
Reissner-Nordström	$-\frac{1}{2X}$	$-\lambda^2 + \frac{Q^2}{X}$
Schwarzschild-(A)dS	$-\frac{1}{2X}$	$-\lambda^2 - \ell X$
Katanaev-Volovich	α	$\beta X^2 - \Lambda$
Achucarro-Ortiz	0	$\Lambda X - \frac{J}{4X^3} + \frac{Q^2}{X}$
Reduced CS	0	$\frac{1}{2}X(c - X^2)$
2D type 0A/0B	$-\frac{1}{X}$	$-2\lambda^2 X + \frac{\lambda^2 q^2}{8\pi}$
exact string BH	?	?

Note: non-standard models possible in 1$^{\text{st}}$ order:

$$-U(X)\frac{(\nabla X)^2}{2} + V(X) \to \mathcal{V}((\nabla X)^2, X)$$

Example: dilaton-shift invariant models

$\mathcal{V} = X\,U((\nabla \ln X)^2)$

No principle complications, but occurs rarely in literature!

4. Gravity as gauge theory: $2^{\text{nd}} \to 1^{\text{st}}$

JT model: (A)dS$_2$ ($SO(1,2)$): *C. Teitelboim, PL* **B126** *(1983) 41*, *R. Jackiw, NP* **B252** *(1985) 343*

$$[P_a, P_b] = \Lambda \epsilon_{ab} J \,, \quad [P_a, J] = \epsilon_{ab} P^b$$

1^{st} order form: *K. Isler, C. Trugenberger, PRL* **63** *(1989) 834*, *A. Chamseddine, D. Wyler, PL* **B228** *(1989) 75*

$$L = X_A F^A = X_a (De)^a + X(\text{d}\omega + \frac{1}{2}\Lambda \epsilon_{ab} e^a e^b)$$

$SO(1,2)$ connection: $A = e^a P_a + \omega J$,
$F = \text{d}A + \frac{1}{2}[A, A]$, e^a, ω: "Cartan variables",
X_A: Lagr. mult. (trafo under coadjoint rep.)

CGHS: central extended Poincaré ($ISO(1,1)$):
D. Cangemi, R. Jackiw, `hep-th/9203056`

$$[P_a, P_b] = \lambda \epsilon_{ab} I \,, \quad [P_a, J] = \epsilon_{ab} P^b \,, \quad [I, J] = 0 = [I, P_a]$$

again first order with $L = X_A F^A$ possible
without central extension: *Verlinde, MG VI*
cf. also *A. Achúcarro,* `hep-th/9207108`

other important pre-cursors:

W. Kummer, D.J. Schwarz, PR **D45** *(1992) 3628*:
all classical sol. of Katanaev-Volovich model

N. Ikeda, `hep-th/9312059`: (non-linear) gauge formulation for $U = 0$ but generic $V(X)$

5. First order action, relation to PSM

Classical and quantum equivalence to:

P. Schaller, T. Strobl, `hep-th/9405110`

$$S^{(FOG)} = \int_{\mathcal{M}_2} [X_a T^a + XR + \epsilon \mathcal{V}(X^a X_a, X)] \tag{1}$$

$T^a = (De)^a$: torsion 2-form
$R^a{}_b = \epsilon^a{}_b R = \epsilon^a{}_b \, d\omega$: curvature 2-form
$\epsilon = -\frac{1}{2}\epsilon_{ab} e^a \wedge e^b$: volume 2-form
X: "dilaton" (Lagrange mult. f. curvature)
X^a: auxiliary fields (— " — torsion)
$\mathcal{V}$: potential defining the model (as before)

Relation to second order:
dilaton: $X = X$
kinetic term: $(\nabla X)^2 = -X^a X_a$
metric: $g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu$
connection: Levi-Civitá $= \omega-$torsion part

Technical Note: Use light-cone components ($\eta_{+-} = 1 = \eta_{-+}$, $\eta_{++} = 0 = \eta_{--}$); define $\epsilon^\pm{}_\pm = \pm 1$

$T^\pm = (\mathrm{d} \pm \omega) \wedge e^\pm$, $\quad \epsilon = e^+ \wedge e^-$, $\quad X^a X_a = 2X^+ X^-$

Typically: $\mathcal{V}(X^+ X^-, X) = X^+ X^- U(X) + V(X)$

Equivalence to specific type of Poisson-σ model
[*P. Schaller, T. Strobl,* hep-th/9405110]

$$\mathcal{S}_{gPSM} = \int_{\mathcal{M}_2} \mathrm{d}X^I \wedge A_I + \frac{1}{2} P^{IJ} A_J \wedge A_I \,.$$

- gauge field 1-forms: $A_I = (\omega, e_a)$, connection, Zweibeine

- target space coordinates: $X^I = (X, X^a)$, dilaton, auxiliary fields

- target space: Poisson manifold

- Poisson tensor: odd dimension $\rightarrow$ kernel!

- Jacobi: $P^{IL} \partial_L P^{JK} + \mathit{perm}\,(IJK) = 0$

$$P^{IJ} = \begin{pmatrix} 0 & X^+ & -X^- \\ -X^+ & 0 & \mathcal{V} \\ X^- & -\mathcal{V} & 0 \end{pmatrix}$$

Equations of motion (first order):

$$\mathrm{d}X^I + P^{IJ} A_J = 0$$
$$\mathrm{d}A_I + \frac{1}{2}(\partial_I P^{JK}) A_K \wedge A_J = 0$$

Gauge symmetries:

$$\delta X^I = P^{IJ} \varepsilon_J$$
$$\delta A_I = -\mathrm{d}\varepsilon_I - \left(\partial_I P^{JK}\right) \varepsilon_K A_J$$

Note 1: if P^{IJ} linear: Lie-Algebra! Otherwise: non-linear gauge symmetries

Note 2: on-shell equivalent to diffeomorphisms+local Lorentz trafos for specific Poisson tensor on previous page

Remark: Schouten-Nijenhuis bracket:

$$[X^I, X^J]_{SN} = P^{IJ}$$

Note 1: like non-commutative geometry

Note 2: Jacobi identity for bracket equivalent to non-linear identity for Poisson tensor on previous page

$\star$: *M. Kontsevich,* `q-alg/9709040`, path integral approach: *A. Cattaneo, G. Felder,* `math.qa/9902090`

6. All classical solutions (locally)

Ansatz: $X^+ \neq 0$ in a patch $\to$ $e^+ = X^+ Z$

Summary of EOM for dilaton gravity:

$$\begin{aligned}
\delta\omega: &\quad \mathrm{d}X + X^- e^+ - X^+ e^- = 0\,, \\
\delta e^\mp: &\quad (\mathrm{d}\pm\omega) X^\pm \mp \mathcal{V} e^\pm = 0\,, \\
\delta X: &\quad \mathrm{d}\omega + \epsilon \frac{\partial \mathcal{V}}{\partial X} = 0\,, \\
\delta X^\mp: &\quad (\mathrm{d}\pm\omega) e^\pm + \epsilon \frac{\partial \mathcal{V}}{\partial X^\mp} = 0\,.
\end{aligned}$$

1. use $\delta\omega$ to get $e^- = \mathrm{d}X/X^+ + X^- Z$
2. read off $\epsilon = e^+ \wedge e^- = Z \wedge \mathrm{d}X$
3. use δe^- to get $\omega = -\,\mathrm{d}X^+/X^+ + Z\mathcal{V}$
4. use δX^- to get $\mathrm{d}Z = \mathrm{d}X \wedge Z U(x)$
5. define “integrating factor”:

$$I(X) := \exp \int^X U(X')\,\mathrm{d}X'$$

6. obtain $Z =: \hat{Z} I(X)$ with $\mathrm{d}\hat{Z} = 0 \to \hat{Z} = \mathrm{d}u$
7. use $g_{\mu\nu} = e^+_\mu e^-_\nu + e^-_\mu e^+_\nu$

general solution for the line element:

$$\mathrm{d}s^2 = I(X)\left(2\,\mathrm{d}u\,\mathrm{d}X + 2X^+X^- I(X)\,\mathrm{d}u^2\right)$$

$X^+X^- = 0$: apparent horizon!

Conservation law:

T. Banks, M. O'Loughlin, NP **B362** *(1991) 649;*
V. Frolov, PR **D46** *(1992) 5383; R. Mann,* `hep-th/9206044`

later generalized by "Vienna group"

W. Kummer+students; for a review cf. e.g.

DG, W. Kummer, D. Vassilevich, `hep-th/0204253`

Derivation in absence of matter: EOMs $X^+\delta e^+ + X^-\delta e^-$ using also the EOM $\delta\omega$ establishes

$$\mathrm{d}(X^+X^-) + \mathcal{V}\,\mathrm{d}X = 0$$

for "standard" $\mathcal{V} = X^+X^-U(X) + V(X)$:

$$\mathcal{C} = I(X)X^+X^- + w(X)\,, \quad \mathrm{d}\mathcal{C} = 0$$

with

$$w(X) := \int^X I(X')V(X')\,\mathrm{d}X'$$

"Generalized Birkhoff theorem" (always 1 Killing)

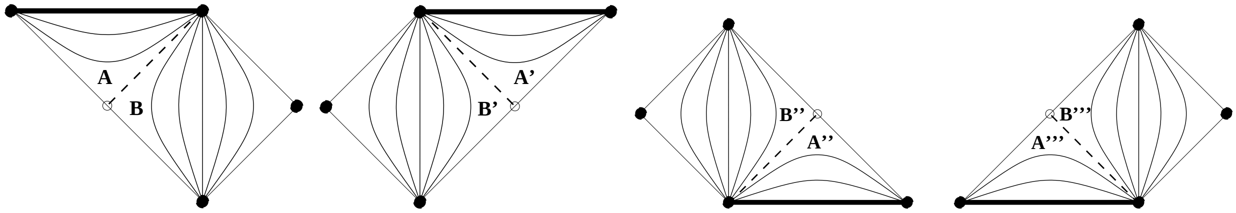
Inserting into line element, $\mathrm{d}r = I(X)\,\mathrm{d}X$:

$$\mathrm{d}s^2 = 2\,\mathrm{d}u\,\mathrm{d}r + 2I(X(r))\,(\mathcal{C} - w(X(r)))\,\mathrm{d}u^2 \tag{2}$$

Eddington-Finkelstein patch!

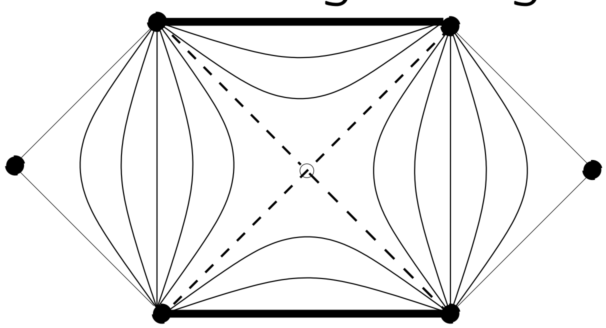
7. Global structure (Penrose diagrams)

Carter-Penrose (CP) diagram of EF patches:



for Schwarzschild-like solutions

Global CP: glue together EF patches:



Only point not covered by EF patches:
bifurcation point: $X^+ = 0 = X^-$

M. Walker, J. Math. Phys. **11** *(1970) 2280*, *T. Klösch, T. Strobl,* `gr-qc/9508020`, `gr-qc/9511081`

open regions with $X^+ = 0 = X^-$: $X =$ const.
"Constant Dilaton Vacua" (very simple)

$$V(X_{CDV}) = 0 \,, \qquad R \propto V'(X_{CDV}) = \text{const.}$$

only Minkowski, Rindler, (A)dS

Non-trivial example: KK reduced CS:

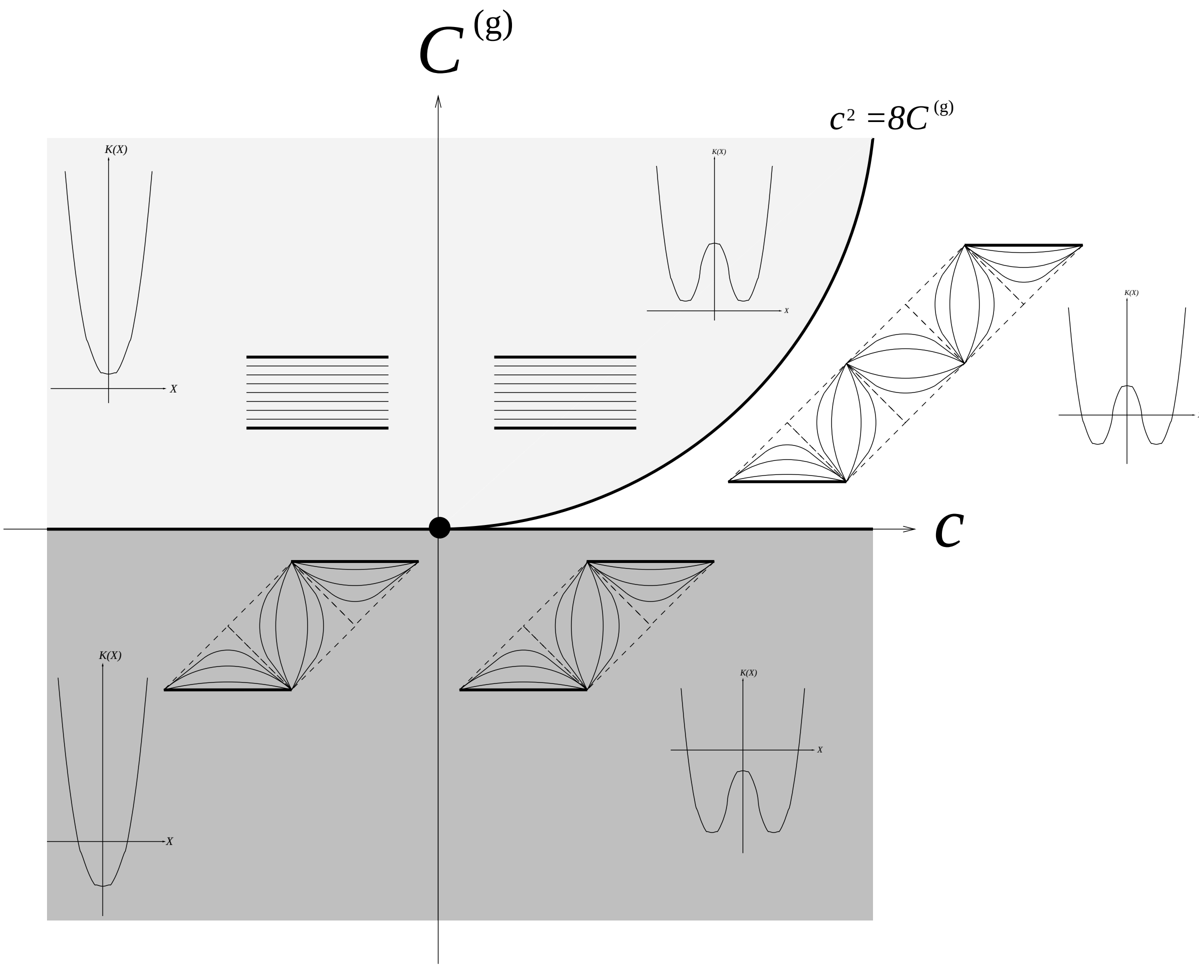
G. Guralnik, A. Iorio, R. Jackiw and S.Y. Pi, `hep-th/0305117`,

DG and W. Kummer, `hep-th/0306036`,

L. Bergamin, DG, A. Iorio, C. Nuñez, `hep-th/0409273`,

talk at MIT by *C. Nuñez* in September 2004

$$U(X) = 0\,, \quad V(X) = \frac{1}{2}X(c - X^2)$$



Kink interpolates between two AdS-CDV

8. Hawking temperature

Naively from surface gravity:

$$T_H = \frac{1}{2\pi}\left|w'(X)\right|_{X=X_h} . \qquad (3)$$

Note: independent from $I(X)$

With minimally coupled matter: same result;
e.g. from trace anomaly

S. Christensen, S. Fulling, PR **D15** *(1977) 2088*

$$< T^{\mu}{}_{\mu} > \propto R\,, \quad \nabla_\mu T^{\mu\nu} = 0$$

Matter coupled ("non-minimally") to dilaton:
Non-conservation equation!

W. Kummer, D. Vassilevich, gr-qc/9907041

$$\nabla^\mu T_{\mu\nu} = -(\partial_\nu \Phi)\frac{1}{\sqrt{-g}}\frac{\delta W}{\delta \Phi}\,, \quad X = e^{-2\Phi}$$

Mass-to-temperature law: need mass!
Sometimes mutually contradicting results for "ADM mass" (e.g. 2D string theory)
clarified in appendix of *DG, D. Mayerhofer,* gr-qc/0404013

Specific heat

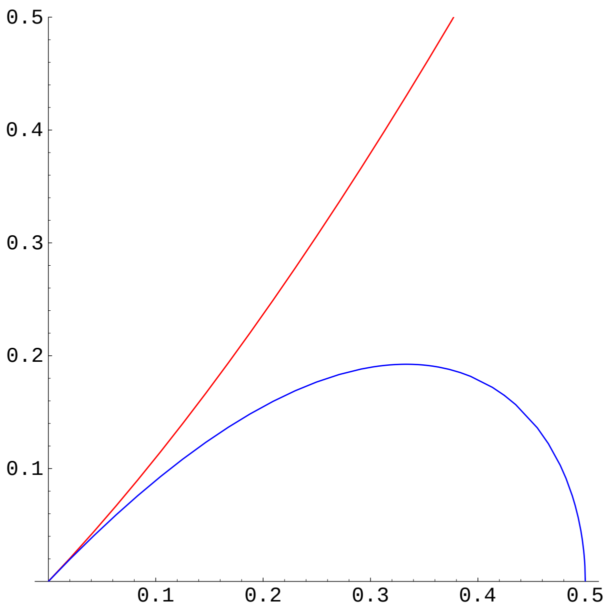
Naive definition:

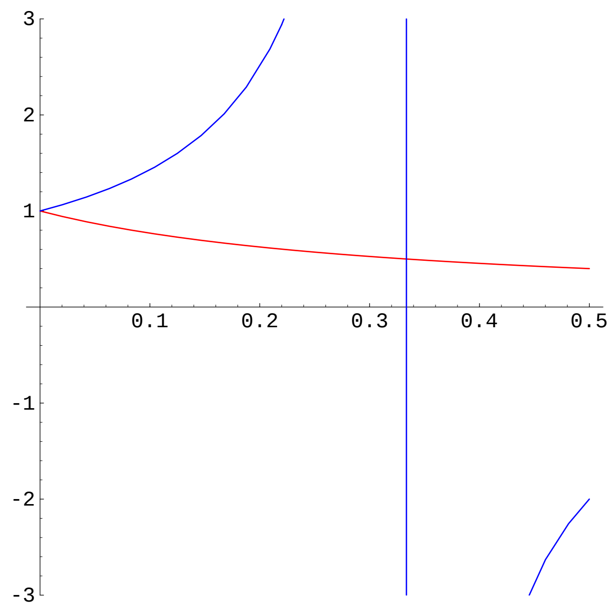
$$C_s = \frac{\mathrm{d}M}{\mathrm{d}T_H} = \gamma(M) T_H$$

$\gamma(M)$: "Sommerfeld function"
(supposing $M \to 0$ implies $T \to 0$)

Example: kink solution of KK reduced CS:

Hawking temperature   Sommerfeld function





Hawking-Page like transition at $C_s \to \infty$
S. Hawking and D. Page, Commun. Math. Phys. **87** *(1983) 577.*

BH Entropy

BH: "Bekenstein-Hawking" or "Black Hole"

Simple thermodynamic considerations:

$$\mathrm{d}S = \frac{\mathrm{d}M}{T}$$

J. Gegenberg, G. Kunstatter, D. Louis-Martinez, `gr-qc/9408015`

$$S = 2\pi \, X|_{\mathcal{C}=w(X)} \equiv \frac{A}{4} \qquad (4)$$

confirmed by more elaborate derivations (CFT methods, near horizon conformal symmetry, Cardy-formula)

S. Carlip `gr-qc/9906126`, `gr-qc/0203001`, `hep-th/0408123`, *S. Solodukhin,* `hep-th/9812056`, *M. Cadoni, S. Mignemi,* `hep-th/9810251`

open question: counting of microstates is fine, but what are actually the microstates of 2D dilaton gravity?

Another important open problem: end point of BH evaporation!

J. Russo, L. Susskind, L. Thorlacius, `hep-th/9206070`, *DG,* `gr-qc/0307005`

9. Path integral quantization (+ matter)

Basic ideas:

W. Kummer, H. Liebl, D. Vassilevich, `hep-th/9809168`

- Matter provides propagating d.o.f.
- Integrate out geometry exactly!
  Possible in first order formulation with EF gauge fixing fermion: gauge fields appear only linearly!
- Treat matter perturbatively
- Reconstruct geometry (Virtual BHs)
- Calculate S-matrix or corrections to classical quantities (like specific heat)

Note: Generalization to SUGRA:

L. Bergamin, DG, W. Kummer, `hep-th/0404004`

Brackets and Algebras

Schouten-Nijenhuis: $[X^I, \bullet]_{SN} = P^{IJ}\partial_J(\bullet)$
NC geometry: $[X^I, X^J]_{SN} = P^{IJ}(X^K)$
Jacobi-id: $P^{IL}P_{,L}{}^{JK} +$ (g)cycl. $= 0$

Commutator: $\hat{X}^I := P^{IJ}\partial_J$
$[\hat{X}^I, \hat{X}^J] = \hat{X}^K C_K{}^{IJ}$, $C_K{}^{IJ} = P_{,K}{}^{IJ}$

Poisson: $\left\{q_i, p^j\right\} = \delta_i^j$
$q_i = (\omega_1, e_1^-, e_1^+)$, $p^i = (X, X^+, X^-)$

Algebra of secondary (first class) constraints:

$$\left\{G^i(x), G^j(x')\right\} = G^k C_k{}^{ij}\delta(x - x')$$

with the same structure functions $C_k{}^{ij}$
crucial: q_i linear in G^i, absent in $C_k{}^{ij}$ $\Rightarrow$
no ordering problems, BRST charge nilpotent without quartic ghost terms

Relation to Virasoro algebra:
$G = G^1$; $H_0 = q_1 G^1 + \varepsilon^a{}_b q_a G^b$; $H_1 = q_i G^i$
$\{G, G'\} = 0$, $\{G, H_0'\} = -G\delta'$, $\{G, H_1'\} = -G\delta'$,
$\{H_0, H_0'\} = (H_1 + H_1')\delta'$,
$\{H_0, H_1'\} = (H_0 + H_0')\delta'$,
$\{H_1, H_1'\} = (H_1 + H_1')\delta'$

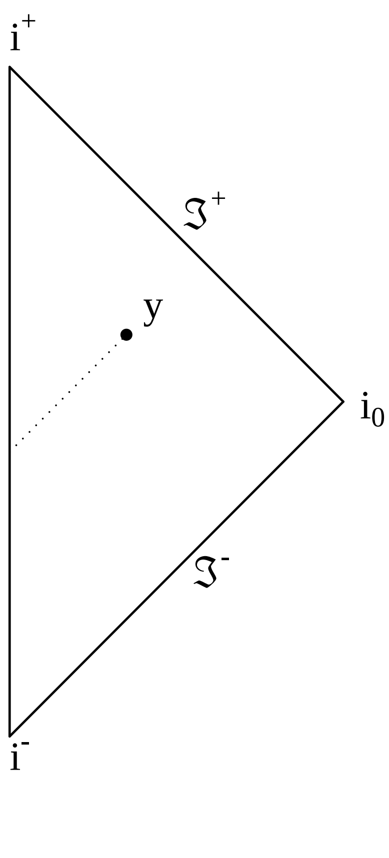
Effective line element

"temporal gauge": $(A_I)_0 = a_0 =$ const. $\Rightarrow$ line element in *outgoing Sachs-Bondi* form:

$$(ds)^2 = 2drdu + K(r,u)(du)^2$$

Killing-norm (SRG, asymp. Minkowski, LO):

$$K(r,u) = \left(1 - \left(\frac{2m}{r} + ar\right)\theta(r_0 - r)\delta(u - u_0)\right),$$



zeros of Killing-norm approximately at $r = 2m$ (*Schwarzschild horizon*) and $r = 1/a$ (*Rindler horizon*) non-vacuum-geometry concentrated on light-like cut; curvature scalar as well; interpretation: virtual black hole (VBH) induced by effective matter interaction; S-matrix: sum over all VBH *DG, W. Kummer, D. Vassilevich*, gr-qc/0001038, review: *DG*, hep-th/0409231

Scattering amplitude

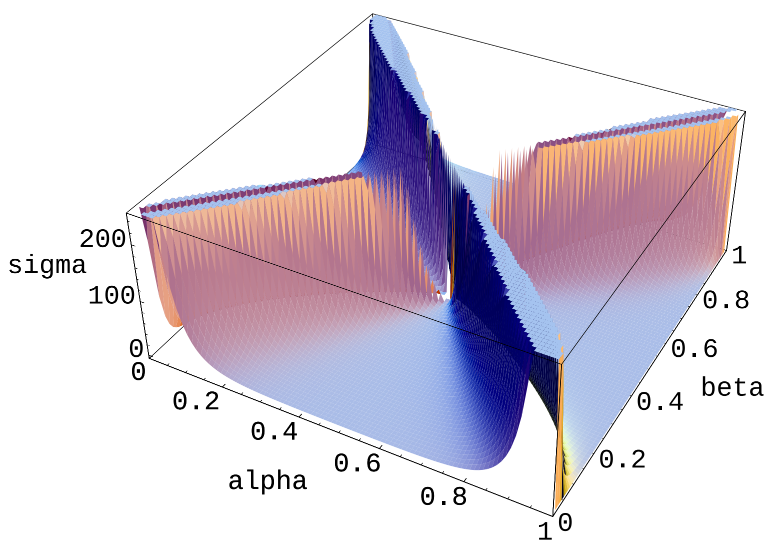
P. Fischer, DG, W. Kummer, D. Vassilevich, gr-qc/0105034, *DG,* gr-qc/0105078, gr-qc/0111097

S-matrix for s-wave scattering: ingoing modes q, q'; outgoing ones k, k'; $E := q + q'$

$$T \propto \tilde{T}\delta(k + k' - q - q')E^3/|kk'qq'|^{3/2}$$

interesting part: *scale independent* $\tilde{T}$, momentum transfer $\Pi := (k + k')(k - q)(k' - q)$

$$\tilde{T}(q, q'; k, k') := \frac{1}{E^3}\left[\Pi \ln \frac{\Pi^2}{E^6} + \frac{1}{\Pi} \sum_{p \in \{k,k',q,q'\}} p^2 \ln \frac{p^2}{E^2}\left(3kk'qq' - \frac{1}{2}\sum_{r \neq p}\sum_{s \neq r,p}\left(r^2 s^2\right)\right)\right],$$



$k = \alpha E$

$k' = (1 - \alpha)E$

$q = \beta E$

$q' = (1 - \beta)E$

Quantum corrected C_V

DG, W. Kummer, D. Vassilevich, `hep-th/0305036`

Reminder: e.g. Schwarzschild: $C_V \propto -M^2$
Convenient: CGHS: C_V classically trivial!

first quantum correction to specific heat:

$$C_V = M^2 96\pi^2/\lambda^2 > 0\,, \qquad \lim_{M\to 0} = \text{ok}$$

horizon shifted to smaller value of dilaton $\Rightarrow$
backreactions stabilize system
note: use Unruh vacuum!

to do: quantum corrected C_V generically (or at least for Schwarzschild)

problem: nonmininmal coupling $F \propto X$ prohibits naive application of Polyakov's 1-loop effective action
moreover: spherical reduction anomaly
V. Frolov, P. Sutton, A. Zelnikov, `hep-th/9909086`

10. Summary

- Conceptual problems in quantum gravity: use lower dimensional models, e.g. 2D
- 2D dilaton gravity in first order formulation (“Vienna approach”)
- All classical solutions (globally!)
- BH thermodynamics (info. paradox)
- Quantization in presence of matter (VBHs)
- Generalization to SUGRA, inclusion of gauge fields: straightforward!

Some recent developments

- NC JT model (*S. Cacciatori et al.*, hep-th/0203038)
- UR boosts (*H. Balasin, DG*, gr-qc/0312086)
- KK reduced (SU)CS (*cf. talk by C. Nuñez*)
- 2D string theory (*several recent papers 03/04*)

Some open problems

- Lift techniques to D>2?
- Simple derivation of S-matrix?
- Generic NC dilaton gravity?
- Action for exact string BH?