Flat space holography and complex SYK

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with Afshar, González, Vassilevich 1911.05739

Outline

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Asymptotic symmetries in gravity

- Natural boundary conditions not applicable in gravity: metric must not vanish asymptotically

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$$\mathcal{L}_\xi g_{\mu\nu} \stackrel{!}{=} \mathcal{O}(\delta g_{\mu\nu})$$

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$$\xi^\mu(r_b,\,x^i) = \xi^\mu_{(0)}(r_b,\,x^i) + \text{subleading terms}$$

$\xi^\mu_{(0)}(r_b,\,x^i)$: generates asymptotic symmetries (improper diffeos)
subleading terms: generate proper diffeos

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Definition of asymptotic symmetry algebra

Lie bracket quotient algebra of asymptotic Killing vectors modulo proper diffeos

Asymptotic symmetries and holography

Asymptotic symmetries = boundary condition preserving transformations modulo proper gauge transformations

Some references:

- covariant phase space: Lee, Wald '90, Iyer, Wald '94 and Barnich, Brandt '02
- reviews: see Compère, Fiorucci '18, Harlow, Wu '19 and refs. therein
- canonical analysis: Arnowitt, Deser, Misner '59, Regge, Teitelboim '74 and Brown, Henneaux '86
- review: see Bañados, Reyes '16 and refs. therein
- boundary excitations/edge modes: see e.g. Balachandran et al '91, Carlip '94
- more recent work: Freidel, Livine, Pranzetti '19; Freidel, Geiller, Pranzetti '20

Asymptotic symmetries and holography

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  - basic ingredient of AdS/CFT tests based on symmetries
  - captures universal UV features of QFTs (conformal symmetries)
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- Holography beyond AdS/CFT
  - asymptotic holography beyond AdS/CFT?
  - near horizon holography?
  - asymptotic symmetries important input for structure of dual QFT

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Brief history:

- general relativity in limit of low curvature $\stackrel{?}{=}$ special relativity

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  no, get infinite extension of Poincaré by super-translations $P(x)$

  x: angular coordinates

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- (extended) BMS_4 algebra ($J_a(x)$: diff S^2 or restriction thereof)

$$\{J_a(x),\, J_b(x')\} = \big(J_a(x')\partial_b - J_b(x)\partial'_a\big)\,\delta(x-x')$$
$$\{J_a(x),\, P(x')\} = \Big(\frac{s}{2}\,P(x')\partial_a - P(x)\partial'_a\Big)\,\delta(x-x')$$
$$\{P(x),\, P(x')\} = 0$$

s: spin of super-translations (in original BMS_4: $s=1$)

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  s: spin of super-translations (in original BMS_4: $s=1$)
- get same algebra as near horizon symmetries (in any dimension ≥ 3)
  Donnay, Giribet, González, Pino '15 $s=0$ ('scalar super-translations')
  DG, Perez, Troncoso, Sheikh-Jabbari, Zwikel '19 arbitrary s

BMS$_3$

- Barnich, Gomberoff, González '12 BMS$_3$ from CFT$_2$ by contraction

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- Change the Virasoro basis $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad\qquad M_n = \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

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$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c_L}{12}\left(n^3 - n\right)\delta_{n+m,\,0}$$
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with central charges $c_L = c - \bar{c}$ and $c_M = \frac{1}{\ell}\left(c + \bar{c}\right)$

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- Contraction means $\ell \to \infty$ and yields BMS_3 (M_n: super-translations)
- Example: Einstein gravity

$$c = \bar{c} = \frac{3\ell}{2G} \qquad \Rightarrow \qquad c_L = 0 \quad c_M = \frac{3}{G}$$

Status of 3d flat space holography

- asymptotic symmetries (Ashtekar, Bicak, Schmidt '96)
- central extensions in asymptotic symmetries (Barnich, Compère '06)
- dual field theory: BMS-invariant QFT (Carrollian CFT_2)
- concrete proposal: flat space chiral gravity (Bagchi, Detournay, DG '12)
- Cardyology works (Bagchi et al '12; Barnich '12)
- Hawking–Page-like phase transitions (Bagchi, Detournay, DG, Simón '13)
- (holographic) entanglement entropy (Basu, Bagchi, DG, Riegler '14)
- 1-loop part. fct. $\simeq$ BMS character (Barnich, Gonzalez, Maloney, Oblak '15)
- stress "tensor" correlators match (Bagchi, DG, Merbis '15)
- BMS bootstrap (Bagchi, Gary, Zodinmawia '16)
- most general boundary conditions (DG, Merbis, Riegler '17)
- HEE via geodesics (Jiang, Song, Wen '17; Hijano, Rabideau '17)
- Semi-classical BMS_3 blocks (Hijano '18)
- BMS characters & modular invariance (Bagchi, Saha, Zodinmawia '19)
- quantum energy conditions (DG, Parekh, Riegler '19)
- geometric actions (Merbis, Riegler '19)
- ...

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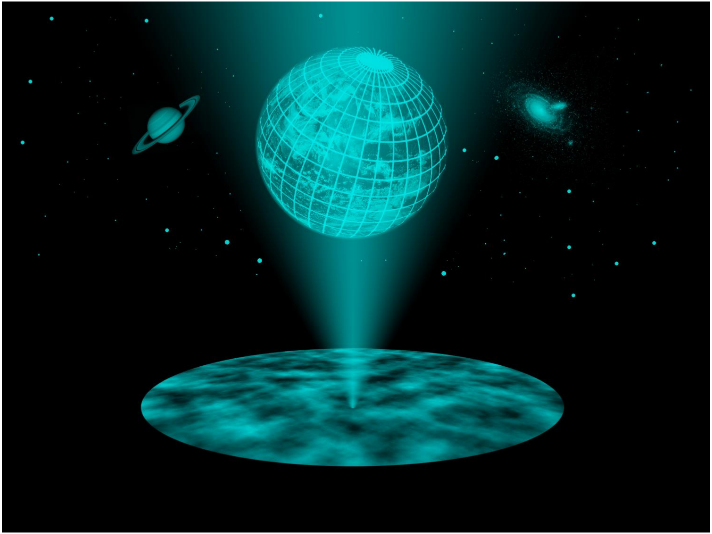

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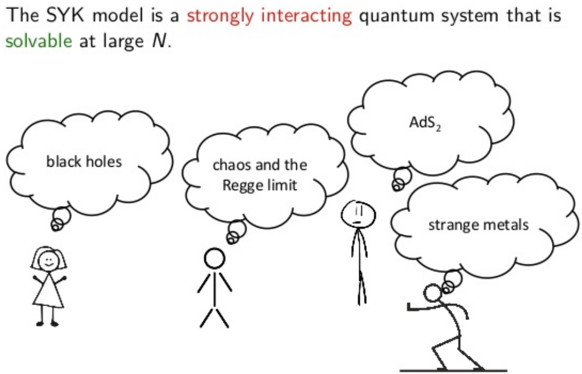
- BMS_2 perhaps useful for near horizon holography
- construct SYK-like models with asymptotically flat gravity side



slide from Stanford's talk at Strings 2017

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Ignore difficulties and proceed*

* van Nieuwenhuizen: task of theoretical physicists is to break no-go theorems

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Kinematics and BMS_2

Dynamics yielding BMS_2

Relation to SYK/JT

Outlook

Asymptotically Ricci-flat metrics

- Gauge-fix to Eddington–Finkelstein coordinates

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + K(u,\,r)\,\mathrm{d}u^2$$

Not obvious that this is possible with proper gauge trafos!

Same remark applies to *any* gauge fixing, e.g. in AdS_3

Asymptotically Ricci-flat metrics

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- Demand Ricci-flatness

$$K(u,\,r) = 2\mathcal{P}(u)\,r + 2\mathcal{T}(u)$$

Note: for constant $\mathcal{P}$ and $\mathcal{T}$ Killing horizon

$$r_h = -\frac{\mathcal{T}}{\mathcal{P}}$$

Assume in most of talk constant $\mathcal{P}$ and $\mathcal{T}$

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- Whatever the gravity theory is going to be, require the following boundary conditions for metric

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + \big(\underbrace{\mathcal{O}(r) + \mathcal{O}(1)}_{\text{state-dependent}} + o(1)\big)\;\mathrm{d}u^2$$

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Determine next asymptotic Killing vectors

Asymptotic Killing vectors

- Class of metrics

$$\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}r + 2\left(\mathcal{P}(u)\,r + \mathcal{T}(u)\right)\,\mathrm{d}u^2$$

preserved by asymptotic Killing vectors

$$\xi(\epsilon,\,\eta) = \epsilon(u)\partial_u - \big(\epsilon'(u)r + \eta(u)\big)\partial_r$$

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- $\eta(u)$ generates radial 'super-translations'

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- Metric functions transform non-trivially

$$\mathcal{L}_\xi \mathcal{P} = \epsilon\mathcal{P}' + \epsilon'\mathcal{P} + \epsilon''$$
$$\mathcal{L}_\xi \mathcal{T} = \epsilon\mathcal{T}' + 2\epsilon'\mathcal{T} + \eta' - \eta\mathcal{P}$$

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- Looks promising!

$\mathcal{P}$ like $u(1)$ current
$\mathcal{T}$ like Virasoro generator

BMS_2 algebra

- Lie-bracket algebra of asymptotic Killing vectors

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> Call this algebra BMS$_2$
> Can (and will) have non-trivial central extensions

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Dismiss winding mode and focus on warped Witt algebra

Outline

Motivation for BMS_2

Kinematics and BMS_2

Dynamics yielding BMS_2

Relation to SYK/JT

Outlook

Dilaton gravity in two dimensions (review hep-th/0204253)

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and guarantees well-defined variational principle $\delta I = 0$ with fineprint

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- Interesting option: couple 2d dilaton gravity to matter

Selected list of models (see review hep-th/0604049)

Black holes in (A)dS$_2$, asymptotically flat or arbitrary spaces (Wheeler property)

Model	$U(X)$	$V(X)$
1. Schwarzschild (1916)	$-\frac{1}{2X}$	$-\lambda^2$
2. Jackiw-Teitelboim (1984)	0	ΛX
3. Witten **Black Hole** (1991)	$-\frac{1}{X}$	$-2b^2 X$
4. CGHS (1992)	0	-2Λ
5. $(A)dS_2$ ground state (1994)	$-\frac{a}{X}$	BX
6. Rindler ground state (1996)	$-\frac{a}{X}$	BX^a
7. **Black Hole** attractor (2003)	0	BX^{-1}
8. Spherically reduced gravity ($N > 3$)	$-\frac{N-3}{(N-2)X}$	$-\lambda^2 X^{(N-4)/(N-2)}$
9. All above: ab-family (1997)	$-\frac{a}{X}$	BX^{a+b}
10. Liouville gravity	a	$be^{\alpha X}$
11. Reissner-Nordström (1916)	$-\frac{1}{2X}$	$-\lambda^2 + \frac{Q^2}{X}$
12. Schwarzschild-$(A)dS$	$-\frac{1}{2X}$	$-\lambda^2 - \ell X$
13. Katanaev-Volovich (1986)	α	$\beta X^2 - \Lambda$
14. BTZ/Achucarro-Ortiz (1993)	0	$\frac{Q^2}{X} - \frac{J}{4X^3} - \Lambda X$
15. KK reduced CS (2003)	0	$\frac{1}{2}X(c - X^2)$
16. KK red. conf. flat (2006)	$-\frac{1}{2}\tanh(X/2)$	$A \sinh X$
17. 2D type 0A string **Black Hole**	$-\frac{1}{X}$	$-2b^2 X + \frac{b^2 q^2}{8\pi}$
18. exact string **Black Hole** (2005)	lengthy	lengthy

Gauge theoretic formulation as Poisson-sigma model (PSM)

- 2d analogue of Chern–Simons formulation of 3d gravity: PSM
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- still need to choose gauge algebra and bilinear form

Cangemi–Jackiw version of Callan–Giddings–Harvey–Strominger

- Choose Maxwell algebra

$$[P_a,\, P_b] = \epsilon_{ab}\, Z \qquad\qquad [P_a,\, J] = \epsilon_a{}^b\, P_b$$

with bilinear form

$$\langle J,\, Z\rangle = -1 \qquad\qquad \langle P_a,\, P_b\rangle = \eta_{ab}$$

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- translate our bc's into BF-formulation

Boundary conditions in BF formulation

- Ansatz (worked nicely for Jackiw–Teitelboim; inspired by 3d)

$$\mathcal{A} = b^{-1}\big(\,\mathrm{d}+a\big)b \qquad\qquad B = b^{-1}xb$$

with group element $b = \exp(-r\,P_+)$ and

$$a = \big(\mathcal{T}(u)P_+ + P_- + \mathcal{P}(u)J\big)\;\mathrm{d}u$$
$$x = x^+(u)P_+ + x_1(u)P_- + YJ + x_0(u)Z$$

where $\delta\mathcal{T} \neq 0 \neq \delta\mathcal{P}$

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- yields metric shown before, dilaton

$$X = x_1(u)\,r + x_0(u)$$

and Maxwell field $A = r\ \mathrm{d}u$

get BMS$_2$ asymptotic symmetries!

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$$\delta A_\nu = \xi^\mu\partial_\mu A_\nu + A_\mu\partial_\nu\xi^\mu + \partial_\nu\sigma \qquad \xi(\epsilon,\,\eta) = \epsilon(u)\partial_u - \big(\epsilon'(u)r + \eta(u)\big)\partial_r$$

  provided $\eta = \sigma'$
  either η has no 0-mode or σ not single-valued (winding modes)

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- focus on case $\delta_\sigma \oint A = 0$ (no winding modes) $\Rightarrow$ warped Witt algebra

Twisted warped boundary action (see also Afshar '19)

- Variation of Euclidean BF action ($t = iu$)

$$\delta I_{\text{BF}} = \text{bulk-EOM} - \kappa \oint \mathrm{d}t \, \langle x, \, \delta a_t \rangle$$

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  Physical interpretation:

  Gravity: I_{tw} describes dynamics of edge modes

  Field theory: I_{tw} describes dynamics of collective low T modes (in large N limit)

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- defining $1/x_1 \sim \partial_t f$ and $x_0/x_1 \sim \partial_t g$ result is

$$I_{\text{tw}}[h, \, g] = \kappa \int_0^\beta \mathrm{d}\tau \Big(\mathcal{T} h'^{\,2} - g' \Big(i\mathcal{P} h' + \frac{h''}{h'} \Big) \Big)$$

with $\tau := f(t)$, $h(\tau) := -f^{-1}(\tau)$ and $\tau \sim \tau + \beta$ (prime means $\mathrm{d}/\mathrm{d}\tau$)

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twisted warped action is flat space analogue of Schwarzian action!

- Schwarzian action: group action for Virasoro coadjoint orbits
- twisted warped action: group action for twisted warped coadjoint orbits

$$[L_n, \, L_m] = (n - m) \, L_{n+m}$$
$$[L_n, \, J_m] = -m \, J_{n+m} - i\kappa (n^2 - n) \, \delta_{n+m, \, 0}$$
$$[J_n, \, J_m] = 0$$

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- Goal: cancel boundary term by adding boundary action I_{tw}
- follow JT story in BF formulation González, DG, Salzer '18
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Twisted warped action resembles effective action for complex SYK

Outline

Hamiltonian formulation

- twisted warped Hamiltonian action

$$I_{\text{tw}} = -\kappa \int_0^\beta \mathrm{d}t \left(p_i \dot{q}_i - p_1 p_2 - e^{q_1} p_3\right) \qquad i = 1, 2, 3$$

where $q_3 = \exp(i\mathcal{P}h)$ and $q_2 = g + ih\mathcal{T}/\mathcal{P}$ (rest: auxiliary fields)

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five integration constants g_0, g_1, g_2, h_0, h_1; periodicity $\tau_0 = \beta/(2\pi)$

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Thermodynamics

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- useful property for scaling limit from complex SYK

Scaling limit from complex SYK (see e.g. Gu, Kitaev, Sachdev, Tarnopolsky '19)

- Effective action governing collective low T modes of complex SYK

$$I_{\text{cSYK}}[h,\,g] = \frac{NK}{2}\int\limits_0^\beta \mathrm{d}\tau\,\Big(g' + \frac{2\pi i\mathcal{E}}{\beta}\,h'\Big)^2 - \frac{N\gamma}{4\pi^2}\int\limits_0^\beta \mathrm{d}\tau\,\Big\{\tan\Big(\frac{\pi}{\beta}h\Big);\,\tau\Big\}$$

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$\{f;\,\tau\} := f'''/f' - \frac{3}{2}(f''/f')^2$ Schwarzian derivative
N (large) number of complex fermions
$N\gamma$ specific heat at fixed charge
K zero temperature compressibility
$\mathcal{E}$ spectral asymmetry parameter
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- inserting these limits into $I_{\text{cSYK}}[h,\,g]$ yields twisted warped action

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with $\kappa^2\sim N^a\gamma K$ kept finite

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- then take limit $\hat{K} \to 0$, $c \to \infty$ keeping fixed $\kappa = \sqrt{-\frac{c\hat{K}}{24}}$

Conclusions

For more details see Afshar, González, DG, Vassilevich 1911.05739

- CGHS a la Cangemi–Jackiw bulk model for flat space holography

$$I[X,\, Y,\, g_{\mu\nu},\, A_\mu] = \frac{\kappa}{2}\int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\big(XR - 2Y + 2Y\epsilon^{\mu\nu}\partial_\mu A_\nu\big)$$

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- could be useful toy model for flat space holography

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