Near horizon dynamics of three dimensional black holes

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work with Wout Merbis, 1906.10694

Outline

Overture

Hamiltonian reduction

Near horizon boundary conditions

Near horizon Hamiltonian

KdV deformation

Conclusions

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Main message

- Near horizon boundary action for 3-dimensional black holes

$$S_{\text{NH}}[\Phi^+,\,\Phi^-] = \int \mathrm{d}t\,\mathrm{d}\sigma\big(\Pi^+\dot{\Phi}^+ + \Pi^-\dot{\Phi}^- - \mathcal{H}_{\text{NH}}(\Phi^+,\,\Phi^-)\big)$$

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  to reduce clutter: drop $\pm$ decorations in rest of talk

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Manifestation of "softness" of near horizon excitations

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Purpose of talk: explain and derive results summarized above

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Einstein gravity in three dimensions as Chern–Simons theory

Einstein gravity in three dimensions useful toy model:

$$I_{\textrm{EH3}}[g] = \frac{1}{16\pi G} \int_{\mathcal{M}} \mathrm{d}^3x\sqrt{-g}\,\Big(R + \frac{2}{\ell^2}\Big) + \hat{I}_{\partial\mathcal{M}}$$

- no local physical degrees of freedom $\Rightarrow$ simple!

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- Brown–Henneaux asymptotic symmetries: 2 Virasoros ($\mathrm{AdS}_3/\mathrm{CFT}_2$)

$$[L_n,\, L_m] = (n-m)\,L_{n+m} + \frac{c}{12}\,(n^3 - n)\,\delta_{n+m,\,0} \qquad\qquad c = \frac{3\ell}{2G}$$

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- Gauge theoretic formulation as Chern–Simons theory $[k = \ell/(4G)]$

$$I_{\text{CS}}[A] = \frac{k}{4\pi} \int_{\mathcal{M}} \mathrm{Tr}\,\big(A\wedge\mathrm{d}A + \tfrac{2}{3}\,A\wedge A\wedge A\big) + I_{\partial\mathcal{M}}$$

$SO(2,2)$ connection A usually split into two $\mathrm{SL}(2,\mathbb{R})$ connections; drop all $\pm$ decorations & work with single sector

Hamiltonian analysis of Chern–Simons theory

- Hamiltonian action of Chern–Simons theory on cylinder
  adapted coordinates: r: radius, $\sigma \sim \sigma + 2\pi$: angle, t: time

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- for formulating boundary conditions related convenient Ansatz:

$$A(t,\,\sigma,\,r) = b^{-1}(r)\left(\mathrm{d} + a(t,\,\sigma)\right)b(r) \qquad a = a_t\,\mathrm{d}t + a_\sigma\,\mathrm{d}\sigma$$

with vanishing variation $\delta b = 0$ and allowed variations $\delta a \neq 0$

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- Gauss decomposition $G = e^{XL_+}e^{\Phi L_0}e^{YL_-}$ yields boundary action

$$I_{\mathrm{CS}}[\Phi,\,X,\,Y] = -\frac{k}{4\pi}\int_{\partial\mathcal{M}}\mathrm{d}t\,\mathrm{d}\sigma\left(\tfrac{1}{2}\,\dot{\Phi}\Phi' - 2e^{\Phi}X'\dot{Y}\right) + I_{\partial\mathcal{M}}$$

used standard basis for SL$(2,\,\mathbb{R})$: $[L_n,\,L_m] = (n-m)\,L_{n+m}$ for $n, m = 0, \pm 1$

also used Polyakov–Wiegmann identity to show b-independence of action and chose $b = 1$ at $\partial\mathcal{M}$

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$r \to 0$: Rindler horizon

κ: surface gravity

$\mathcal{J}^+(t,\,\sigma) + \mathcal{J}^-(t,\,\sigma)$: metric transversal to horizon

$\dots$: terms of higher order in r

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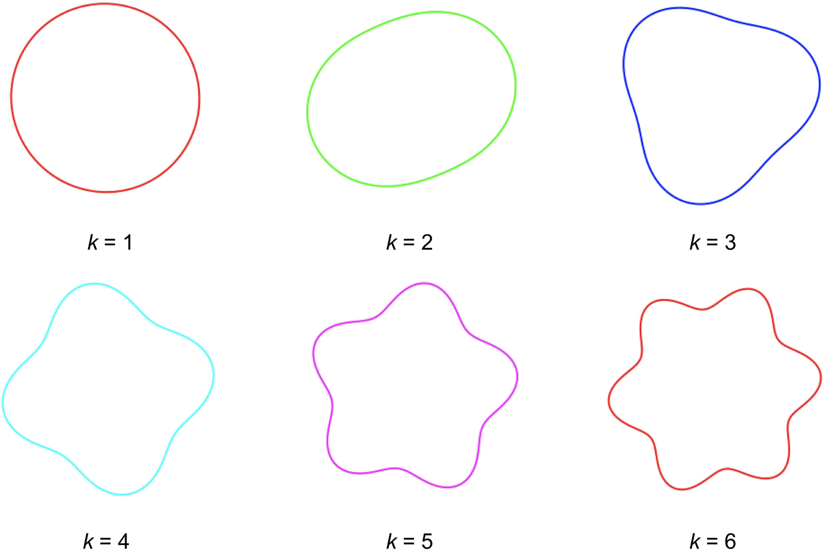
- simplifying assumption: constant surface gravity $\Rightarrow$ "holographic Ward identities" imply time-independence of state-dependent fct's

$$\dot{\mathcal{J}}^\pm = 0$$

Black holes can be deformed into black flowers Afshar et al. 16

Horizon can get excited by area preserving shear-deformations



Near horizon Chern–Simons connection

- same boundary conditions in Chern–Simons language:

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- like Brown–Henneaux: 2 towers of conserved boundary charges $\mathcal{J}^\pm$

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replaced Poisson brackets by commutators as usual, $i\{\,,\,\} \to [\,,\,]$; note: algebra isomorphic to Heisenberg algebras

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- near-horizon (Cardy-like) entropy formula: $S = 2\pi\left(J_0^+ + J_0^-\right)$

Unique features of near horizon boundary conditions

1. All states allowed by bc's have same temperature

   By contrast: asymptotically AdS or flat space bc's allow for black hole states at different masses and hence different temperatures

Unique features of near horizon boundary conditions

1. All states allowed by bc's have same temperature
2. All states allowed by bc's are regular

   (in particular, they have no conical singularities at the horizon in the Euclidean formulation)

   By contrast: for given temperature not all states in theories with asymptotically AdS or flat space bc's are free from conical singularities; usually a unique black hole state is picked

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3. There is a non-trivial reducibility parameter (= Killing vector)

   By contrast: for any other known (non-trivial) bc's there is no vector field that is Killing for all geometries allowed by bc's

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4. Technical feature: in Chern–Simons formulation of 3d gravity simple expressions in diagonal gauge

$$A^{\pm} = b^{\mp 1}\big(\,\mathrm{d} + a^{\pm}\big)\, b^{\pm 1}$$
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5. Leads to soft Heisenberg hair (see next slide!)

Soft Heisenberg hair

- Black flower excitations = hair of black holes
  Algebraically, excitations from descendants

$$|\text{black flower}\rangle \sim \prod_{n_i^\pm > 0} J^+_{-n_i^+} J^-_{-n_i^-} |\text{black hole}\rangle$$

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- Near horizon Hamiltonian = boundary charge associated with unit time-translations*

$$H = Q[\partial_t] = \kappa \left(J_0^+ + J_0^- \right)$$

  commutes with all generators $J_n^\pm$

* units defined by specifying κ

Soft Heisenberg hair

- Black flower excitations = hair of black holes
  Algebraically, excitations from descendants
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Near horizon boundary action

- recall general boundary action

$$I_{\mathrm{CS}}[\Phi,\, X,\, Y] = -\frac{k}{4\pi}\int_{\partial\mathcal{M}}\mathrm{d}t\,\mathrm{d}\sigma\,\big(\tfrac{1}{2}\,\dot{\Phi}\Phi' - 2e^{\Phi}X'\dot{Y}\big) + I_{\partial\mathcal{M}}$$

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- still need to discuss $I_{\partial\mathcal{M}}$, since it encodes the boundary Hamiltonian!

Simplest choice of boundary term

- well-defined variational principle if

$$\delta I_{\partial\mathcal{M}} = \frac{k}{2\pi}\int_{\partial\mathcal{M}} \mathrm{d}t\,\mathrm{d}\sigma\,\mathrm{Tr}\left(a_t\,\delta a_\sigma\right)$$

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- simplest choice (near horizon boundary conditions for a_t):

$$\delta\zeta = 0$$

make this choice to obtain near horizon Hamiltonian!

Near horizon Hamiltonian

- solving integrability condition

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for $\mathcal{H}$ yields boundary Hamiltonian density

$$\mathcal{H}_{\mathrm{NH}} = \zeta\,\mathcal{J} = \zeta\,\Phi'$$

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- near horizon Hamiltonian given by zero mode generator

$$H_{\text{NH}} = \frac{k}{2\pi}\oint \mathrm{d}\sigma\,\mathcal{H}_{\text{NH}} = \frac{k}{2}\,\zeta\,J_0$$

recovers result expected from near horizon symmetry analysis

Mode decomposition

- near horizon equations of motion

$$\dot{\Phi}' = 0$$

solved by

$$\Phi(t,\,\sigma)\big|_{\textrm{EOM}} = \Phi_0(t) + J_0\,\sigma + \sum_{n\neq 0}\frac{J_n}{in}\,e^{in\sigma}$$

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- off-shell mode-decomposition in near horizon boundary action:
$$I_{\text{NH}}[\Phi_0,\,J_n] = \frac{k}{2}\int \mathrm{d}t\,\Big(-\frac{1}{2}\,\dot{\Phi}_0 J_0 + \sum_{n>0}\frac{i}{n}\,\dot{J}_n J_{-n} - \zeta J_0\Big)$$

Floreanini–Jackiw symplectic structure

reminder:

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$$i\{J_n,\, J_m\} = \frac{2}{k}\, n\, \delta_{n+m,\,0}$$

plus an extra relation

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- Hamiltonian $H_{\rm NH} \sim J_0$ commutes with all canonical variables $\Rightarrow$ expected softness property recovered!

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Other choices for boundary action

- would like to lift soft hair degeneracy
  - reason 1: because it allows to recover Brown–Henneaux story
  - reason 2: because lifting soft hair degeneracy may help to address the fantasy that soft hair excitations could correspond to black hole microstates (at least in semi-classical limit and far away from extremality)
  - reason 3: because we can and it is fun

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- start by recovering Brown–Henneaux boundary conditions and the Schwarzian action

Recovering Brown–Henneaux and the Schwarzian action

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- boundary action analogous, but Hamiltonian density changes

$$\mathcal{H}_{\text{BH}} = -\frac{k\mu}{8\pi}\,\big((\Phi')^2 + 2\Phi''\big)$$

no longer have soft hair, since $\mathcal{H}_{\text{BH}}$ is not a boundary term and the associated Hamiltonian does not commute with all generators of the asymptotic symmetries!

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- expressing action instead in terms of $X' = e^{-\Phi}$ yields

$$I_{\text{BH}}[X] = \frac{k}{4\pi}\int \mathrm{d}t\,\mathrm{d}\sigma\left(\frac{\dot{X}''}{X'} - \frac{3}{2}\,\frac{X''\dot{X}'}{X'^2} - \mu\{X,\,\sigma\}_{\text{Sch}}\right)$$

  = geometric action of Virasoro group on coadjoint orbit

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$$H_N \sim \oint \mathrm{d}\sigma \, R_{N+1}(\mathcal{J})$$

where R_{N+1} is a Gelfand–Dikii differential polynomial:

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- for general N Hamiltonian density reads

$$\mathcal{H}_N \sim \tfrac{1}{N+1}\,\mathcal{J}^{N+1} + \sum_{i=1}^{N-1} h_{i,\,N}\mathcal{J}^{N-i-1}(\partial_\sigma^i \mathcal{J})^2 + \mathcal{H}_N^{\mathrm{nl}} \qquad \mathcal{J} = \Phi'$$

non-linear term in derivatives $\mathcal{H}_N^{\mathrm{nl}}$ exists only for $N \geq 5$; the $h_{i,\,N}$ are computable rational coefficients

Scaling properties

- for $N > 1$ field equations have anisotropic scale invariance

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- note that we rescaled by $1/\varepsilon$ to have non-trivial limit $\varepsilon \to 0^+$!

KdV scaling limit for near horizon Hamiltonian

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- achieved goal: Hamiltonian no longer commutes with everything!

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Outline

Overture

Hamiltonian reduction

Near horizon boundary conditions

Near horizon Hamiltonian

KdV deformation

Conclusions

Relations to fluff proposal? (Afshar, Grumiller, Sheikh-Jabbari, Yavartanoo '17)

- conjectured semi-classical set of BTZ microstates

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$$\sum n_i^\pm = c\,\Delta^\pm \qquad\qquad \Delta^\pm = \frac{1}{2}\big(\ell M_{\mathrm{BTZ}} \pm J_{\mathrm{BTZ}}\big) = \frac{c}{24}\,(J_0^\pm)^2$$

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Fluff proposal intriguing, but not (yet) derived from first principles

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Carrollian limit

- Floreanini–Jackiw action

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- this is the Carrollian limit (compare with Donnay, Marteau and Penna)

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Ultrarelativistic strings

- other consideration: start with bosonic string theory

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Confirms suspicion that nearly tensionless strings key in near horizon description of generic black holes

Thanks for your attention!